

Structure Function Working Group **— Theory Review —**

James Stirling
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Convenors:

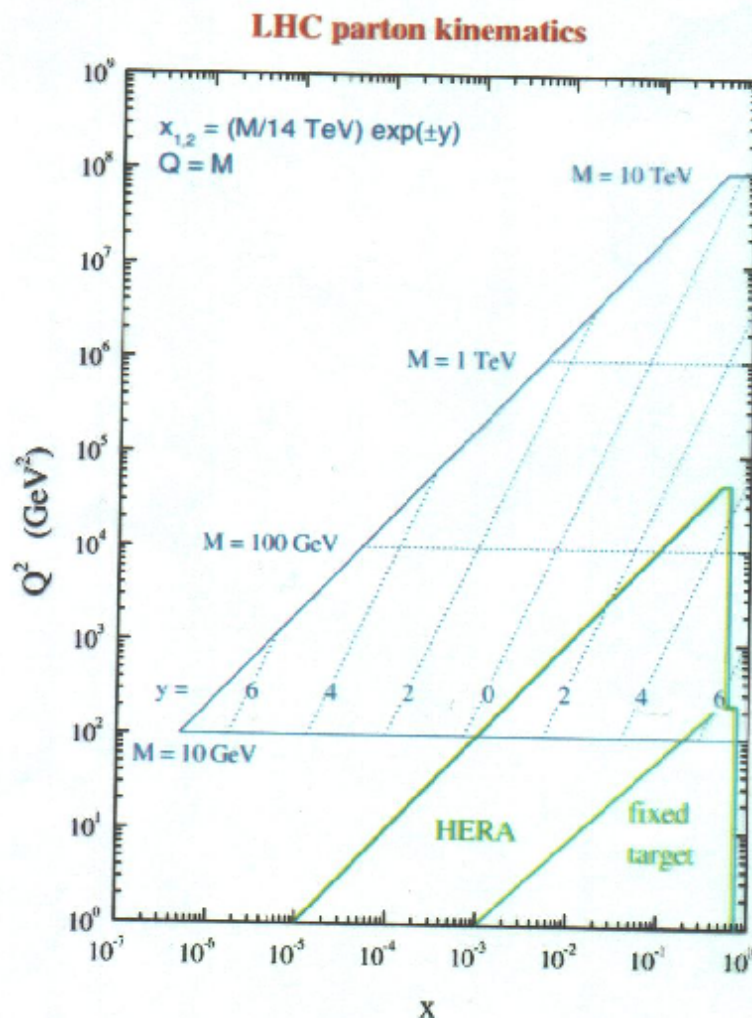
Ken Long, Richard Nisius, James Stirling

Alan Martin – the working group will reveal

- new data
- new (global) descriptions
- how to estimate uncertainties
- new calculations, new ideas

higher orders, higher twists

in order to match the precision of present and future DIS data, we need to go beyond **NLO**, **leading twist**, and **leading log resummations** (in $\ln x$, $\ln(1-x)$)

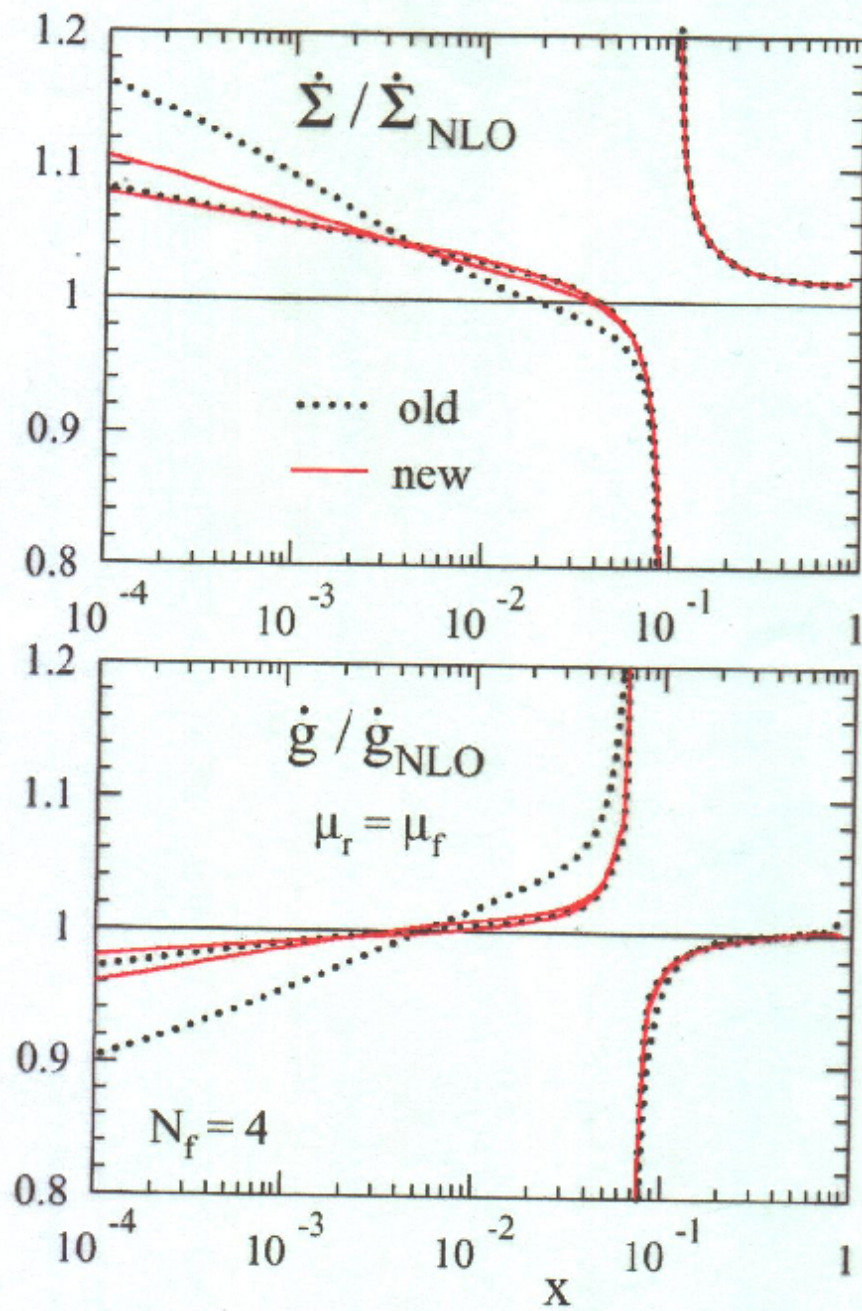


NNLO – the top priority

DGLAP: $P(x, \alpha_s) = P^{(0)} + \alpha_s P^{(1)}(x) + \alpha_s^2 P^{(2)}(x) + \dots$

Coeff. Function: $\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_s \hat{\sigma}^{(1)} + \alpha_s^2 \hat{\sigma}^{(2)} + \dots$

- **Vogt** update on NNLO DGLAP
 - improved approximations for $P^{(2)}(x)$ by van Neerven, Vogt using recent $N = 10, 12$ moment calculations by Retey, Vermaseren (combined with sum rule and symmetry constraints and known leading behaviour for $x \rightarrow 0, 1$) ; certainly sufficient for $x > 10^{-2}$ phenomenology, probably OK for $x > 10^{-4}$
 - full NNLO splitting functions expected by end of year!
- but note: $\hat{\sigma}^{(2)}$ still not calculated for many of the cross sections used in global fits, e.g. $d\sigma/dE_T^{jet}$, $d\sigma/dy_W$, $d^2\sigma^{DY}/dMdy$, ...

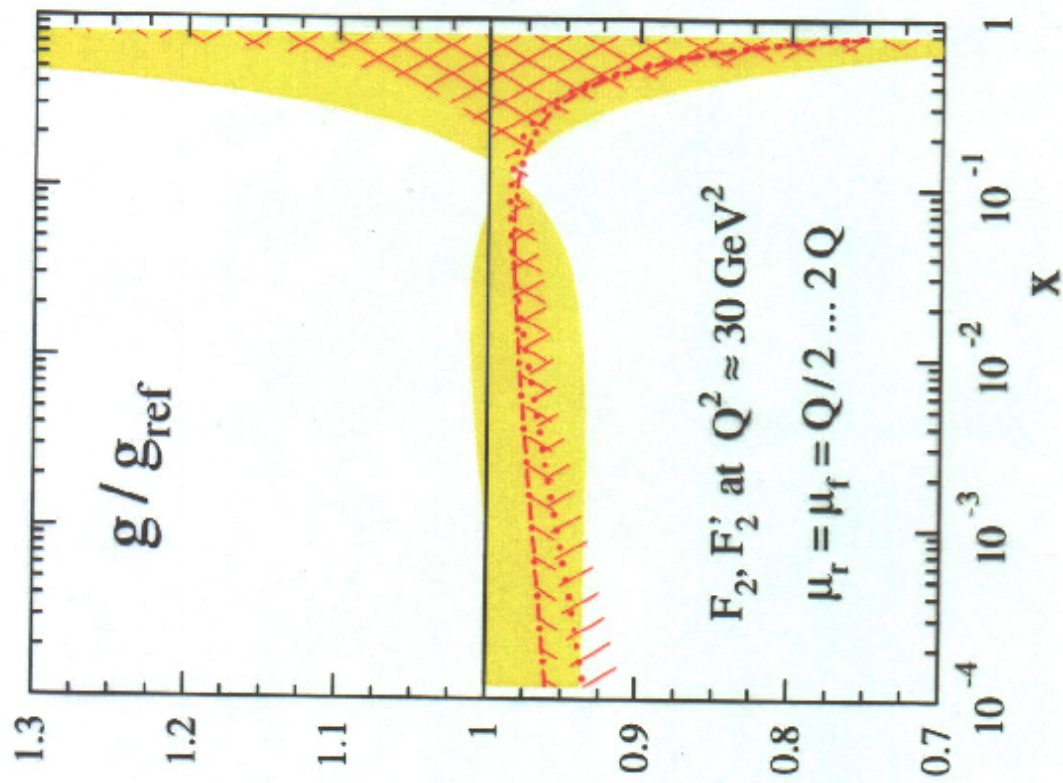
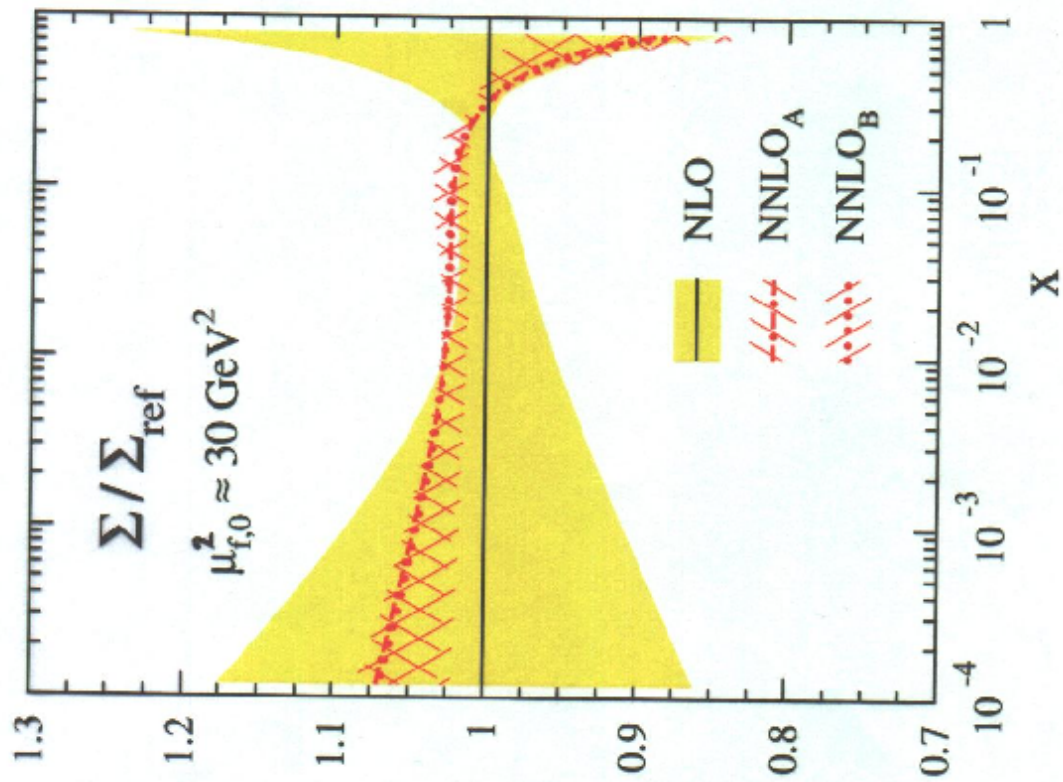


$$\dot{q} \equiv d \ln q / d \ln \mu_f^2$$

$$\alpha_s(\mu_r^2 = \mu_f^2) = 0.2 \leftrightarrow \mu_f^2 \approx 30 \text{ GeV}^2$$

$$x\Sigma = 0.6 x^{-0.3} (1-x)^{3.5} (1+5x^{0.8})$$

$$xg = 1.0 x^{-0.37} (1-x)^5$$



Appendix: full 3-loop calculation

S. Moch, J.A.M. Vermaseren, M. Zhou: in progress

Approach

**Operator product expansion, analytic in N -space
(specific for structure functions in DIS)**

Task

**Evaluation of about 11000 three-loop diagrams
(known from the fixed- N calculation)**

- **development of algorithms for the integrals**
- **effective implementation**
- **checks using integer- N results at all stages**

