Muon ''depth-intensity'' relation measured by the LVD underground experiment and cosmic-ray muon spectrum at sea level

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We present an analysis of muon events with all muon multiplicities collected during 21804 h of operation of the first Large Volume Detector tower. The measured angular distribution of muon intensity has been converted to the ''depth–vertical-intensity'' relation in the depth range from 3 to 12 km w.e. The analysis of this relation allowed us to derive the power index γ of the primary all-nucleon spectrum: $\gamma = 2.78 \pm 0.05$. The ''depth–vertical-intensity'' relation has been converted to standard rock and the comparison with the data of other experiments has been done. We also present the derived vertical muon spectrum at sea level. [S0556-2821(98)00517-7]

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I. INTRODUCTION

During the last 30 years the cosmic-ray muon energy spectrum has been studied in many experiments using different methods. These methods can be combined into three groups: (i) the direct measurements of the muon energy spectrum at sea level using magnetic spectrometers [1–3] (the spectrum up to about 10 TeV was measured at the zenith angles near horizon), (ii) the measurements of the energy spectrum of cascades produced by muons at shallow depth [4–7], and (iii) the measurement of the depth-intensity curve deep underground [8–12,33].

Since the spectrum of primary cosmic-ray nucleons has a power-law form with the power index γ, the spectrum of π and K mesons produced by primaries should also have a power-law form with the power index $\gamma_{\pi,K}$. If the scaling hypothesis is valid in the fragmentation region at high energies, the value of $\gamma_{\pi,K}$ is approximately equal to γ. The muon spectrum at sea level has a more complex form due to the competition between the interaction and decay of their

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parents. Moreover, the flux of muons produced by pions and kaons strongly depends on the zenith angle because the interaction path length of mesons in the atmosphere varies with the zenith angle θ. The muon spectrum follows the power-law dependence only at high energies $E_\mu \gg E_\pi^{cr}, E_K^{cr}$, where E_π^{cr} and E_K^{cr} are the critical energies of pions and kaons in the atmosphere. For $\theta=0$ (vertical) this condition is quite satisfied at $E_\mu > 2$ TeV. The power index of the muon spectrum γ_μ in this case is more by 1 than the value of $\gamma_{\pi,K}$.

Despite the numerous experiments which studied the muon spectrum, there are discrepancies in the published results (values of γ, $\gamma_{\pi,K}$, or γ_μ). Most of the experiments carried out in the last 20 years gave values of γ (or $\gamma_{\pi,K}$) in the range 2.60–2.80. However, the dispersion of the results is greater than the statistical and systematic errors published by the authors. Even the values of γ obtained using one method (for example, using the measurement of the depth-intensity curve) have a large dispersion. Such discrepancy can be due to either the difference between the data or between the calculated muon intensities used to fit the data, or both.

The muon intensities deep underground have been calculated by many authors using different methods (Monte Carlo simulation of the muon transport, numerical solution of the kinetic equations, etc., see, for example, Refs. [13–20]). However, the discrepancy between the muon survival probabilities and, hence, between the muon intensities, calculated by different authors, is quite large and can result in the significant discrepancy between the final values of γ.

The LVD (Large Volume Detector), located underground, can measure atmospheric muon intensities from 3000 to 12 000 hg/cm^2 and higher (which corresponds to muon energies at sea level from 1.5 to 40 TeV) at zenith angles from 0° to 90°. This allows us to study muon spectra and their characteristics at energies 1.5–40 TeV (which correspond to energies of primaries of about 10–400 TeV).

In a previous paper [12] we presented our measurement of the muon depth-intensity curve and the evaluation of the power index of the meson spectrum in the atmosphere using the depth-intensity relation for single muon events. The muon survival probabilities, used to obtain the value of $\gamma_{\pi,K}$ in Ref. [12], have been presented in Ref. [21]. They were calculated using the muon interaction cross sections from Refs. [22–24]. After the publication of these results, a new calculation of the cross section of the muon bremsstrahlung and of the corrections to the knock-on electron production cross section have been done [25]. In the present analysis we have taken into account the corrections proposed in Ref. [25] and we have estimated the uncertainties of γ (γ is assumed to be equal to $\gamma_{\pi,K}$) due to the uncertainties of the cross sections used to simulate muon transport through rock. The analysis is based on an increased statistics comparing with the previous publications and refers to events with all muon multiplicities.

