SUSY AND PRECISION ELECTROWEAK DATA

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The present Standard Model fit of precision data has a low confidence level, and is characterized by a few inconsistencies. We look for supersymmetric effects that could improve the agreement among the electroweak precision measurements and with the direct lower bound on the Higgs mass. We find that this is the case particularly if the $3.6\sigma$ discrepancy between $\sin^2 \theta_{\text{eff}}$ from leptonic and hadronic asymmetries is finally settled more on the side of the leptonic ones. After the inclusion of all experimental constraints, our analysis selects light sneutrinos, with masses in the range $55-80$ GeV, and charged sleptons with masses just above their experimental limit, possibly with additional effects from light gauginos. The phenomenological implications of this scenario are discussed.

I report in this talk on our recent work [1] about possible SUSY effects in electroweak precision tests. The results of the electroweak precision tests as well as of the searches for the Higgs boson and for new particles performed at LEP and SLC have now been presented in a close to final form. Taken together with the measurements of $m_t$, $m_W$ and the searches for new physics at the Tevatron, and with some other data from low-energy experiments, they form a very stringent set of precise constraints to compare with the Standard Model (SM) or with any of its conceivable extensions. When confronted with these results, on the whole the SM performs rather well, so that it is fair to say that no clear indication for new physics emerges from the data. However, if we look at the results in detail, there are a number of features that are either not satisfactory or could indicate the presence of small new physics effects.

One problem is that the two most precise measurements of $\sin^2 \theta_{\text{eff}}$ from $A_{LR}$ and $A_{FB}^0$ differ by $3.5\sigma$ [2]. More in general, there appears to be a discrepancy between $\sin^2 \theta_{\text{eff}}$ measured from leptonic asymmetries and from hadronic asymmetries. The result from $A_{LR}$ is actually in good agreement with the leptonic asymmetries measured at LEP, while all hadronic asymmetries are better compatible with the result of $A_{FB}^0$. It is well known that this discrepancy is not likely to be explained by some new physics effect in the $b\bar{b}Z$ vertex. In fact $A_{FB}^0$ is the product of lepton and $b$-asymmetry factors: $A_{FB}^0 \propto A_f A_b$, where $A_f = 2g_A^f g_V^f/(g_A^f + g_V^f)$. The sensitivity of $A_{FB}^0$ to $A_b$ is limited, because the $A_f$ factor is small, so that, in order to reproduce the measured discrepancy, the new effect should induce a large change of the $b$ couplings with respect to the SM. But then this effect should be clearly visible in the direct measurement of $A_b$ performed at SLD using the LR polarized $b$ asymmetry, even within the moderate precision of this result, and it should also appear in the accurate measurement of $R_b \propto g_A^{b1} + g_V^{b1}$. Neither $A_b$ nor $R_b$ show deviations of the expected size. One concludes that most probably the observed discrepancy is due to a large statistical fluctuation and/or to an experimental problem. Indeed, the measurement of $A_{FB}^0$ not only requires $b$ identification, but also distinguishing $b$ from $\bar{b}$, and therefore the systematics involved are different than in the measurement of $R_b$. At any rate, the disagreement between $A_{FB}^0$ and $A_{LR}$ implies that the ambiguity in the measured value of $\sin^2 \theta_{\text{eff}}$ is larger than the nominal error obtained from averaging all the existing determinations.

Another point of focus is the relation between the fitted Higgs mass and the lower limit on this mass from direct searches, $m_H > 113$ GeV, as
it was recently stressed in ref. [3]. The central value of the fitted mass is systematically below the limit. In particular, given the experimental value of the top mass, the measured results for \( m_W \) (with perfect agreement between LEP and the Tevatron) and \( \sin^2 \theta_{eff} \) measured from leptonic asymmetries, taken together with the results on the \( Z_0 \) partial widths, push the central value of \( m_H \) very much down. In fact, if one arbitrarily excludes \( \sin^2 \theta_{eff} \) measured from the hadronic asymmetries, the fitted value of \( m_H \) becomes only marginally consistent with the direct limit, to a level that depends on the adopted value and the error for \( a_{QED} (m_Z) \). Consistency is reinstated if the results from hadronic asymmetries are also included, because they drive the fitted \( m_H \) value towards somewhat larger values.

In conclusion, if one takes all available measurements into account the \( \chi^2 \) of the SM fit is not good, with a probability of about 4%, partly because the measurements of \( \sin^2 \theta_{eff} \) are not in good agreement among them. If, on the other hand, one only takes the results on \( \sin^2 \theta_{eff} \) from the leptonic asymmetries, then the \( \chi^2 \) of the SM fit considerably improves, but the consistency with the direct limit on \( m_H \) becomes marginal.