Status

- **development close to finished**
- **implementation well under way**
- **production started**

Expectation

Calculation finished before the end of this year

fully *quantitative* predictions for higher-twist contributions to DIS structure functions remain elusive

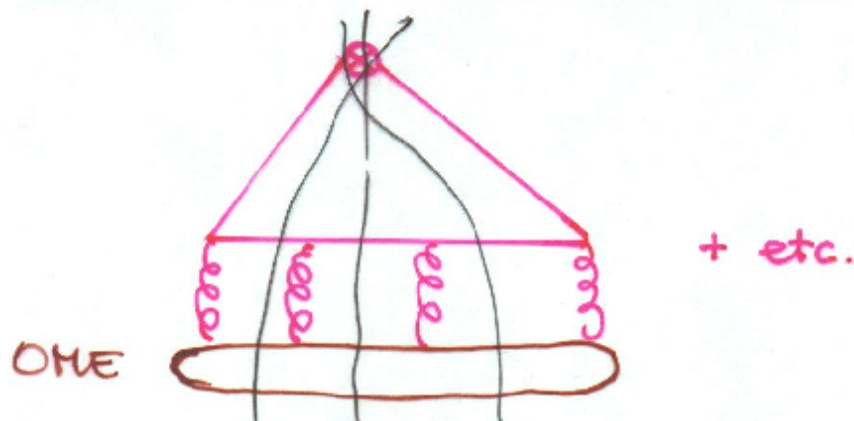
- **Blümlein** twist-4 gluon recombination corrections
 - modelling of $4g$ distribution amplitude; estimate of $C_{4g,2q}$ coefficient function $\Rightarrow$ **screening** and **antiscreening** contributions (cf. GLR)
 - numerical predictions for $\partial F_2 / \partial \log Q^2 \Rightarrow$ depends on R ; small effect in **HERA** region
- **Magnea** resummation and power corrections using DR

$$F(Q^2) \propto \int_0^{Q^2} \frac{dk^2}{k^2} (k^2)^a \alpha_s(k^2)$$

ill-defined because of **Landau pole**

- regulate using $d = 4 - 2\epsilon > 4$ dimensions
- pole moves off real axis $\Rightarrow$ analytic resummed expressions for form factors
- $\epsilon \rightarrow 0$ limit at fixed $\alpha_s(Q^2)$ gives usual power corrections $\sim \Lambda^2/Q^2$

2. 4-Gluon Operator Matrix Elements



$$G_n(x_1, \dots, x_n) = A_n x_1^{\alpha_1} \dots x_n^{\alpha_n} (1 - x_1 - \dots - x_n)^{\beta}$$

assume: $\forall \alpha_i \equiv d$

$$\hat{G}_2(x_1, x_2; x'_1, x'_2) = G_2(x_1, x_2; x'_1, x'_2) \delta(x_1 + x_2 - x'_1 - x'_2)$$

$$\hat{G}_2(x_1, \dots, x_4) = G_1(x_1) G_1(x_2) \chi(x_1, x_2, x_3)$$

↑ correlator.

assume: $\chi \equiv 1$.

$$G_2(x_1) = \frac{Q_0^2}{Q^2} G_1^2(x_1).$$

or:

$$G_2(x_1) = \frac{9}{8} Q_0^2 x_1^2 G_1^2(x_1)$$

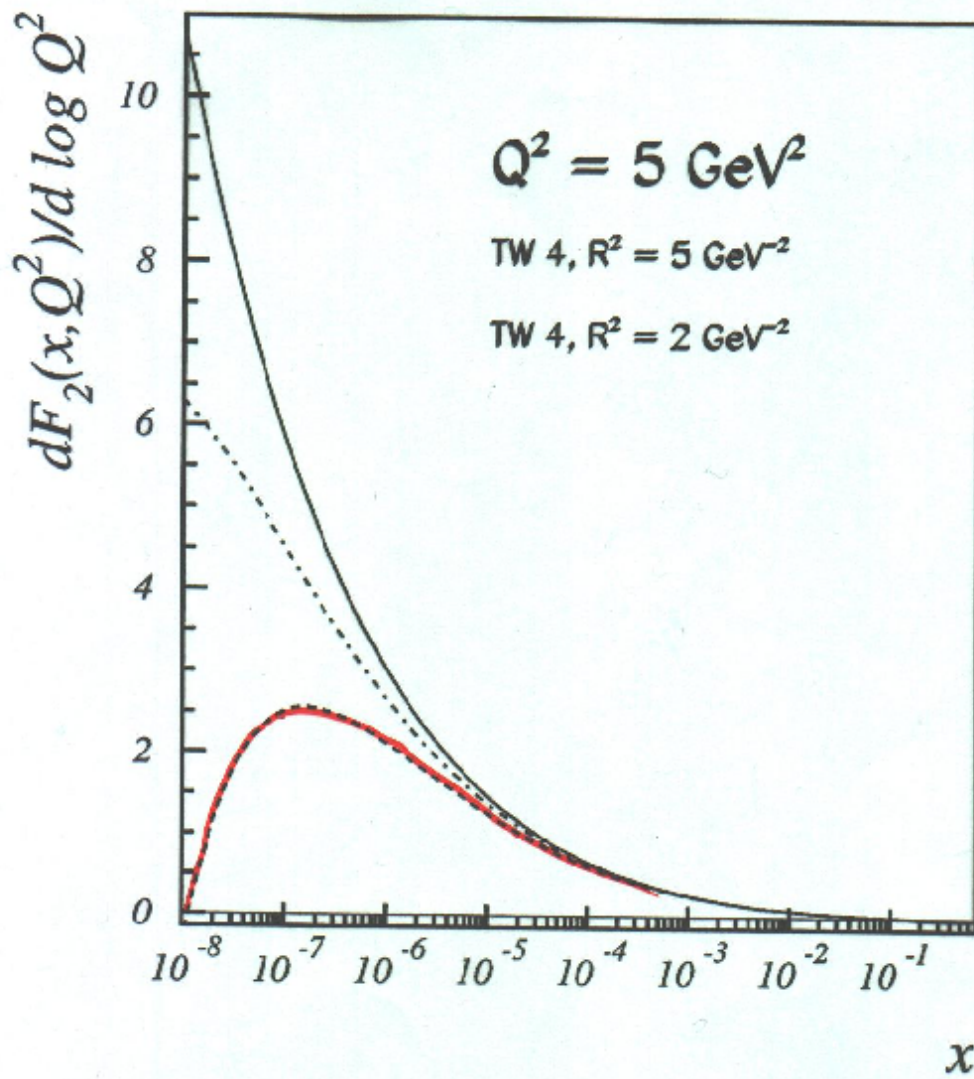


Figure 4a: The slope $dF_2(x, Q^2)/d \log Q^2$ at $Q^2 = 5 \text{ GeV}^2$. Full line: leading order twist-2 contributions (parameterization Ref. [24]). Dash-dotted line: Eq. (7) with twist-4 mass scale $R^2 = 5 \text{ GeV}^{-2}$, and dashed line: $R^2 = 2 \text{ GeV}^{-2}$.

- **Ermolaev** small- x asymptotics of NS structure functions

$$F_1^{NS}, g_1^{NS} \sim \left(\frac{1}{x} \sqrt{\frac{Q^2}{\mu^2}} \right)^{a_{\pm}}$$

$$a_+ = \sqrt{2\alpha_s C_F/\pi}, \quad a_- = \sqrt{1 + 1/(2N^2)} a_+$$

... but what to take for α_s ?

- incorporate running α_s in ladder
- solve using IR evolution equation
- $a_{\pm} \rightarrow \omega_{\pm}(\mu) = 0.37/0.4$ at 1 GeV
- good agreement with xF_3 data

pdf fits and uncertainties

progress in two areas:

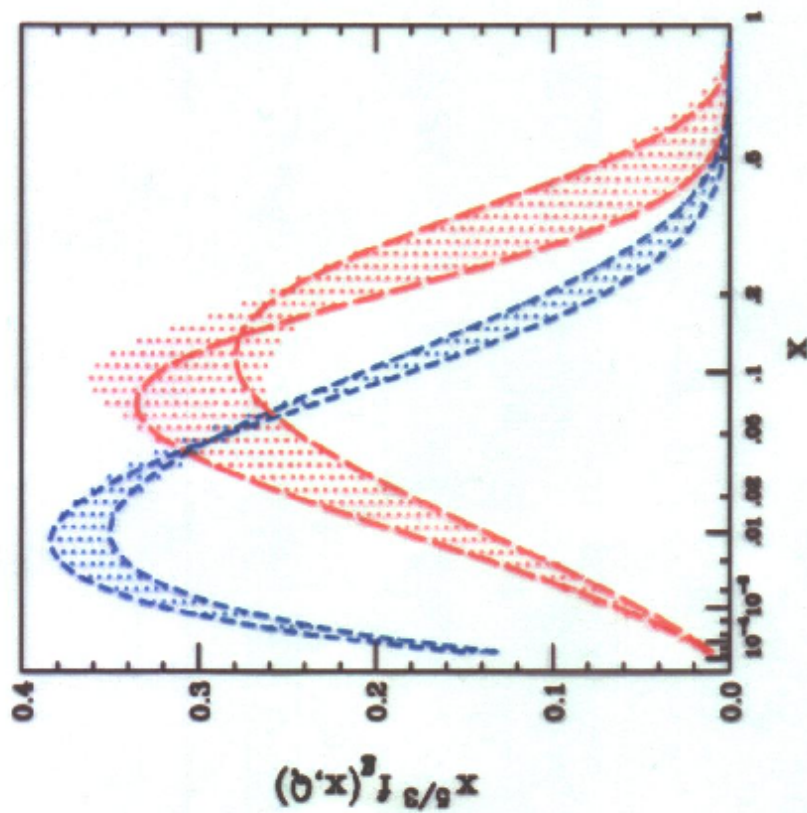
1. further fine-tuning of global 'best NLO-DGLAP fit' using new data (MRST)
2. investigation of techniques for defining and implementing 'errors on pdfs' or, arguably more relevant, 'errors on physical quantities due on pdfs' (H1, ZEUS, Botje, Barone et al., Giele et al., Alekhin et al., CTEQ, MRST, . . .)

- **Stump** uncertainties in (CTEQ) global pdf fits

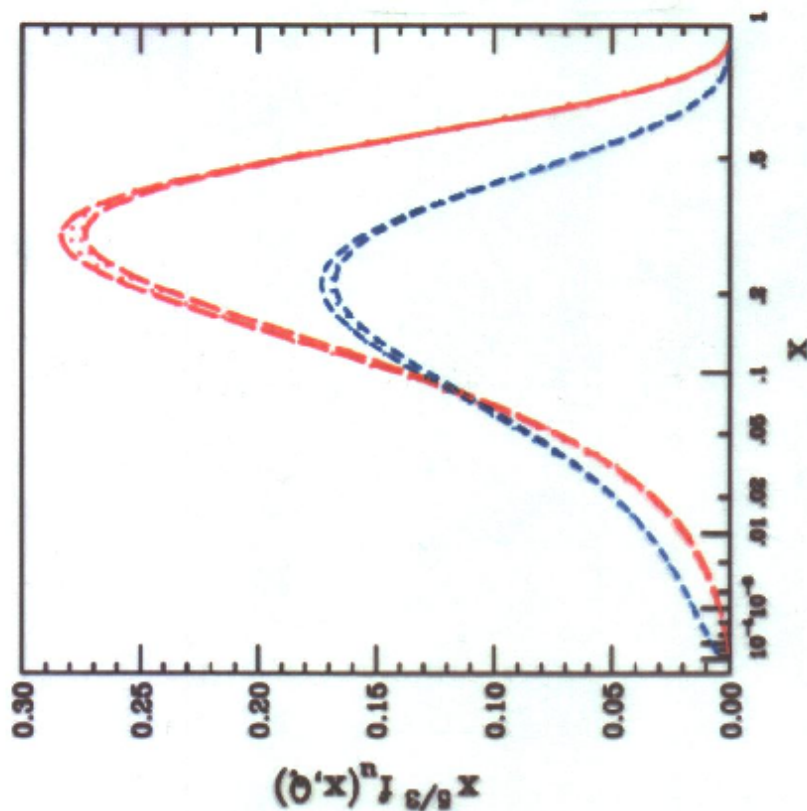
- two methods for quantifying pdf uncertainties

1. **Lagrange Multiplier** (constrained fitting): focus on uncertainty on physical quantity σ_X due to input pdfs by finding 'best global fits' for a given constrained value of σ_X
2. **Hessian Matrix** (traditional error analysis):

$$\chi_{\text{global}}^2 = \chi_0^2 + \sum_{i,j=1}^d (a_i - a_i^0) H_{ij} (a_j - a_j^0) + \dots$$