In Sec. II the detector and the procedure of data processing together with the conversion of muon intensity to vertical intensity are briefly described. In Sec. III the results of the analysis of the ''depth–vertical-muon-intensity'' distribution $[I_\mu(x)]$ are presented. In Sec. IV the ''depth–vertical-intensity'' relation in standard rock is compared with the data of other underground experiments. In Sec. V we present the derived muon spectrum at sea level. Section VI contains our conclusions.

II. LVD AND DATA PROCESSING

The LVD (Large Volume Detector) is located in the underground Gran Sasso Laboratory at a minimal depth of about 3000 hg/cm^2. The LVD consists of five towers. The first tower has been running since June, 1992 and the second one since June, 1994. The data presented here were collected with the first LVD tower during 21 804 h of live time.

The first LVD tower contains 38 identical modules [28]. Each module consists of eight scintillation counters and four layers of limited streamer tubes (tracking detector) attached to the bottom and to one vertical side of the supporting structure. A detailed description of the detector was given in Ref. [28]. One LVD tower has dimensions of $13\times 6.3\times 12\,\mathrm{m}^3$.

The LVD measures the atmospheric muon intensities from 3000 hg/cm^2 to more than 12 000 hg/cm^2 (which correspond to the median muon energies at sea level from 1.5 to 40 TeV) at zenith angles from 0° to 90° (on average, larger depths correspond to higher zenith angles).

In the analysis we have used muon events with all multiplicities, as well as a sample of single muons. Our basic results have been obtained with the all muon sample. This sample contains about two million reconstructed muon tracks.

The acceptances for each angular bin have been calculated using the simulation of muons passing through the LVD taking into account muon interactions with the detector materials and the detector response. The acceptances for both single and multiple muons were assumed to be the same.

As a result of the data processing the angular distribution of the number of detected muons $N_\mu(\phi,\cos\theta)$ has been obtained. The angular bin width $1°\times 0.01$ has been used. The analysis refers to angular bins for which the efficiency of the muon detection and track reconstruction is greater than 0.03. We have excluded from the analysis angular bins with a large variation of depth.

The measured $N_\mu(\phi,\cos\theta)$ distribution has been converted to the depth–vertical-muon-intensity relation $I_\mu(x)$ using the formula

$$I_\mu(x_m)=\frac{\sum_{ij} N_\mu[x_m(\phi_j,\cos\theta_i)]\cos\theta_i^{\star\star}}{\sum_{ij}\{A[x_m(\phi_j,\cos\theta_i)]\epsilon[x_m(\phi_j,\cos\theta_i)]\Omega_{ij}T\}}, \tag{1}$$

where the summing up has been done over all angular bins $(\phi_j,\cos\theta_i)$ contributing to the depth x_m, $A[x_m(\phi_j,\cos\theta_i)]$ is the cross section of the detector in the plane perpendicular to the muon track at the angles $(\phi_j,\cos\theta_i)$, $\epsilon[x_m(\phi_j,\cos\theta_i)]$ is the efficiency of muon detection and reconstruction, Ω_{ij} is the solid angle for the angular bin, T is the live time, and $\cos\theta_i^{\star\star}=I_\mu^c(x_m,\cos\theta=1)/I_\mu^c(x_m,\cos\theta_i)$ is the ratio of predicted muon intensity at $\cos\theta=1$ to that at $\cos\theta_i$. To obtain the values of cos $\theta_i^{\star\star}$, Eq. (4) (see below) with the previously estimated parameters of the muon spectrum at sea level [12,21] and the calculated survival probabilities have been used. Actually, the values of cos $\theta_i^{\star\star}$ do not depend on the parameters of the muon spectrum at sea level much (normalization factor A and power index γ) for any reasonable values of γ. The factor cos $\theta_i^{\star\star}$ is different from the simple cos θ law used to convert the muon intensities to vertical intensities in Ref. [12].

In the calculations of cos $\theta_i^{\star\star}$ the contribution of the prompt muons from charmed particle decay has been neglected. As is shown in Refs. [26,27] the ratio of prompt muons to pions according to LVD data does not exceed 2×10^{-3} at 95% confidence level. This means that at median depths [no more than 6–7 km water equivalent (w.e.)] the fraction of prompt muons with respect to conventional muons does not exceed 10% at vertical. And there is also a small probability that this fraction is more than 10% at large depths (8–10 km w.e.).