In Ref. [1] we enlarged the discussion of the data from the SM to the Minimal Supersymmetric Standard Model (MSSM). We looked for regions of the MSSM parameter space where the corrections are sufficiently large and act in the direction of improving the quality of the fit and the consistency with the direct limit on \( m_H \) with respect to the SM, especially in the most unfavourable case for the SM that the results on \( \sin^2 \theta_{eff} \) from the hadronic asymmetries are discarded. We show that, if sleptons (and, to a lesser extent, charginos and neutralinos) have masses close to their present experimental limits, it is possible to considerably improve the overall picture. In particular, the possible MSSM effects become sizeable if we allow the sneutrino masses to be as small as allowed by the direct limits on \( m_{\tilde{\nu}} \) and by those on charged sleptons masses, which are related by \( m_{\tilde{\nu}_i}^2 = m_{\tilde{\tau}}^2 + m_W^2 |\cos 2\beta| \). At moderately large values of \( \tan \beta \) (i.e. for \( |\cos 2\beta| \sim 1 \)), light sneutrinos with masses as low as 55 GeV are not excluded by present limits, while charged sleptons must be heavier than 96 GeV. These low values of the sneutrino mass can still be compatible with the neutralino being the lightest supersymmetric particle. We recall that \( \tan \beta \gtrsim 2 - 3 \) is required by LEP, and large \( \tan \beta \) and light sleptons are indicated by the possible deviation observed by the recent Brookhaven result [4] on the muon \( g - 2 \), if this discrepancy is to be explained by a MSSM effect. We find it interesting that, by taking seriously the small hints that appear in the present data, one can pinpoint a region of the MSSM which match the data better than the SM, and is likely to be within reach of the present run of the Tevatron and, of course, of the LHC.

For this analysis in the MSSM we used the technique of the epsilon parameters \( \epsilon_1, \epsilon_2, \epsilon_3 \) and \( \epsilon_6 \), introduced in ref. [5]. The variations of \( \epsilon_1, \epsilon_2 \) and \( \epsilon_3 \) due to new physics contributions are proportional to the shifts in the \( T, U, \) and \( S \) parameters [6], respectively, if one keeps only oblique contributions (i.e. terms arising from vacuum polarization diagrams), expanded up to the first power in the external momentum squared. But in the MSSM not all important contributions are of this kind. We recall that the starting point of the epsilon analysis is the unambiguous definition of the \( \epsilon_i \) in terms of four basic observables that were chosen to be \( \sin^2 \theta_{eff} \) from \( A_{FB}, \Gamma_{\mu}, m_W \) and \( R_0 \). Given the experimental values of these quantities, the corresponding experimental values of the \( \epsilon_i \) follow, independent of \( m_t \) and \( m_H \), with an error that, in addition to the propagation of the experimental errors, also includes the effect of the present ambiguities in \( \alpha_s (m_Z) \) and \( a_{QED} (m_Z) \).

If one assumes lepton universality, which is well supported by the data within the present accuracy, then the combined results on \( \sin^2 \theta_{eff} \) from all leptonic asymmetries can be adopted together with the combined leptonic partial width \( \Gamma_\ell \). At this level the epsilon analysis is model-independent within the stated lepton universality assumption. As a further step we can observe that by including the information on the hadronic widths arising from \( \Gamma_Z, \sigma_h, R_\ell \), the central values of the \( \epsilon_i \) are not much changed (with respect to the error size) and the errors are slightly de-
creased. Thus one may decide of including or not including these data in the determination of the $\epsilon_i$, without affecting the results.

Different is the case of including the results from the hadronic asymmetries in the combined value of $\sin^2 \theta_{eff}$. In this case, obviously, the determination of $\epsilon_i$ is sizeably affected and one remains with the alternative between an experimental problem or a bizarre effect of some new physics in the $b$ coupling (not present in the MSSM). But if we remain within the first stage of purely leptonic measurements plus $m_W$ and $R_b$, the $\epsilon_i$ analysis is quite general and, in particular, is independent of an assumption of oblique correction dominance.

The comparison with the SM can be repeated in the context of the $\epsilon_i$ (see Fig. 1). The predicted theoretical values of the $\epsilon_i$ in the SM depend on $m_H$ and $m_t$, while they are practically independent of $a_t(m_Z)$ and $a_{QED}(m_Z)$. If we only take the leptonic measurements of $\sin^2 \theta_{eff}$, for $m_H = 113$ GeV and $m_t = 174.3$ GeV one finds that the experimental value of $\epsilon_1$ agrees within the error with the prediction, while both $\epsilon_2$ and $\epsilon_3$ are below the theoretical expectation by about 1 $\sigma$. We recall that $m_W$ is related to $\epsilon_2$ and the fact that the experimental value is below the prediction for this quantity corresponds to the statement that $m_W$ would prefer a value of $m_H$ much smaller than $m_W = 113$ GeV. Similarly the smallness of the fitted value of $\epsilon_3$ with respect to the prediction has to do with the marked preference for a light $m_H$ of $\sin^2 \theta_{eff}$ from all leptonic asymmetries. The agreement between fitted value and prediction for $\epsilon_1$, which, contrary to $\epsilon_2$ and $\epsilon_3$, contains a quadratic dependence on $m_t$, reflects the fact that the fitted value of $m_t$ is in agreement with the measured value. The other variable that depends quadratically on $m_t$ is $\epsilon_3$. The agreement of the fitted and predicted values of $\epsilon_3$ reflects the corresponding present normality of the results for $R_b$.