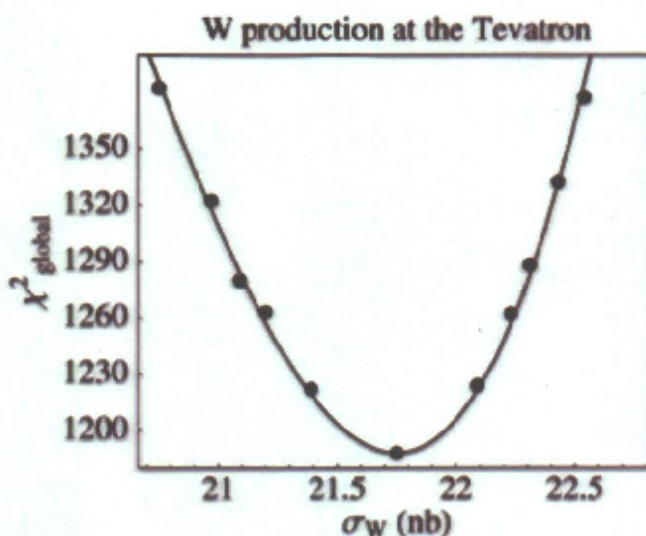
Extreme gluon distributions,
 $Q = 2$ and 100 GeV
 Curves: extremes at some x
 Shaded region: envelope



Extreme u -quark distributions,
 $Q = 2$ and 100 GeV
 Curves: extremes at some x
 Shaded region: envelope

Examine χ^2 's of individual experiments for "alternate hypotheses". Three step analysis

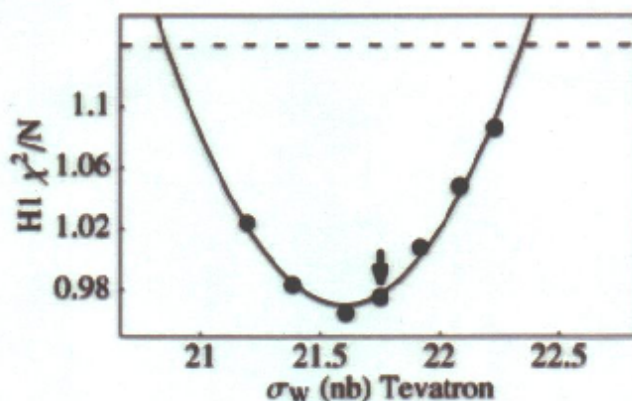
(1) Use Lagrange multiplier method to generate best fits with various values of X .



(2) Examine χ^2 for individual experiments for the alternate fits.

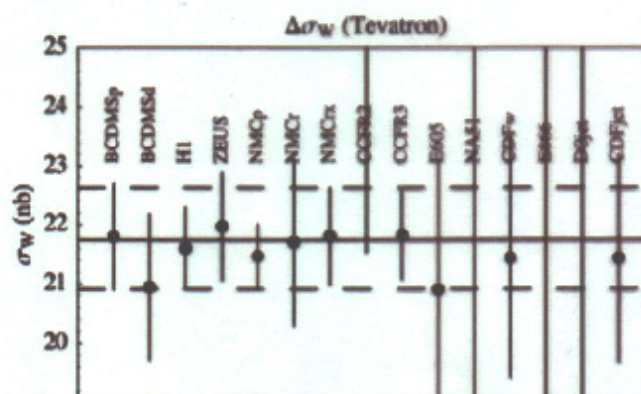
Example shows χ^2 for H1 data versus σ_W (Tevatron), including correlated systematic errors.

The 90% confidence level criterion $\Rightarrow$ range of X .



(3) Compare the individual ranges.

Use the "intersection" to estimate the global uncertainty on X .



We find $10 \leq T \leq 15$.

for $a_1, \dots, a_d$ parameters defining the pdfs; use to define eigenvectors, eigenvalues in parameter space, $\pm 1\sigma$ parameter variations, etc.

- difficulty: what constitutes a reasonable tolerance, i.e. deviation from global minimum?

- **Collins** the meaning of $\Delta\chi^2$ in global fits – a critical assessment of 'good' and 'bad' fits

- rather than look at $\Delta\chi_{\text{tot}}^2$, consider quality of fit to individual data subsets
- in parameter space, find sets that are 'best fits' to individual data sets — how far away are these from the overall best fit?
- reject, as incompatible, data sets whose χ^2 dramatically improves when moving away from $\chi_{\text{tot}}^2(\text{min})$ in parameter space

→ results in a procedure for identifying incompatible data sets for rejection from global fit (also **Giele et al.**)

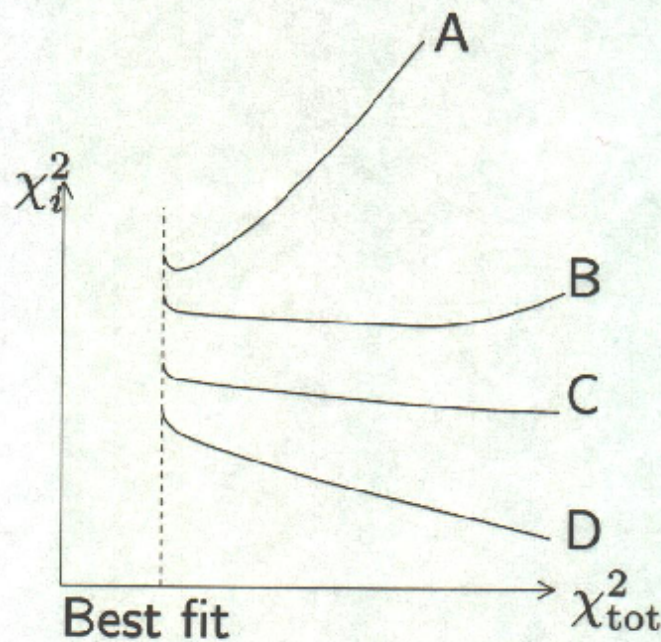
Warning! rejected data sets may contain important physics (mass effects, higher-order corrections, new physics, ...)

How to visualize variation of $\chi^2(\text{expt})$

For each χ_{tot}^2 , find parameters minimizing χ_i^2 .
Lagrange multiplier λ :

$$0 = \frac{\partial}{\partial \text{params}} [\chi_{\text{tot}}^2(\mathbf{p}) + (\lambda - 1)\chi_i^2(\mathbf{p})].$$

Possible results:

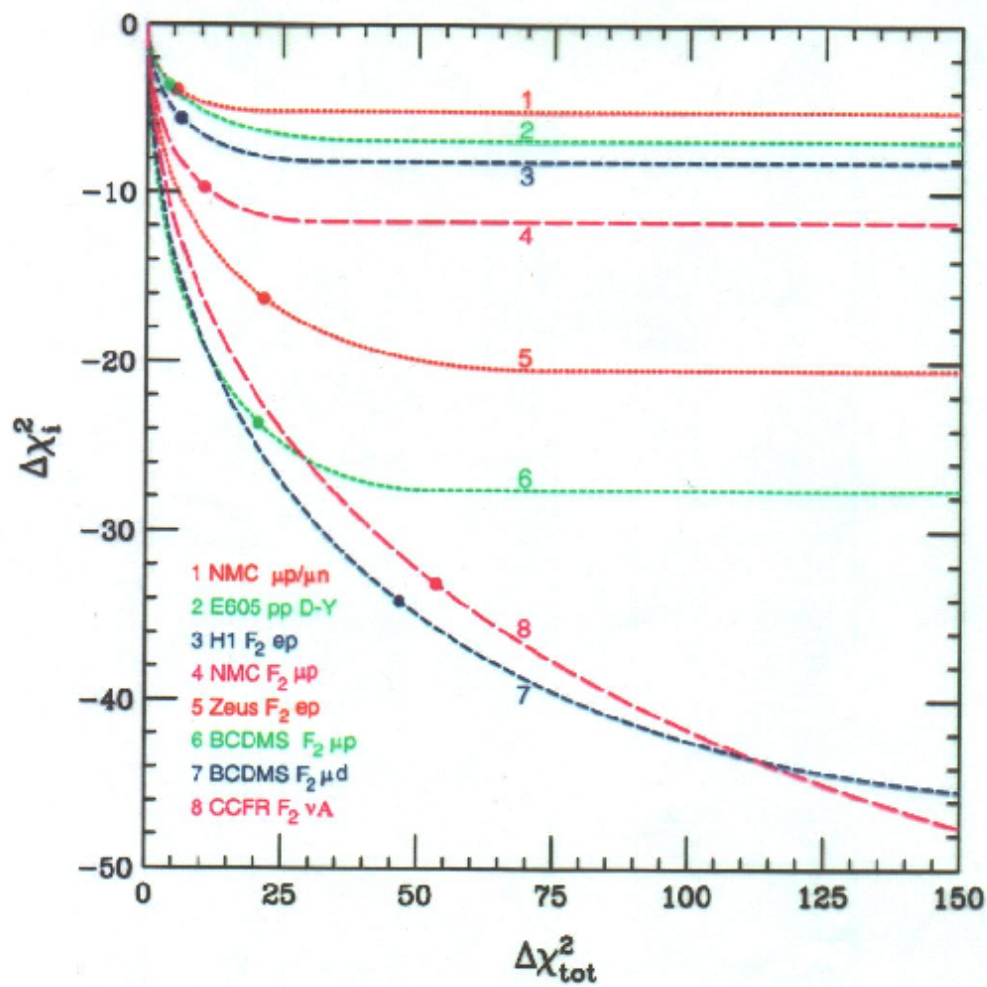


A Agrees with global fit and fixes all parameters.

B, C Agree with global fit; do not fix all parameters.

D Disagrees with global fit.

Apply to CTEQ5



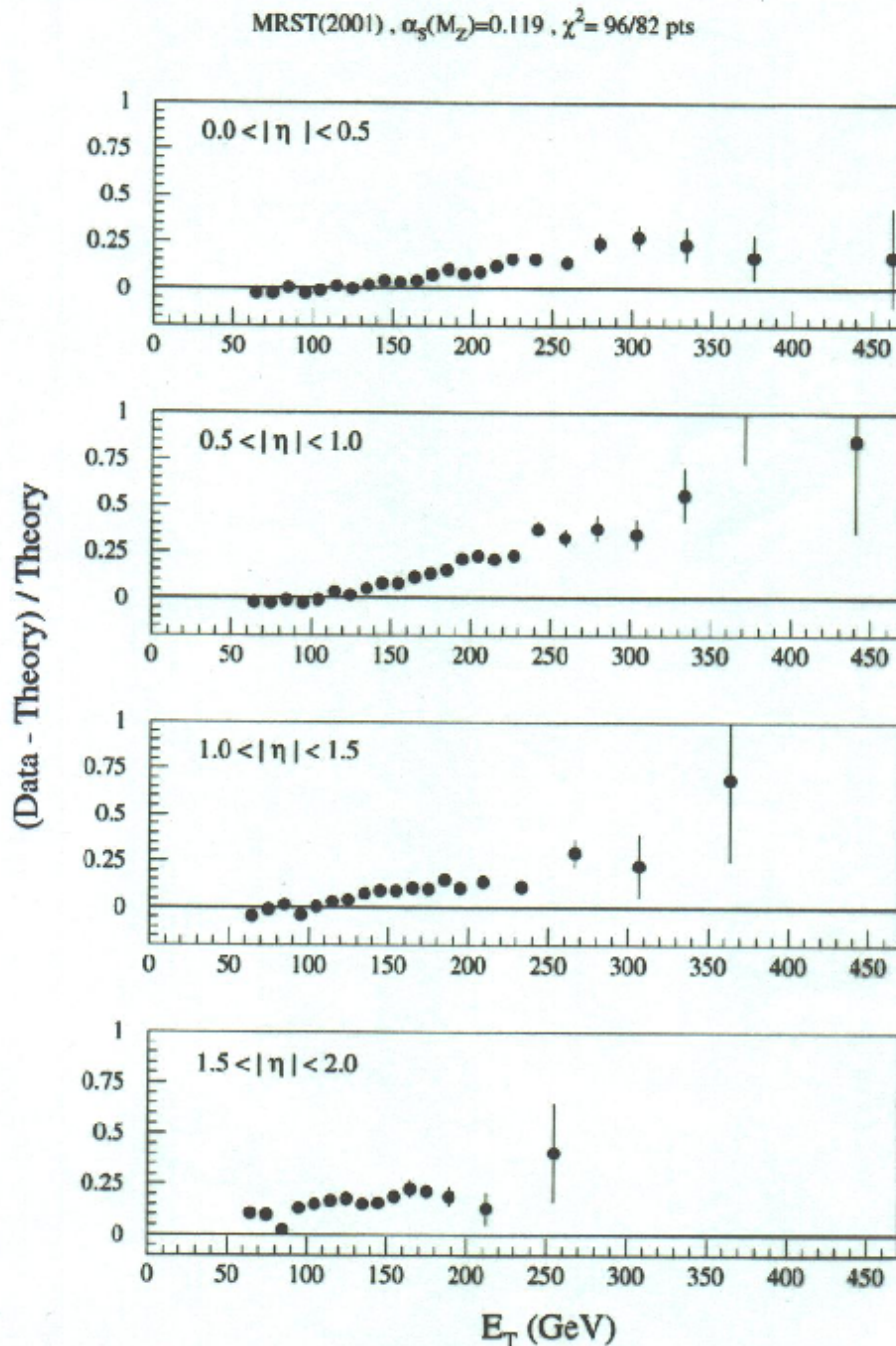
Expt	N	χ^2	χ^2/N	$\frac{\chi^2 - N}{\sqrt{2N}}$
1. NMC D/H	123	111	0.90	-0.8
2. E605	119	92	0.77	-1.8
3. H1 F_2 '96	172	108	0.63	-3.4
4. NMC H	104	108	1.04	0.3
5. ZEUS F_2 '94	186	249	1.34	3.3
6. BCDMS H	168	146	0.87	-1.2
7. BCDMS D	156	222	1.42	3.7
8. CCFR F_2	87	74	0.85	-1.0

- Thorne MRST global fit update
 - standard NLO updated using new DIS, DY, Tevatron jet data → MRST2001
 - Tevatron jet data incorporated (correlated systematic errors important); requires sizeable glue at large x , HERA F_2 fits suffer slightly as a result
 - uncertainties? – focus on $\delta\sigma_{\text{obs}}$ rather than δf
 - how exactly is σ_{obs} constrained? which parts of the fit fail when trying to force higher/lower σ_{obs} ?
 - estimate $\delta\sigma_{\text{pdf}}(W) \simeq \pm 2\%$ at Tevatron, LHC

Good fit to jet data. Normalization about correct.