For the depth-intensity relation a bin width of 200 m w.e. has been chosen. For depths more than 9 km w.e. the bin width was increased to 500 m w.e. to increase the statistics for each bin. The conversion of muon intensity to the middle points of each depth bin has been done using the formula

$$I_\mu^m(x_i)=I_\mu^m(x_m)\frac{I_\mu^c(x_i)}{I_\mu^c(x_m)}, \tag{2}$$

where $I_\mu^m(x_m)$ and $I_\mu^c(x_m)$ are the measured and calculated muon intensities at the weighted average depth x_m which corresponds to a depth bin with a middle value of x_i; $I_\mu^m(x_i)$ and $I_\mu^c(x_i)$ are the derived and calculated muon intensities at the depth x_i which is the middle point of the depth bin. The values of x_m have been obtained by averaging the depths for all angular bins contributing to the given depth bin with a weight equal to the detected number of muons. To calculate the muon intensities at x_m and x_i we have again used Eq. (4) with the previously estimated parameters of the muon spectrum [12,21] and the simulated muon survival probabilities. Since the width of depth bins is quite small (200 m w.e. for depth bins with high statistics) and the number of angular bins contributing to each depth bin is quite large (several hundreds), the conversion factor does not exceed 10%.

III. DEPTH–VERTICAL-INTENSITY RELATION IN GRAN SASSO ROCK

The depth–vertical-muon-intensity relation derived as described in the previous section has been fitted with calculated distributions with two free parameters of the muon spectrum at sea level: normalization constant A and the power index of the primary all-nucleon spectrum γ. To calculate the muon intensities underground we have used the formula

$$I_\mu(x,\cos\,\theta)=\int_0^\infty P(E_{\mu0},x)\frac{dI_{\mu0}(E_{\mu0},\cos\,\theta)}{dE_{\mu0}}dE_{\mu0}, \tag{3}$$

where $P(E_{\mu0},x)$ is the survival probability of the muon with an initial energy $E_{\mu0}$ at sea level to reach the depth x and $dI_{\mu0}(E_{\mu0},\cos\,\theta)/dE_{\mu0}$ is the muon spectrum at sea level at zenith angle θ. This spectrum has been taken according to Ref. [29]

$$\frac{dI_{\mu0}(E_{\mu0},\cos\,\theta)}{dE_{\mu0}} = A\times0.14\times E_{\mu0}^{-\gamma}\left(\frac{1}{1+1.1E_{\mu0}\cos\,\theta^\star/115\ \text{GeV}} + \frac{0.054}{1+1.1E_{\mu0}\cos\,\theta^\star/850\ \text{GeV}}\right), \tag{4}$$

where the values of cos θ have been substituted by cos $\theta^\star$ which have been taken from Ref. [30]. In Ref. [30] cos $\theta^\star=E_{\pi,K}^{\text{cr}}(\cos\,\theta=1)/E_{\pi,K}^{\text{cr}}(\cos\,\theta)$, where $E_{\pi,K}^{\text{cr}}$ are the critical energies of pions and kaons. This formula has been obtained under a simple assumption of scaling in high-energy hadron-nucleus interactions. Under this assumption the power index of the primary spectrum γ is equal to that of the meson (pion + kaon) spectrum $\gamma_{\pi,K}$. To fit the depth–vertical-intensity relation measured by LVD we have put cos θ=cos $\theta^\star=1$, however, to convert the muon intensity to vertical intensity (as was described in the previous section) we have used the values of cos $\theta^\star$ from Ref. [30].

We have used the muon survival probabilities $P(E_{\mu0},x)$ calculated with the muon cross sections from Refs. [23–25] and have taken the stochasticity of all processes of muon interaction with matter into account. They differ from those presented in Ref. [21] and used also in Ref. [12] (see also references therein) because of the new corrections for the muon bremsstrahlung and knock-on electron production cross sections proposed in Ref. [25] (see also the discussion in Ref. [31]).

The measured depth-intensity curve has been fitted to the calculated function [see Eq. (3)] with two free parameters: additional normalization constant A and the power index of the primary all-nucleon spectrum γ. As a result of the fitting procedure the following values of free parameters have been obtained: $A=1.95\pm0.31$, $\gamma=2.78\pm0.02$. These values are in good agreement with the results of the analysis of the depth-angular distributions [26,27]. We note that the energy in Eq. (4) is expressed in GeV and the intensity is expressed in $\text{cm}^{-2}\,\text{s}^{-1}\,\text{sr}^{-1}$. The errors of the parameters include both statistical and systematic uncertainties. The systematic uncertainty takes into account possible uncertainties in the depth, rock composition, density, etc., but does not take into account the uncertainty in the cross sections used to simulate muon transport through rock. If we add the uncertainty in the muon interaction cross sections, the error of γ will increase from 0.02 to 0.05 and the error of A to 1.0 (for a discussion on the uncertainty due to different cross sections see Ref. [31]). A similar analysis performed for single muons reveals almost the same value of the power index while the absolute intensity is 10% smaller: $A=1.65\pm0.30$, $\gamma=2.77\pm0.02$. We note that the estimates of the parameters A and γ are strongly correlated. The larger the value of γ is, the larger

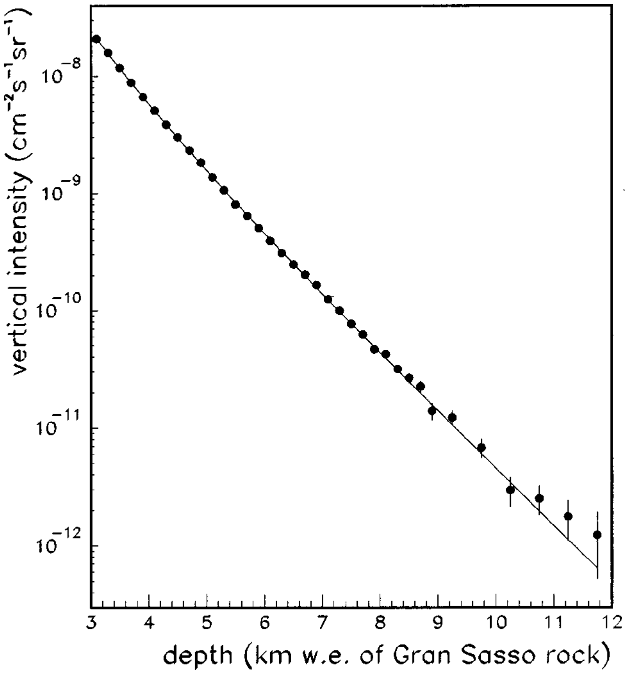

FIG. 1. Depth–vertical-muon-intensity curve in Gran Sasso rock measured by LVD together with the best fit using Eq. (4) with the parameters $\gamma=2.78$ and $A=1.95$.

the normalization factor A should be. The depth–vertical-muon-intensity relation is shown in Fig. 1 for the all muon sample together with the best fit. The muon intensities are also presented in Table I (column 2).

If the formula from Ref. [32] is used for the muon spectrum at sea level instead of Eq. (4), the best fit values of γ will be decreased by 0.04–0.05 and will be in agreement with the previously published values for single muons [12,21] analyzed using the formula from Ref. [32]. The value of γ obtained with LVD data is in reasonable agreement with the results of many other surface and underground experiments (see, for example, Refs. [2–4,6,7,10,11]).

TABLE I. Vertical muon intensities measured by LVD vs depth in Gran Sasso (GS) and standard (st) rocks. Errors include both statistical and systematic uncertainties.