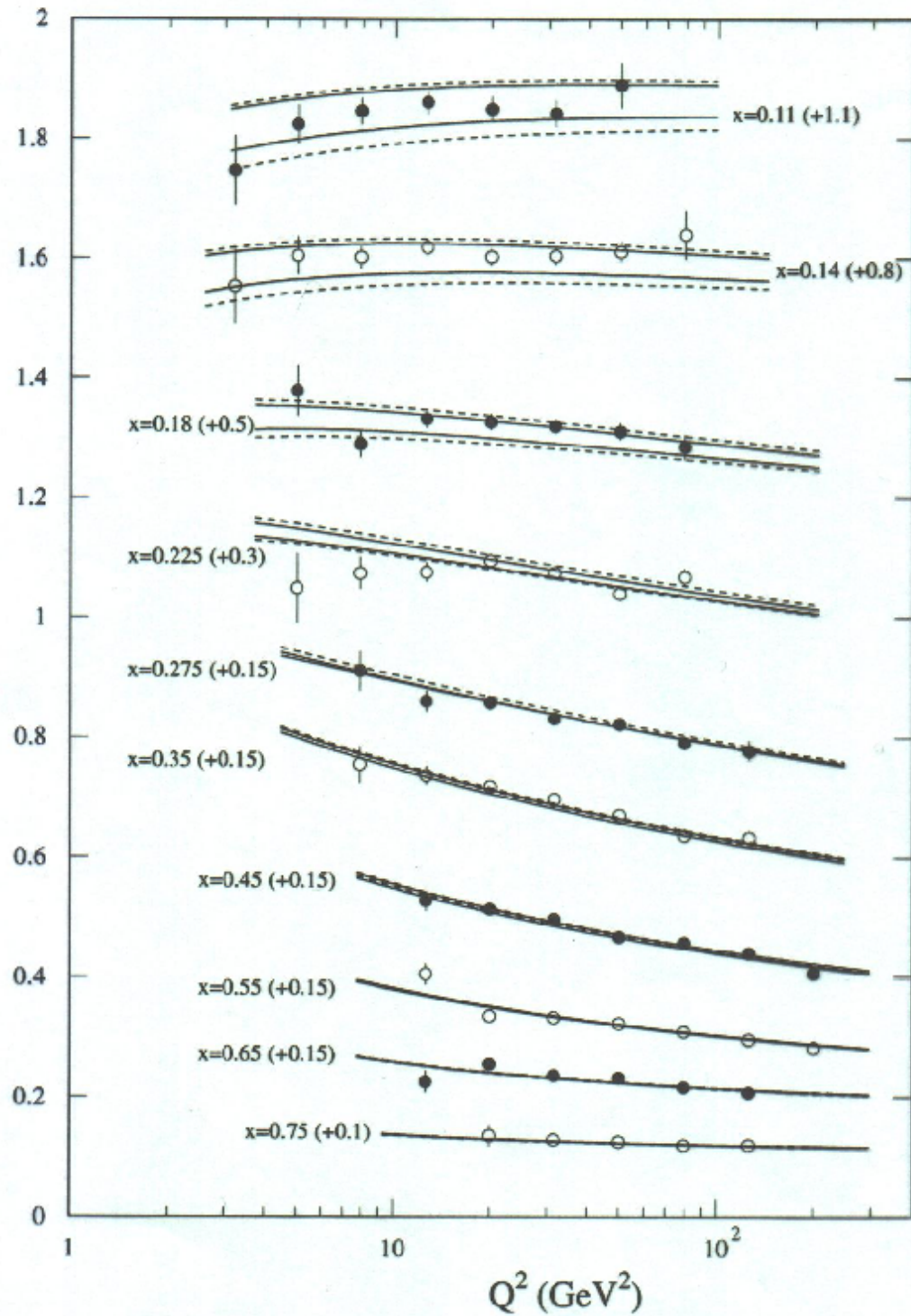
Jet data prefer $\alpha_s(M_Z^2) = 0.117$. Compromise between this and other data.

Fit to new D0 jet data



Deterioration in fit for increase and decrease in $\sigma_W(\text{TeV})$

CCFR XF_3 data , $W_{\text{TeV}}(1.021)$, $W_{\text{TeV}}(0.973)$, $W_{\text{TeV}}(1.034)$, $W_{\text{TeV}}(0.956)$ fit



phenomenology

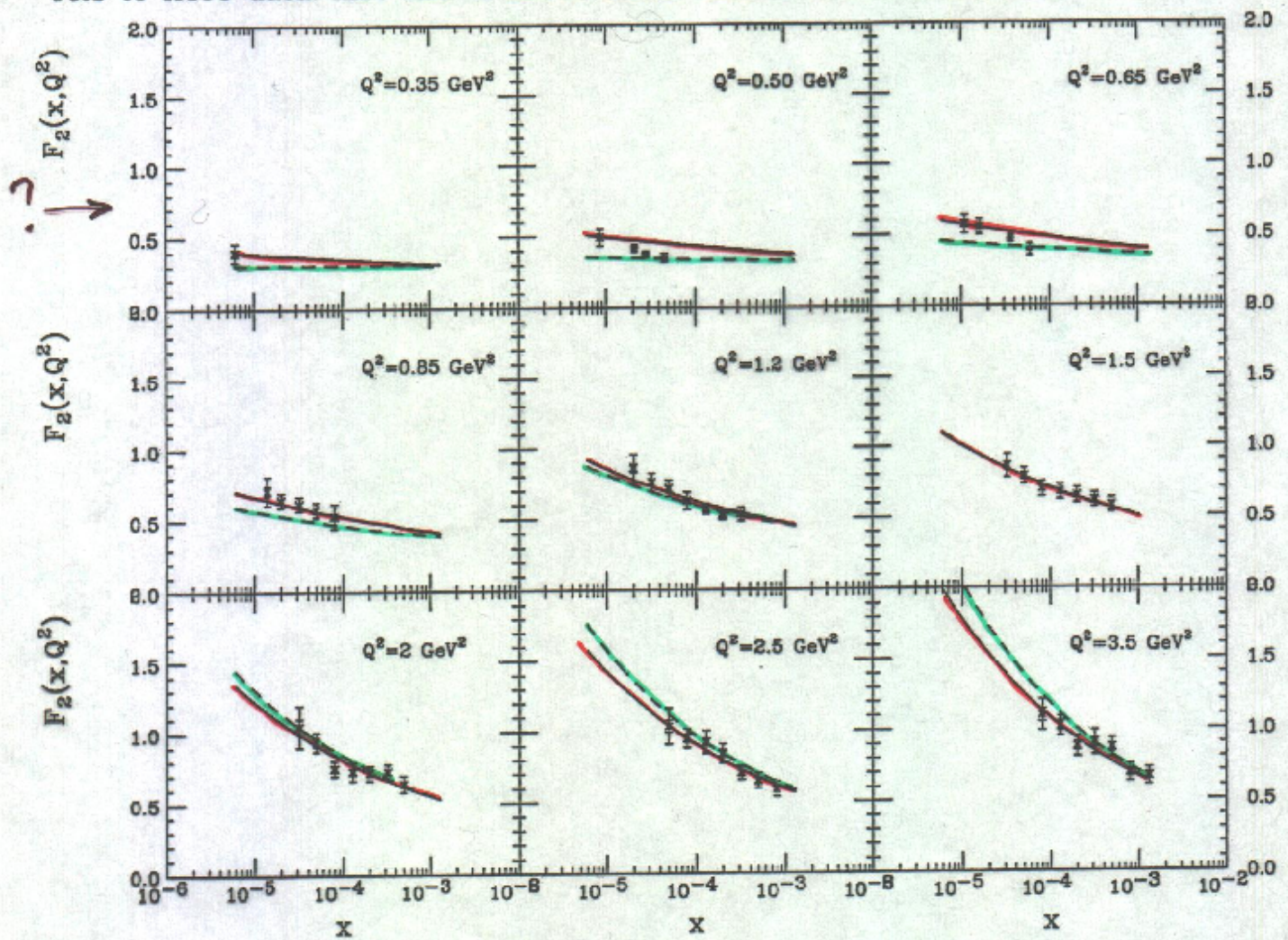
- **Kotikov** pdfs at small x in (analytic) NLO DGLAP
 - extends GRV, Ball-Forte, ... DLLA analysis, incorporating non-singular, NLO and higher-twist corrections in analytic format, assuming flat input at Q_0^2
 - reasonable few-parameter description of HERA small- x F_2
- **Szczurek** using dijet correlations to probe the unintegrated gluon distribution $f_g(x, k_T^2)$
 - $\gamma^* g \rightarrow q\bar{q}$: non-zero k_T for g smears out back-to-back azimuthal correlation of dijet pair
 - model predictions for $d\sigma/d\phi_{jj}$ await data

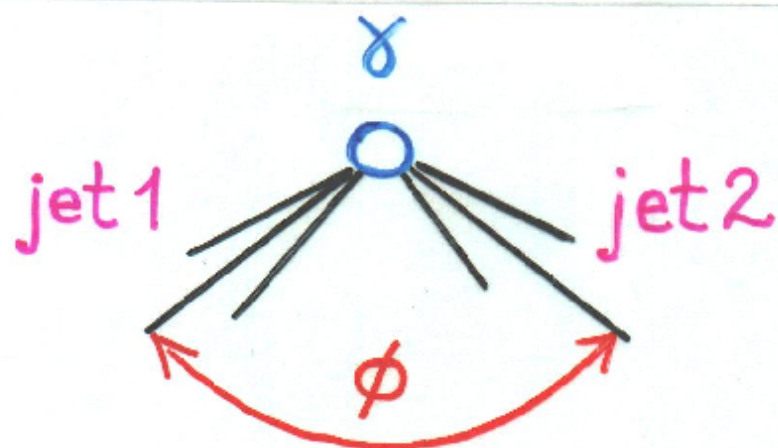
ZEUS95+H195 data

— LO Tw2
— LO Tw2 + Tw4 + Tw6
renormalon

preliminary

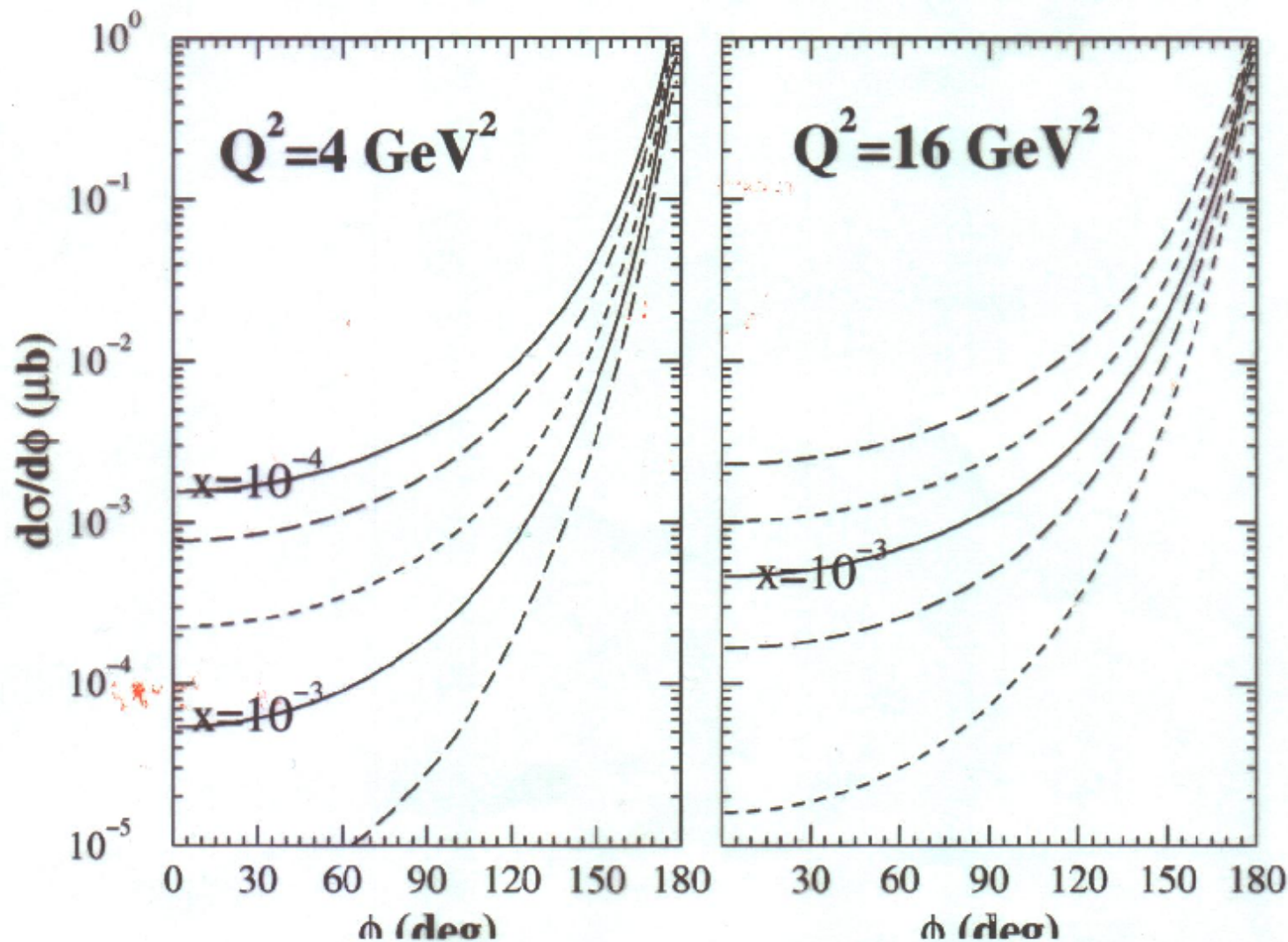
Fits to H195 data. Also shown ZEUS95 data. dashes=LO solid=LO+HTren. nf=4





Azimuthal correlations in DIS

$$p_{1,\perp}^{\text{HCM}}, p_{2,\perp}^{\text{HCM}} > p_{t,\text{cut}} = 4 \text{ GeV}$$



- **Kumano** parton distributions in nuclei

- fit fixed-target F_2 data on nuclear targets using ansatz

$$\begin{aligned}f_i^A(x, Q^2) &= w_i(x, A) f_i^N(x, Q^2) \\w_i(x, A) &= 1 + (1 - A^{-1})^{1/3} h_i(x)\end{aligned}$$

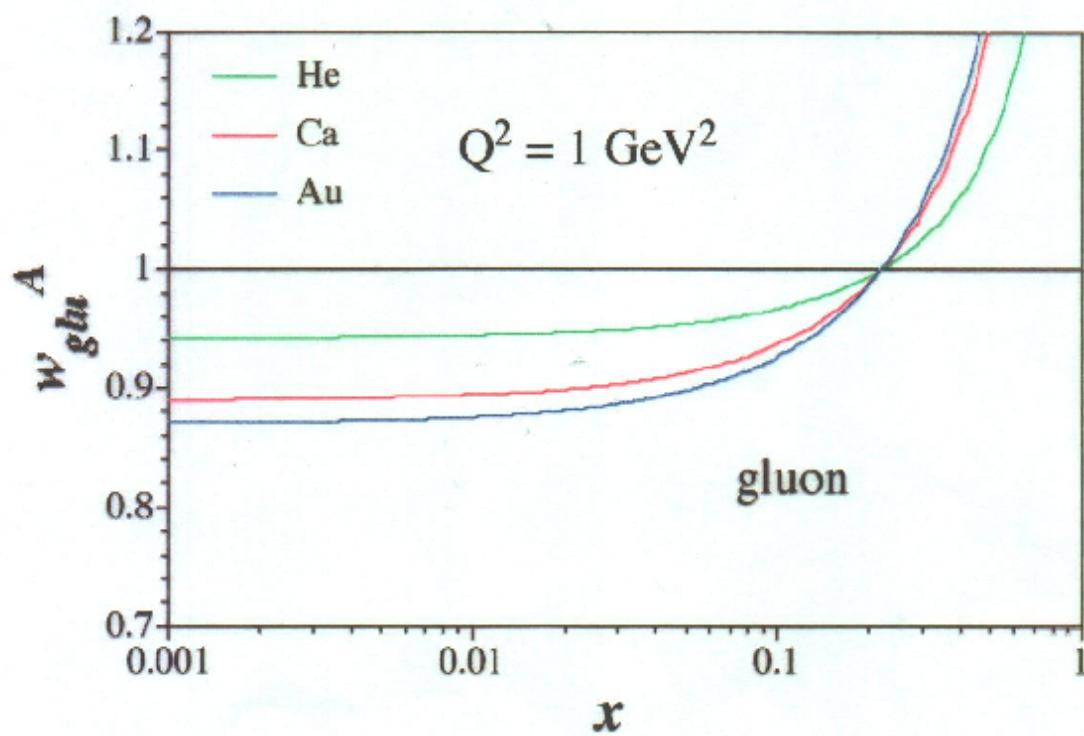
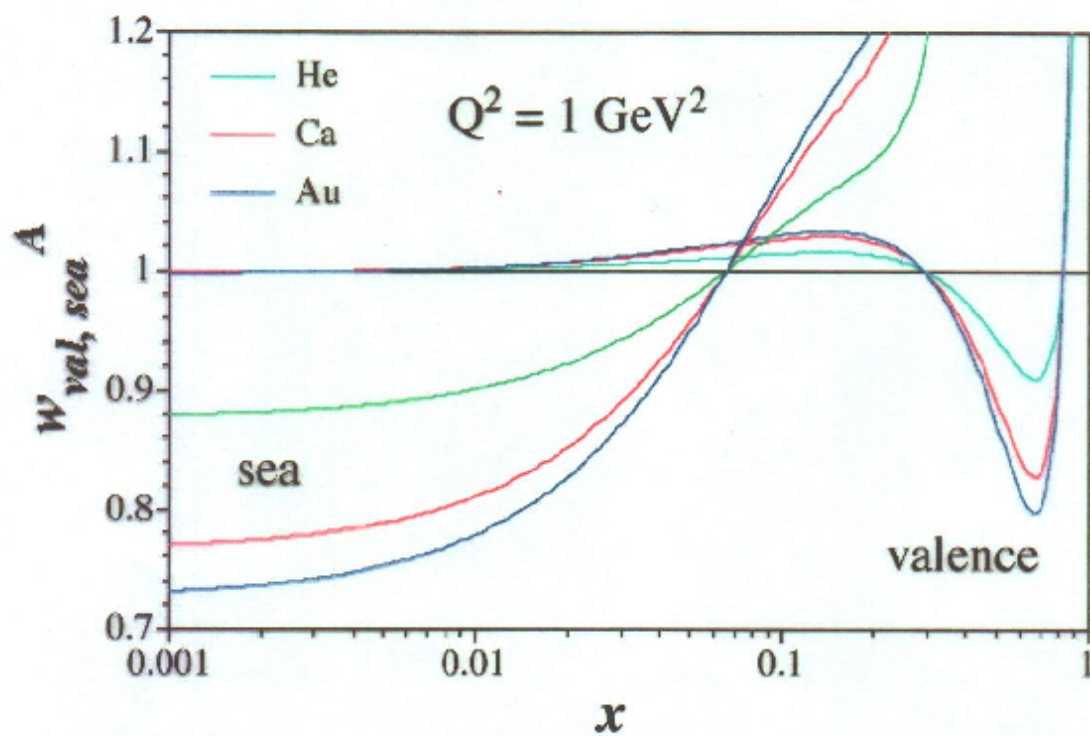
with simple parametric form for h_i

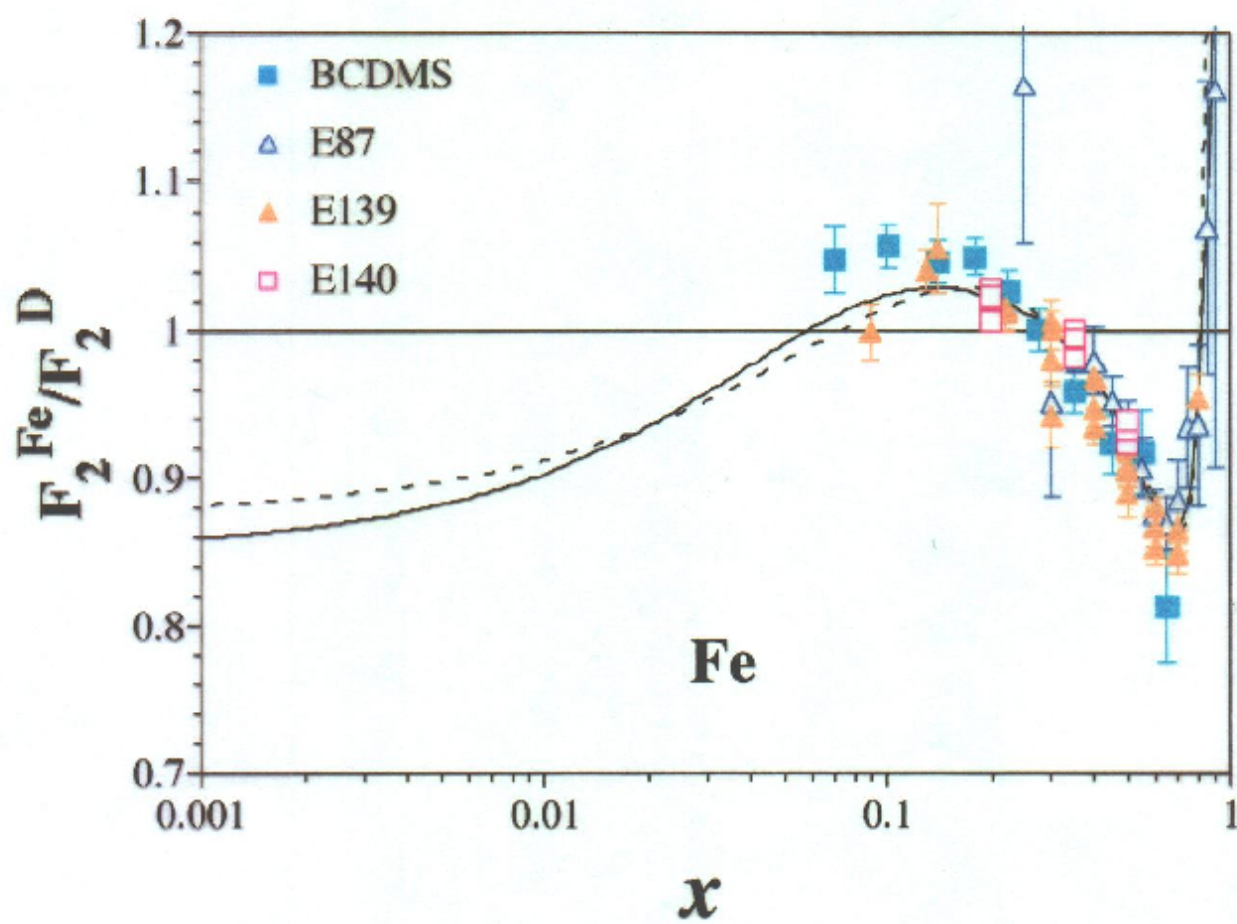
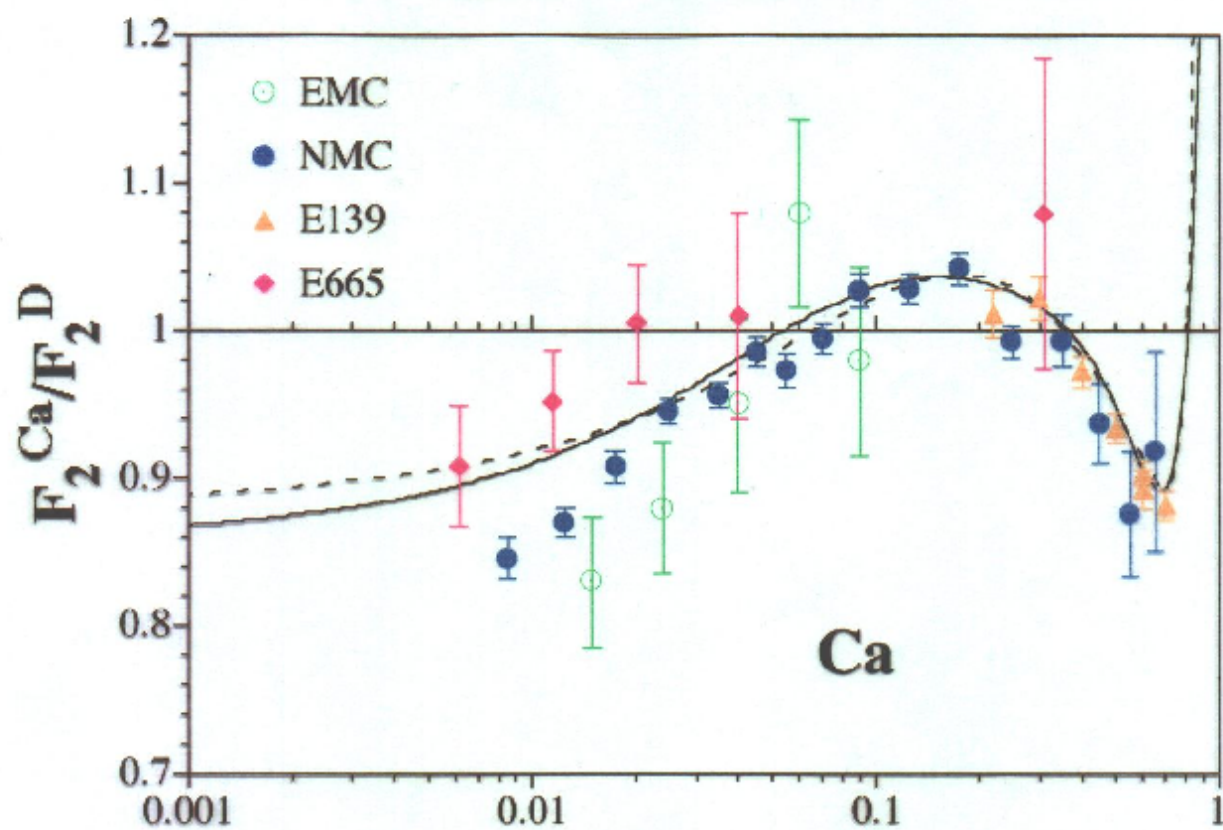
- 9 parameters, $\chi^2/\text{dof} = 580/302$ (mainly due to inconsistencies between data sets, e.g. E665 *vs.* NMC)
- need comparison with DY, prompt- γ data to further constrain $g^A, \bar{q}^A$ (see also Eskola)