x (km w.e.)	I_μ (cm^2 s sr)$^{-1}$ GS rock	I_μ (cm^2 s sr)$^{-1}$ st rock
3.1	$(2.09\pm0.02)\times10^{-8}$	$(2.17\pm0.02)\times10^{-8}$
3.3	$(1.58\pm0.02)\times10^{-8}$	$(1.65\pm0.02)\times10^{-8}$
3.5	$(1.18\pm0.01)\times10^{-8}$	$(1.23\pm0.01)\times10^{-8}$
3.7	$(8.82\pm0.09)\times10^{-9}$	$(9.24\pm0.09)\times10^{-9}$
3.9	$(6.66\pm0.07)\times10^{-9}$	$(7.04\pm0.07)\times10^{-9}$
4.1	$(5.08\pm0.06)\times10^{-9}$	$(5.37\pm0.06)\times10^{-9}$
4.3	$(3.87\pm0.04)\times10^{-9}$	$(4.12\pm0.04)\times10^{-9}$
4.5	$(3.00\pm0.03)\times10^{-9}$	$(3.19\pm0.04)\times10^{-9}$
4.7	$(2.34\pm0.03)\times10^{-9}$	$(2.47\pm0.03)\times10^{-9}$
4.9	$(1.83\pm0.02)\times10^{-9}$	$(1.99\pm0.03)\times10^{-9}$
5.1	$(1.37\pm0.02)\times10^{-9}$	$(1.50\pm0.02)\times10^{-9}$
5.3	$(1.07\pm0.02)\times10^{-9}$	$(1.16\pm0.02)\times10^{-9}$
5.5	$(8.07\pm0.13)\times10^{-10}$	$(8.99\pm0.14)\times10^{-10}$
5.7	$(6.42\pm0.11)\times10^{-10}$	$(6.79\pm0.12)\times10^{-10}$
5.9	$(5.07\pm0.09)\times10^{-10}$	$(5.52\pm0.10)\times10^{-10}$
6.1	$(3.94\pm0.08)\times10^{-10}$	$(4.34\pm0.09)\times10^{-10}$
6.3	$(3.11\pm0.07)\times10^{-10}$	$(3.53\pm0.08)\times10^{-10}$
6.5	$(2.48\pm0.06)\times10^{-10}$	$(2.68\pm0.07)\times10^{-10}$
6.7	$(2.03\pm0.07)\times10^{-10}$	$(2.27\pm0.07)\times10^{-10}$
6.9	$(1.66\pm0.06)\times10^{-10}$	$(1.88\pm0.07)\times10^{-10}$
7.1	$(1.25\pm0.05)\times10^{-10}$	$(1.40\pm0.05)\times10^{-10}$
7.3	$(1.00\pm0.04)\times10^{-10}$	$(1.14\pm0.05)\times10^{-10}$
7.5	$(7.70\pm0.34)\times10^{-11}$	$(9.32\pm0.39)\times10^{-11}$
7.7	$(6.27\pm0.30)\times10^{-11}$	$(6.85\pm0.32)\times10^{-11}$
7.9	$(4.68\pm0.25)\times10^{-11}$	$(5.65\pm0.29)\times10^{-11}$
8.1	$(4.27\pm0.25)\times10^{-11}$	$(4.45\pm0.24)\times10^{-11}$
8.3	$(3.19\pm0.25)\times10^{-11}$	$(3.80\pm0.26)\times10^{-11}$
8.5	$(2.67\pm0.26)\times10^{-11}$	$(2.97\pm0.25)\times10^{-11}$
8.7	$(2.26\pm0.26)\times10^{-11}$	$(2.51\pm0.27)\times10^{-11}$
8.9	$(1.40\pm0.22)\times10^{-11}$	$(2.11\pm0.26)\times10^{-11}$
9.25	$(1.23\pm0.17)\times10^{-11}$	$(1.40\pm0.16)\times10^{-11}$
9.75	$(6.8\pm1.3)\times10^{-12}$	$(7.9\pm1.2)\times10^{-12}$
10.25	$(3.0\pm0.8)\times10^{-12}$	$(4.5\pm1.0)\times10^{-12}$
10.75	$(2.5\pm0.7)\times10^{-12}$	$(2.7\pm0.8)\times10^{-12}$
11.25	$(1.8\pm0.7)\times10^{-12}$	$(2.2\pm0.7)\times10^{-12}$
11.75	$(1.2\pm0.7)\times10^{-12}$	$(1.4\pm0.8)\times10^{-12}$

IV. DEPTH–VERTICAL-MUON-INTENSITY RELATION IN STANDARD ROCK

The simulations carried out for Gran Sasso and standard rocks allow us to obtain the formula for the conversion of the depth in Gran Sasso rock $x_{\rm GS}$ to that in standard rock $x_{\rm st}$. This was done by comparing the values of $x_{\rm st}$ and $x_{\rm GS}$ for the same muon intensity $I_\mu(x_{\rm st})=I_\mu(x_{\rm GS})$. The muon intensities have been calculated with the value of γ which fit the LVD data well. The depth in standard rock can be evaluated from the depth in Gran Sasso rock using the formula

$$x_{\rm st}=-9.344+1.0063x_{\rm GS}+1.7835\times10^{-6}x_{\rm GS}^2 -5.7146\times10^{-11}x_{\rm GS}^3, \qquad (5)$$

where the depth is measured in hg/cm^2. This formula is valid for depth range 1–12 km w.e.: it has been used to convert the depth–muon-intensity relation measured in Gran Sasso rock to that in standard rock to allow comparison with the data of other experiments.