Weight functions $w_i(x,A)$

for He, Ca, Au

“cubic fit”





photon structure

- new **data** from **LEP**, **HERA**, no new **theory** analyses (so compare with existing pdf models: GRV, SaS, GS, ...)
- new small- x and high Q^2 F_2^γ data from **OPAL**
- a look to the future: photon structure at a e^+e^- **linear collider**
- new jet data from **H1** and **ZEUS**



MEASUREMENT OF F_2^γ

$$\frac{d^2 \sigma_{e\gamma \rightarrow eX}}{dx dQ^2} = \frac{2\pi\alpha^2}{x Q^4} [(1 + (1-y)^2) F_2^\gamma(x, Q^2) - y^2 F_L^\gamma(x, Q^2)]$$

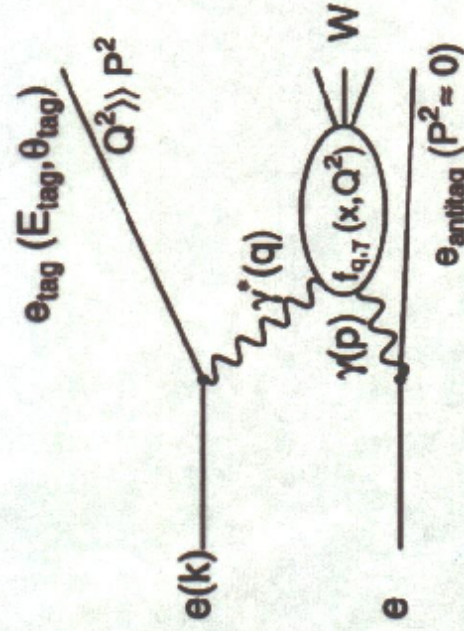
where $Q^2 = 2 E_{\text{beam}} E_{\text{tag}} (1 - \cos \theta_{\text{tag}})$

$$x = \frac{Q^2}{Q^2 + W^2 + P^2} \approx \frac{Q^2}{Q^2 + W^2}$$

$$y = 1 - \frac{E_{\text{tag}}}{E_{\text{beam}}} \cos^2\left(\frac{\theta_{\text{tag}}}{2}\right) \ll 1$$

x must be determined from poorly measured
hadronic final state, W .

$\Rightarrow$ UNFOLDING



$$F_2^\gamma = F_2^{PL} + F_2^{hadr} \Rightarrow$$

point-like
PERTURBATIVE



$$\gamma \rightarrow ff$$

hadron-like

NON-PERTURBATIVE

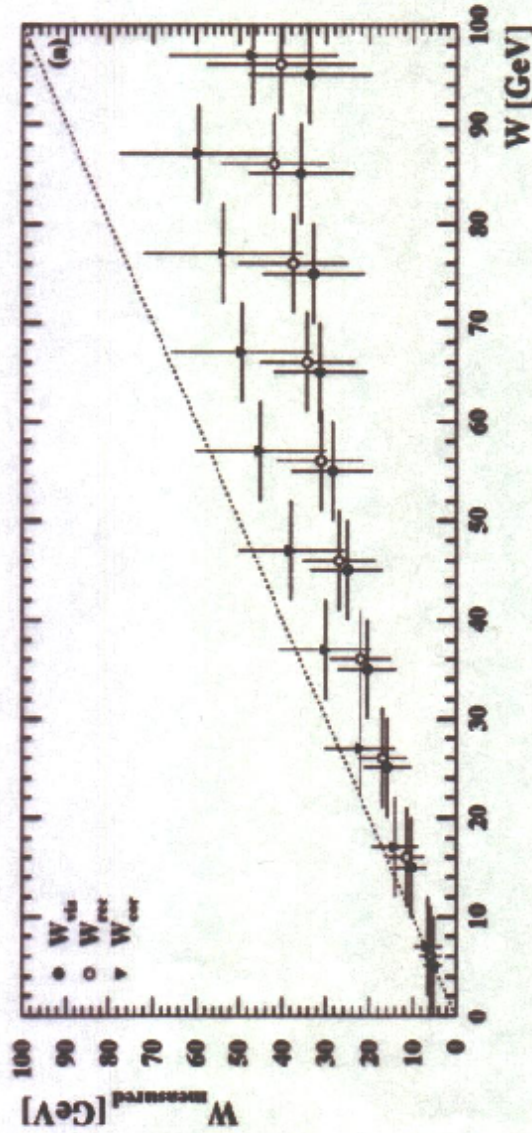


$$\gamma \rightarrow V(J^{PC} = 1^{--})$$

LATEST LOW x MEASUREMENT - W CORRECTIONS

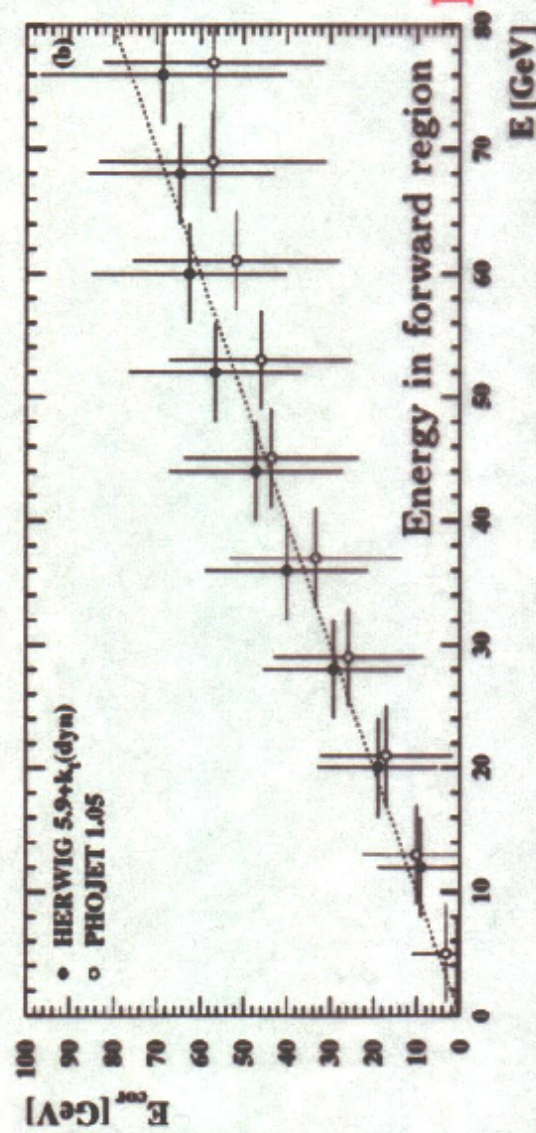
Hadronic final state
not fully reconstructed
 $W_{\text{vis}} < W$ and $x_{\text{vis}} > x$

Using kinematics of tagged
electron and transverse
momentum conservation
 $W_{\text{vis}} \Rightarrow W_{\text{rec}}$

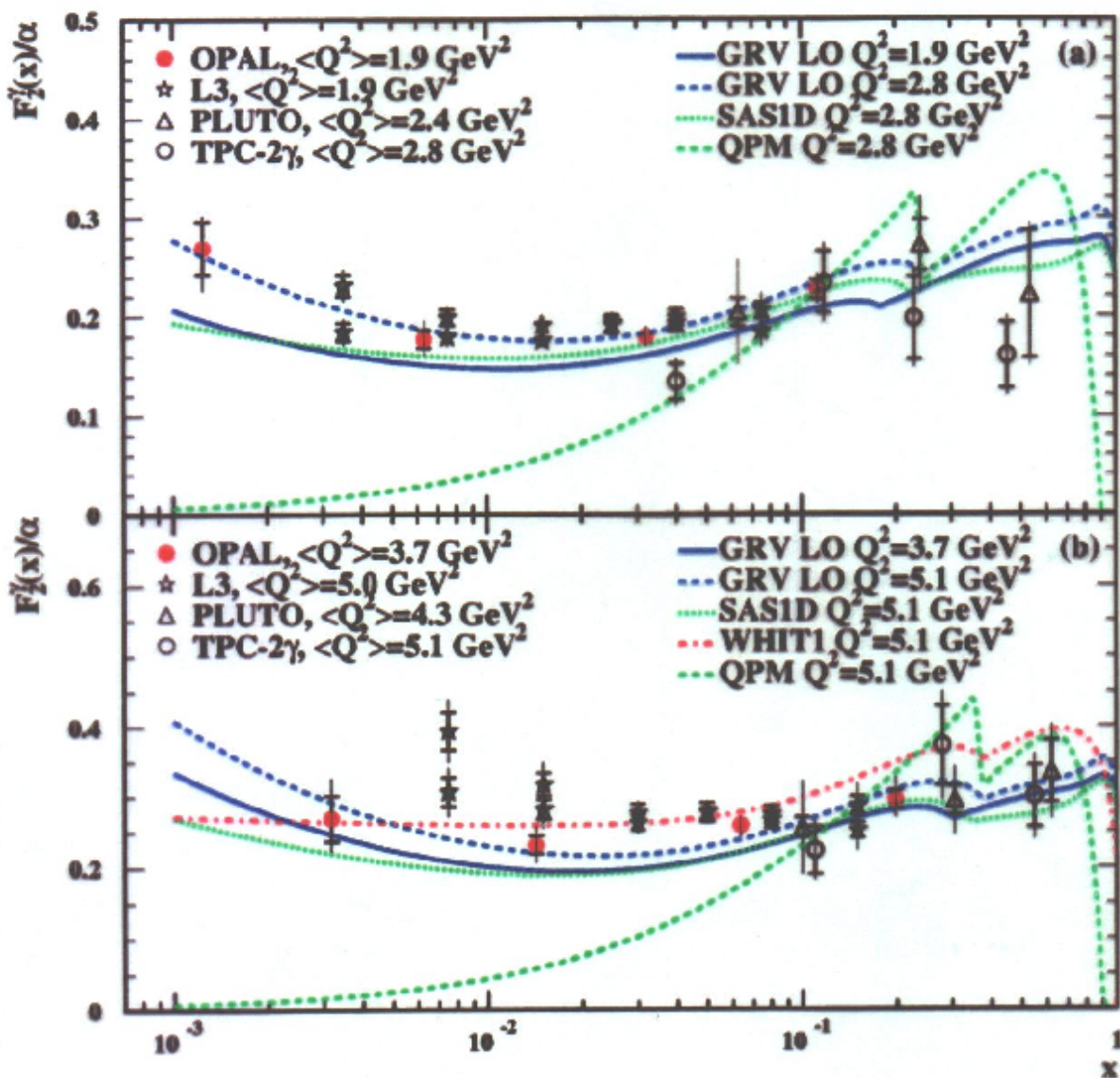


Doubling energy in forward
region (see lower plot)
 $W_{\text{rec}} \Rightarrow W_{\text{cor}}$

BUT unfolding still needed...



LOW x MEASUREMENT - LOW Q^2

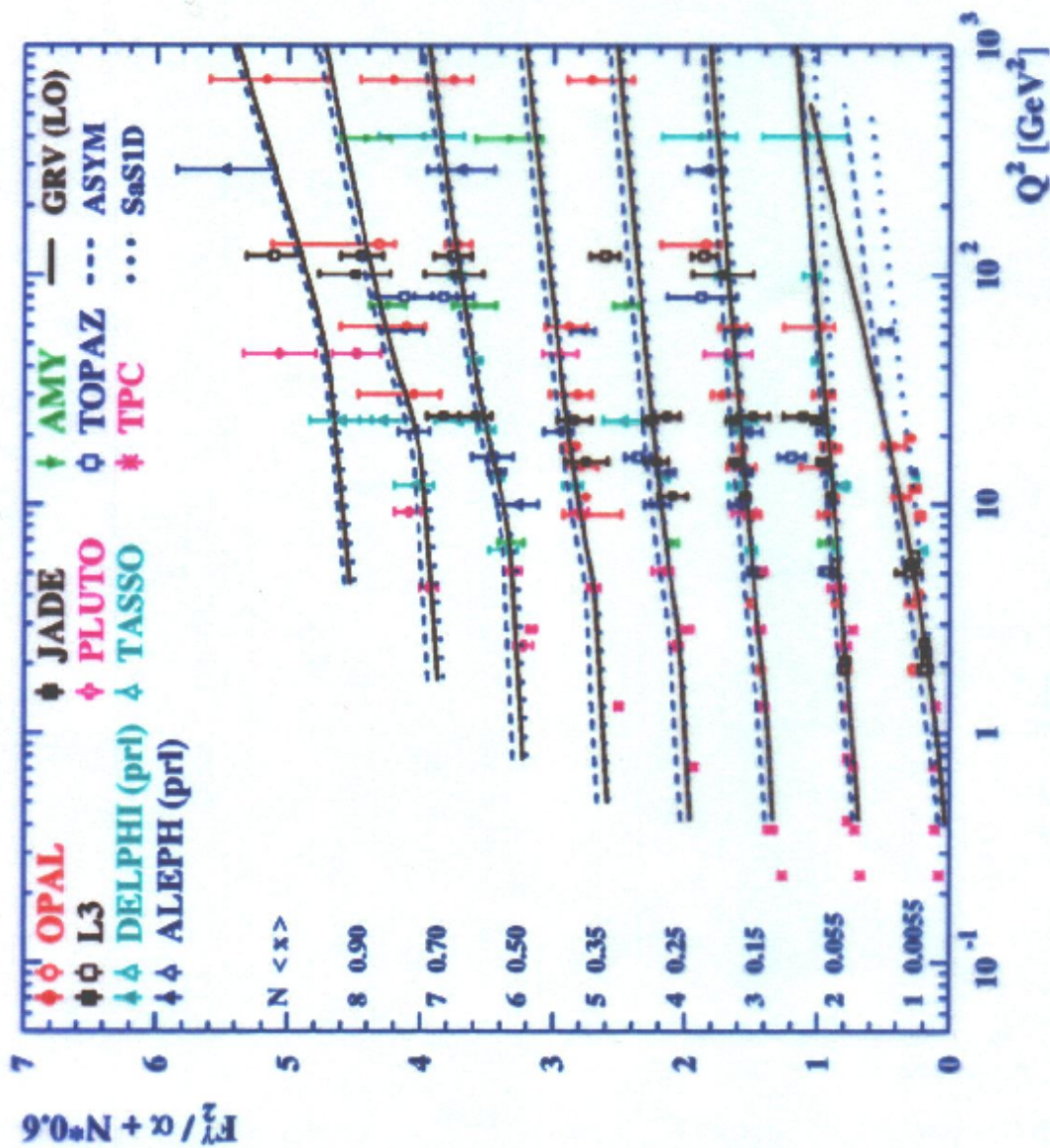


- Shapes of GRV LO and SaS1D parameterisations generally consistent with OPAL and L3 data.
- Evidence for a low x rise??
- Photon must contain significant hadron-like component at low x .





x DEPENDENCE OF Q^2 EVOLUTION OF F_2^γ



Photon Structure at a Future e^+e^- Linear Collider

28th April, DIS2001

Albert De Roeck
CERN

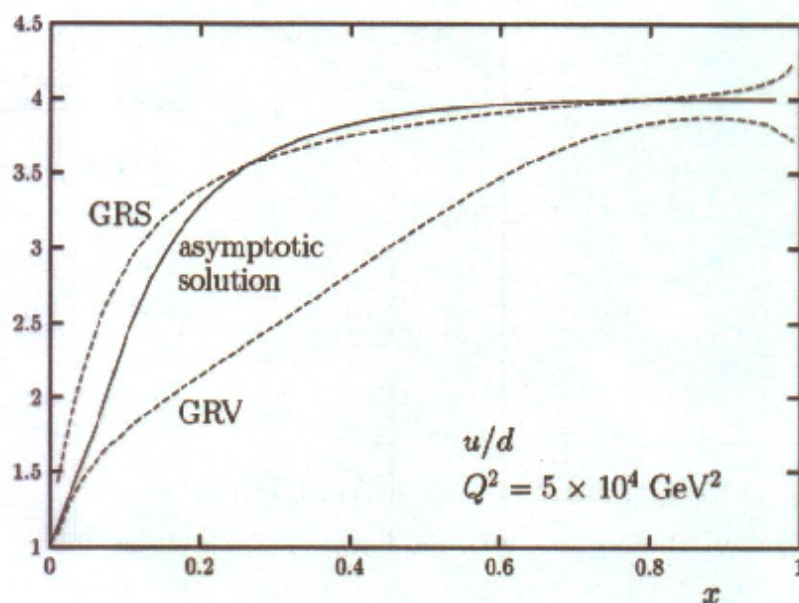
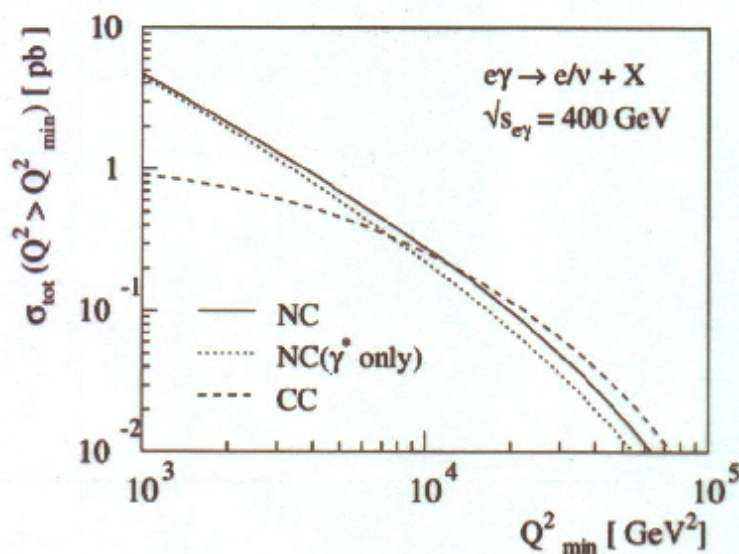
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- Polarised Structure Functions
M.Stratmann, W.Vogelsang, J. Kwiecinski, B. Ziaja
- Summary

Electron Photon Scattering

A. Gehrmann-De Ridder, H. Spiesberger, P. Zerwas

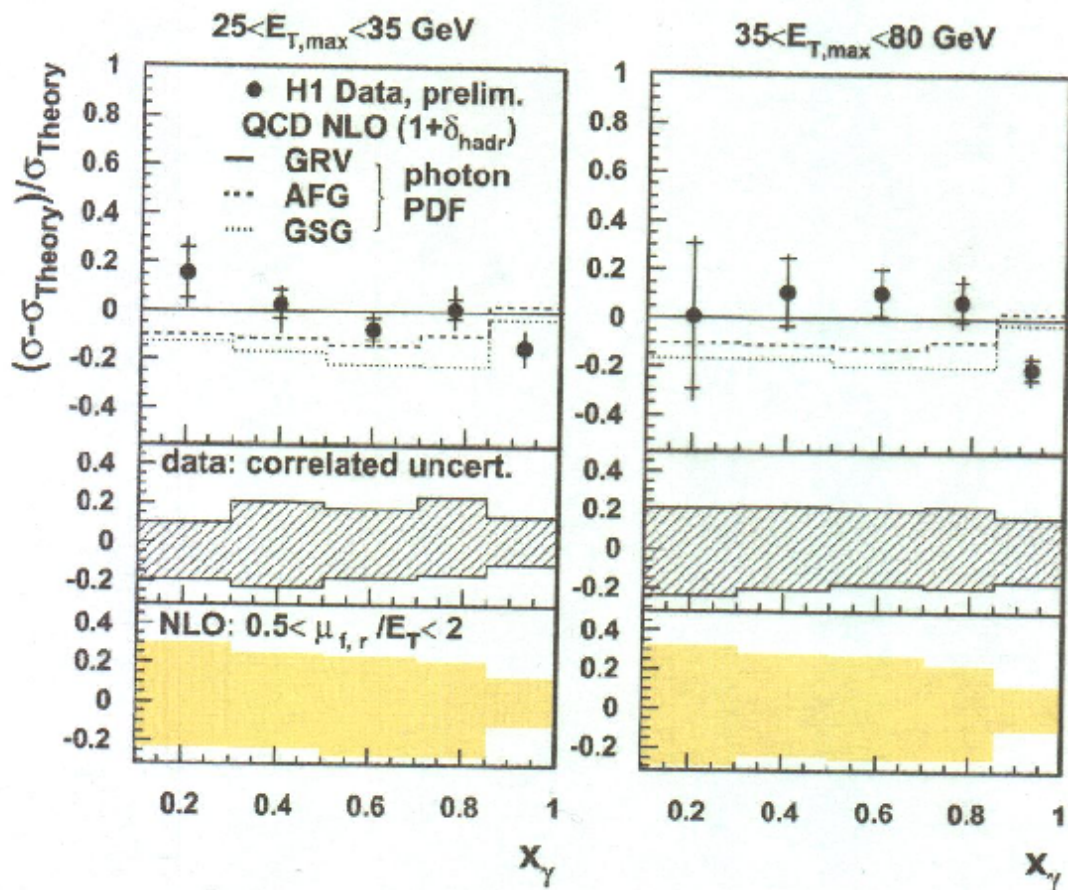
PC mode: Very high Q^2 region ($Q^2 > 10^4 \text{ GeV}^2$)
 $\Rightarrow$ EW neutral and charged current SF's



Determine separately up- and down-type quarks in photon!

Measurement of $\frac{d\sigma}{dx_\gamma}$

Difference between data and theory:

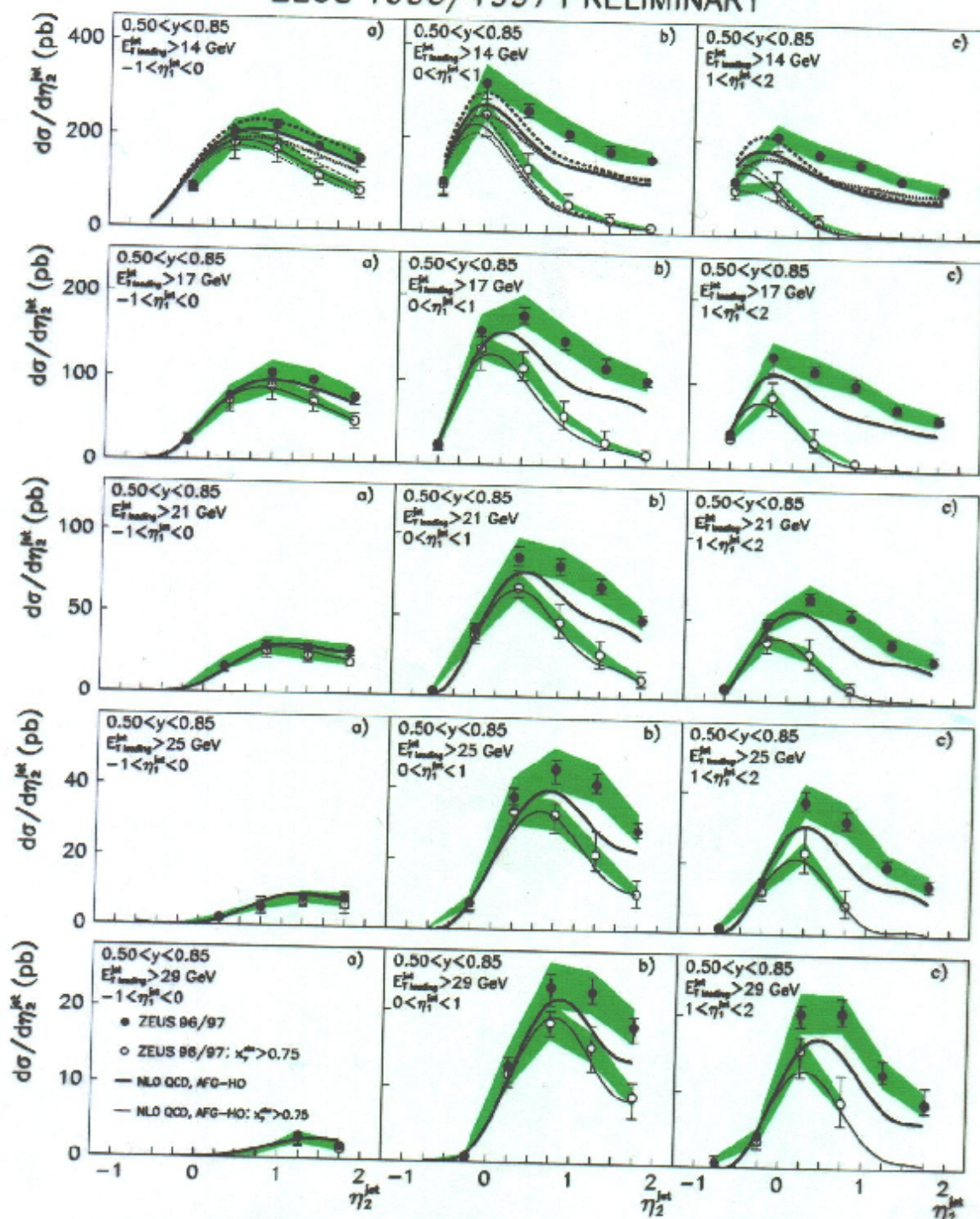


Upper plot: only uncorrelated part of the systematic errors are shown.