The depth–vertical-intensity relation in standard rock for the all muon sample is presented in Fig. 2. It can be fitted with a three parameter function

$$I_\mu(x)=A\left(\frac{x_0}{x}\right)^{\alpha}e^{-x/x_0}, \qquad (6)$$

where $A=(2.15\pm0.08)\times10^{-6}$ cm^{-2} s^{-1} sr^{-1}, $x_0=(1155^{+60}_{-30})$hg/cm^2.

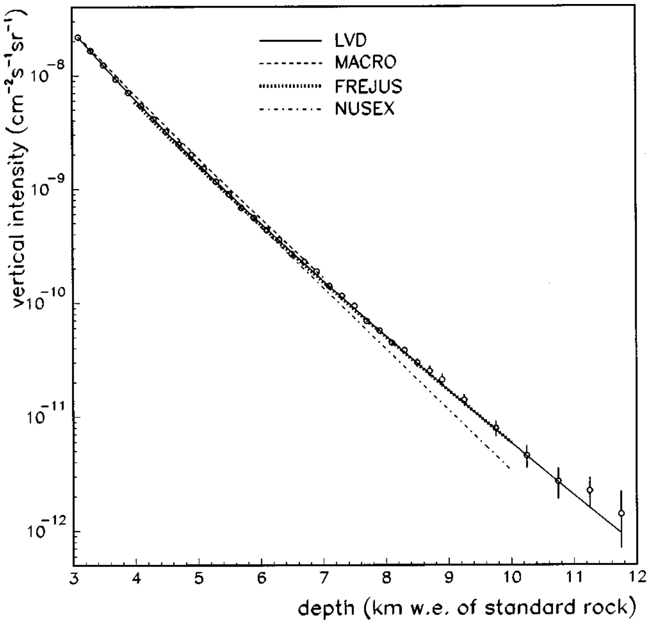

FIG. 2. Depth–vertical-muon-intensity curve in standard rock. LVD data are presented together with the best fit using three-parameter function [see Eq. (6)] and the best fits to the data of other experiments: MACRO [11] (dashed curve), Frejus [33] (dotted curve), and NUSEX [10] (dash-dotted curve).

The best fit function is also shown in Fig. 2 by the solid curve. The LVD data converted to standard rock agree quite well with the best fit functions for the data of MACRO [11] (dashed curve in Fig. 2) and Frejus [33] (dotted curve which almost coincides with the solid curve) underground experiments, despite the difference in the formulas used for depth conversion. The LVD data also do not contradict the function which fits the data of the NUSEX experiment [10] (dash-dotted curve). The muon intensities measured by LVD and converted to standard rock are also presented in Table I (3rd column).

V. MUON ENERGY SPECTRUM AT SEA LEVEL

Using Eq. (4) with the estimates of the free parameters, the LVD data for Gran Sasso rock can be converted to the vertical muon spectrum at sea level. A simple normalization procedure has been applied,

$$I^m_{\mu 0}(x=0,E_{m0})=\frac{I^m_\mu(x)I^c_{\mu 0}(x=0,E_{m0})}{I^c_\mu(x)}, \tag{7}$$

where $I^m_\mu(x)$ and $I^c_\mu(x)$ are the measured and calculated vertical muon intensities at depth x in Gran Sasso rock and $I^m_{\mu 0}(x=0,E_{m0})$ and $I^c_{\mu 0}(x=0,E_{m0})$ are the derived and calculated [using Eq. (4)] differential vertical muon intensities at energy E_{m0} at sea level. The median energies $E_{m0}(x)$ which determine the muon intensity at depth x have been calculated using the equation

$$\frac{I_\mu(x)}{2}=\int_0^{E_{m0}}P(E_{\mu 0},x)\frac{dI_{\mu 0}(E_{\mu 0})}{dE_{\mu 0}}dE_{\mu 0}, \tag{8}$$

TABLE II. Vertical muon spectrum at sea level derived from LVD data. Errors include both statistical and systematic uncertainties.

$E_{\mu 0}$ (GeV)	I_μ(cm^{-2} s^{-1} sr^{-1} GeV^{-1})
1932	$(1.30\pm0.01)\times10^{-11}$
2793	$(3.42\pm0.03)\times10^{-12}$
3909	$(9.72\pm0.11)\times10^{-13}$
5371	$(2.88\pm0.04)\times10^{-13}$
7246	$(9.73\pm0.16)\times10^{-14}$
9685	$(3.24\pm0.07)\times10^{-14}$
12770	$(1.15\pm0.04)\times10^{-14}$
16750	$(4.74\pm0.32)\times10^{-15}$
21980	$(1.62\pm0.22)\times10^{-15}$
28580	$(6.7\pm1.5)\times10^{-16}$
42660	$(1.6\pm0.5)\times10^{-16}$

where $dI_{\mu 0}(E_{\mu 0})/dE_{\mu 0}$ is the muon energy spectrum at sea level. The values of $I^m_{\mu 0}(x=0,E_{m0})$ derived from LVD data are presented in Table II. The derived (full circles) and calculated (middle solid curve) muon spectra at sea level are shown in Fig. 3 together with the data of MSU [7] (diamonds), ASD [4] (open circles), and the best fit of MACRO [11] (dashed line). The upper and lower solid curves in Fig. 3 represent the errors in the parameters and the additional 10% error in the absolute normalization of the muon flux. Muon intensities at sea level derived from LVD data are also presented in Table II.

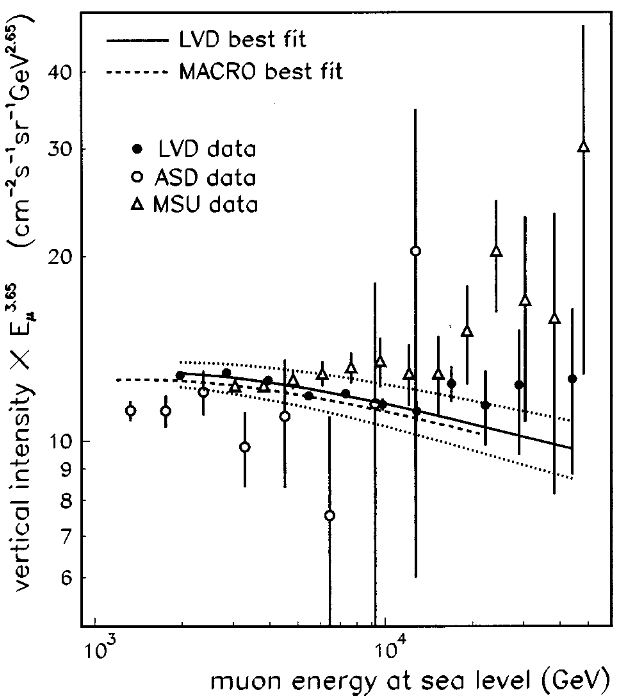

FIG. 3. Vertical muon energy spectra at sea level: ●: LVD data, ○: ASD data [4], ◇: MSU data [7], solid curves: LVD best fit together with the curves representing the errors of the parameters and of the absolute flux normalization, dashed curve: MACRO best fit [11].

VI. CONCLUSIONS

The angular distribution of muon intensity measured by LVD has been converted to the depth–vertical-muon-intensity relation. An analysis of this relation in the depth range 3000–12 000 hg/cm^2 has been done and the parameters of the muon spectrum at sea level have been obtained: $A = 1.9 \pm 1.0$, $\gamma = 2.78 \pm 0.05$. The errors include both statistical and systematic errors with the systematic error due to the uncertainty of the dominating muon interaction cross sections. A similar analysis performed for single muon events revealed almost the same power index value, while the absolute intensity is 10% smaller. The depth-intensity relation has been converted to standard rock and fitted with a three-parameter function. This relation agrees well with the data of other underground experiments. Using the measured depth–intensity curve and the estimated parameters of the muon spectrum at sea level we have derived the vertical muon energy spectrum at sea level.

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