NLO predictions describe the x_γ distribution

Jet production from ZEUS

ZEUS 1996/1997 PRELIMINARY



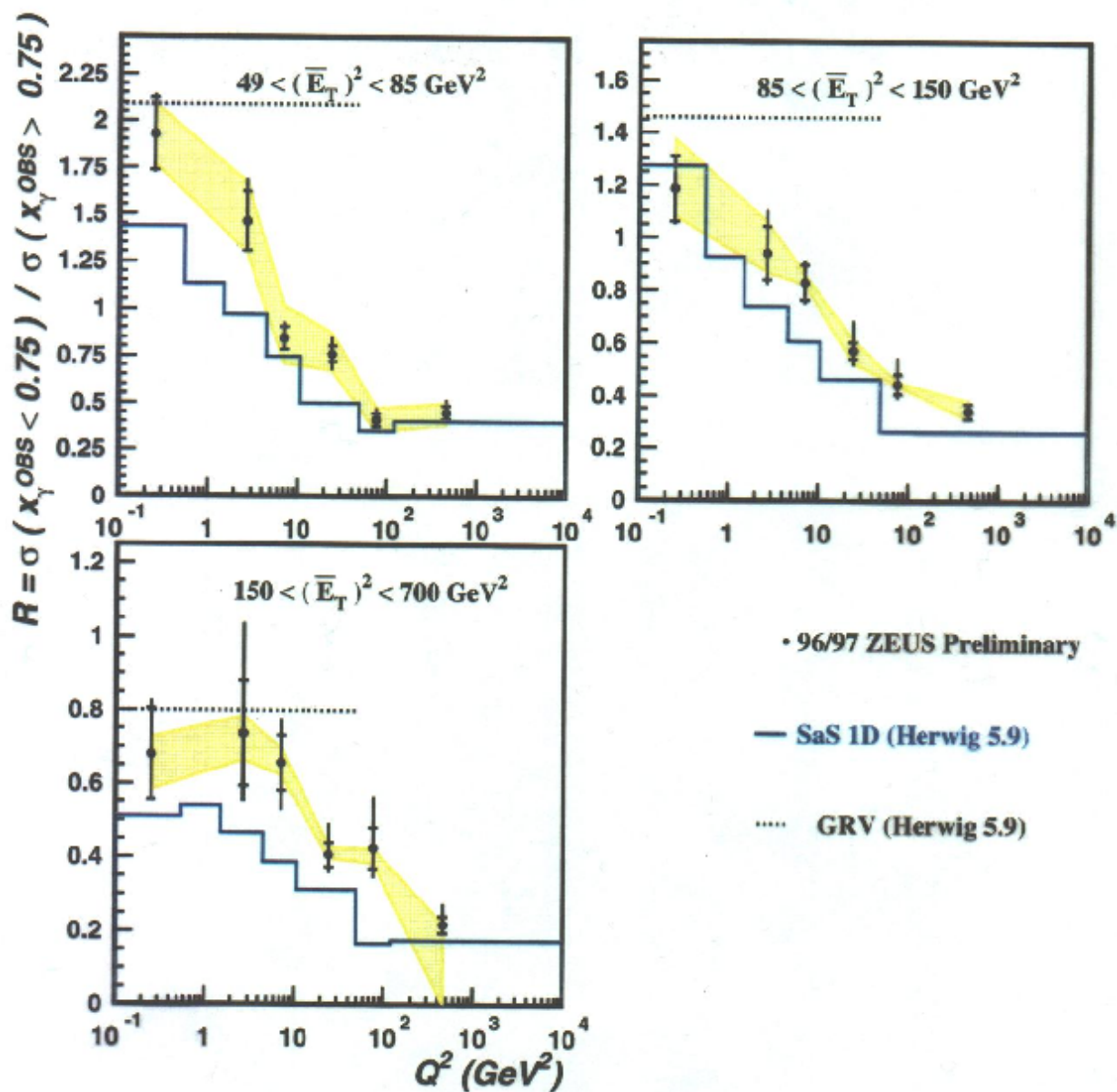
The predictions are too low at medium x

ZEUS / H1

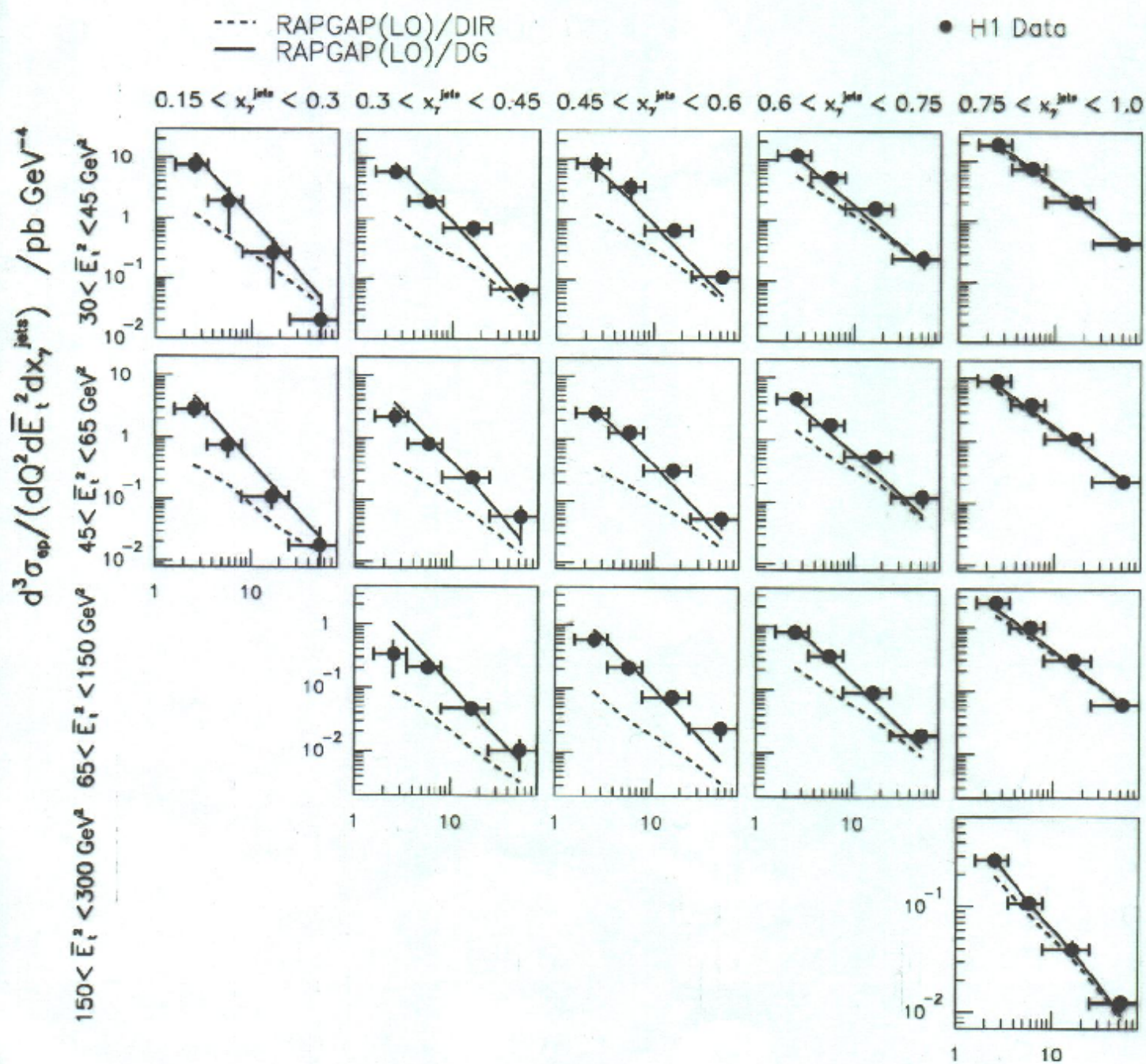
	ZEUS		H1	
δ PDFs	AFG		GRV	15%
p PDFs	CTEQ5.1		CTEQ5.1	/
α_s	0.116		0.118	5%
Shad	/		with	5%

Difference between the data ~ 10%

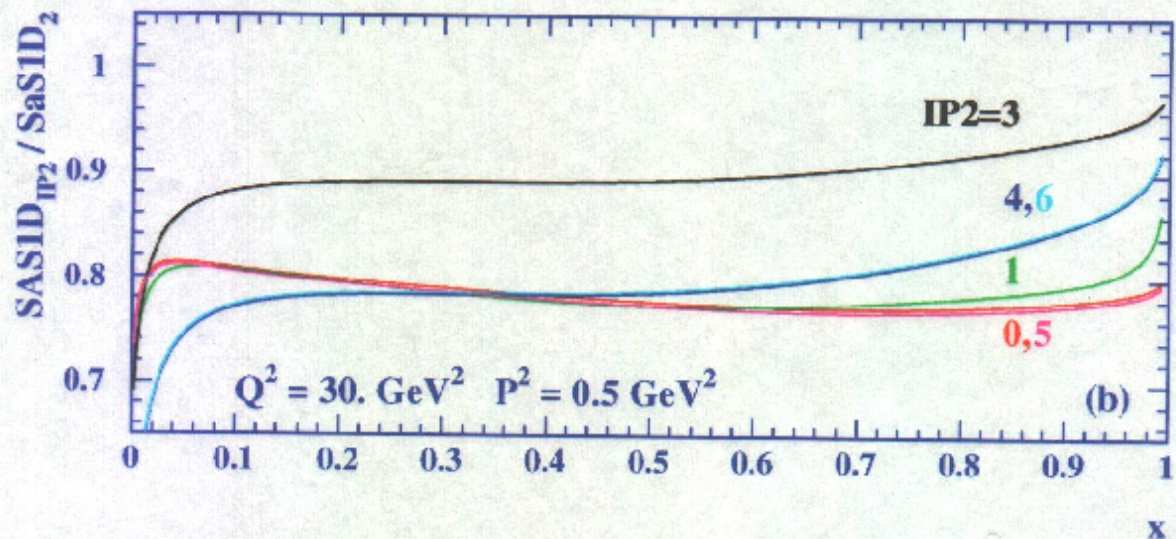
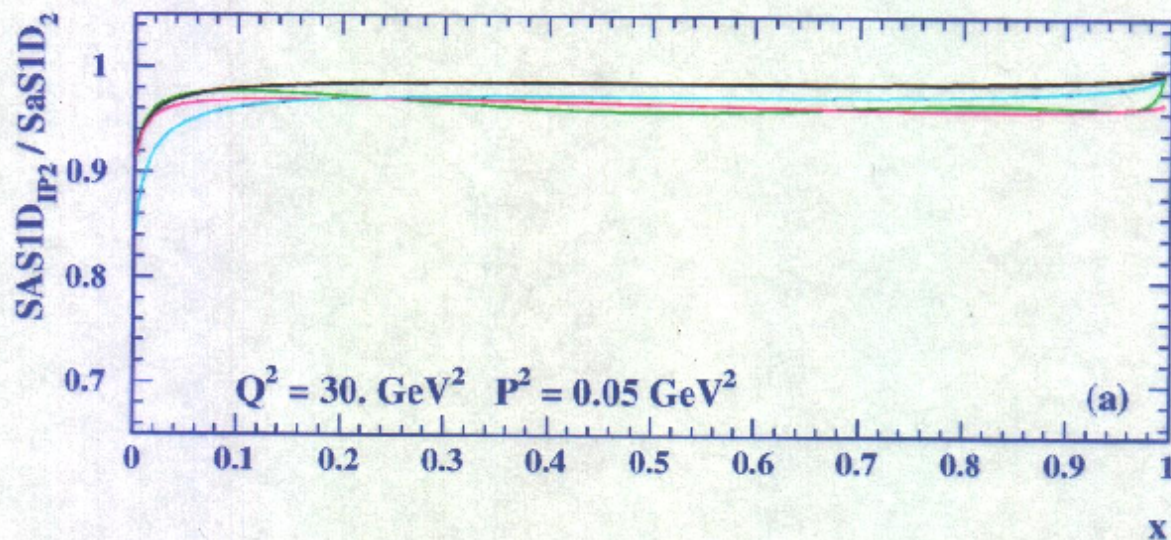
Ratios of Cross Sections



- ratios fall with $Q^2 \Rightarrow \gamma^*$ -structure suppressed in each $(\overline{E}_T)^2$ region.
- SaS1D below data ratio.



The P^2 suppression predicted by SaS



There is a significant variation in the predictions.

summary

- remarkably, after ~ 20 years, NLO-DGLAP still works well
- experimental coverage and precision requires better theory
 - heavy quarks $\leftarrow$ done!
 - consistent NNLO
 - incorporation of $\log x$ and $\log(1 - x)$ resummations
 - better understanding of higher twists
 - nuclear target effects
- ... to enable us to properly define and quantify $\delta f_{\text{exp,th}}$ and their impact on cross section predictions
- where are the young theorists to do this exciting and important work?!