Now we want to investigate whether low-energy supersymmetry can reconcile a Higgs mass above the direct experimental limit with a good $\chi^2$ fit of the electroweak data, in the case of $\sin^2 \theta_{eff}$ near the value obtained from leptonic asymmetries. Our approach is to discard the measure-

![Figure 1](image-url)

Figure 1. One-sigma ellipses in the $\epsilon_3 - \epsilon_2$ (left) and in the $\epsilon_1 - \epsilon_3$ (right) planes obtained from: a. $m_W$, $\Gamma_t$, $\sin^2 \theta_{eff}$ from all leptonic asymmetries, and $R_b$; b. the same observables, plus the hadronic partial widths derived from $\Gamma_Z$, $\sigma_b$ and $R_b$; c. as in b., but with $\sin^2 \theta_{eff}$ also including the hadronic asymmetry results. The solid straight lines represent the SM predictions for $m_H = 113$ GeV and $m_t$ in the range 174.3 ± 5.1 GeV. The dotted curves represent the SM predictions for $m_t = 174.3$ GeV and $m_H$ in the range 113 to 500 GeV.
mement of $A_{FB}$, which cannot be reproduced by conventional new physics effects, fix the Higgs mass above its present limit, and look for supersymmetric corrections that can fake a very light SM Higgs boson. As we have discussed in the previous section and as summarized in fig. 1, this can be achieved if the new physics contributions to the $\epsilon$ parameters amount to shifting $\epsilon_2$ and $\epsilon_3$ down by slightly more than $1\,\sigma$, while leaving $\epsilon_1$ essentially unchanged.

Squark loops cannot induce this kind of shifts in the $\epsilon$ parameters, since their leading effect is a positive contribution to $\epsilon_1$. Thus, we will assume that all squarks are heavy, with masses of the order of one TeV. Since the mass of the lightest Higgs $m_H$ receives a significant contribution from stop loops, we can treat $m_H$ as an independent parameter and, in our analysis, we fix $m_H = 113$ GeV. Varying the pseudoscalar Higgs mass $m_A$ does not modify the results of our fit, and therefore we fix $m_A = 1$ TeV. The choice of the right-handed slepton mass has also an insignificant effect on the fit. Therefore, we are left with four relevant supersymmetric free parameters: the weak gaugino mass $M_2$, the higgsino mass $\mu$, the ratio of the Higgs vacuum expectation values $\tan\beta$ (which are needed to describe the chargino–neutralino sector), and a supersymmetry-breaking mass for the left-handed sleptons, $\tilde{m}_{\mu_\ell}$ (lepton flavour universality is assumed). The choice of the $B$-ino mass parameter $M_1$ does not significantly affect our results and, for simplicity, we have assumed the gaugino unification relation $M_1 = \frac{1}{2} M_2 \tan^2 \theta_W$.

As described in Ref. [1], we have computed the supersymmetric one-loop contributions to $\epsilon_1$, $\epsilon_2$, and $\epsilon_3$. Figure 2 shows the range of the $\epsilon$ parameters that can be spanned by varying $M_2$, $\mu$, $\tan\beta$, and $\tilde{m}_{\mu_\ell}$, consistently with the present experimental constraints. We have imposed a limit on charged slepton masses of 96 GeV, on chargino masses of 103 GeV, and on the cross section for neutralino production $\sigma(e^+\nu_e + \nu_\mu \to \chi_1^0 \chi_2^0 \to \mu^+\nu^-)$ up to 0.1 pb. We have also required that the supersymmetric contribution to the mu anomalous magnetic moment, $a_\mu = (g-2)/2$, lie within the range $0 < \delta a_\mu < 7.5 \times 10^{-9}$. As apparent from fig. 2, light particles in the chargino–neutralino sector and light left-handed sleptons shift the values of $\epsilon_1$ in the favoured direction, and by a sufficient amount to obtain a satisfactory fit.

In fig. 3 we show an alternative presentation of our results directly in terms of the shifts in the observables $m_W$, $\sin^2 \theta_{eff}$ and $\Gamma_\ell$ induced by supersymmetry.\footnote{A good approximation of the relations between shifts in the physical observables and in the $\epsilon$ parameters is given by $\delta m_W = (0.33 \delta \epsilon_1 - 0.37 \delta \epsilon_2 - 0.32 \delta \epsilon_3) \times 10^8$ MeV; $\delta \Gamma_\ell = (1.01 \delta \epsilon_1 - 0.22 \delta \epsilon_2) \times 10^{3}$ keV; $\delta \sin^2 \theta_{eff} = -0.33 \delta \epsilon_1 + 0.43 \delta \epsilon_2$.} For reference, we also display in fig. 3 the difference between the measured values of the observables (excluding the hadronic asymmetries) and the corresponding SM predictions for $m_H = 113$ GeV, $m_t = 174.3$ GeV. Supersymmetric contributions can bring the theoretical predictions in perfect agreement with the data. An interesting observation is that particle effects can increase $m_W$ by $\delta m_W$ up to $\sim 100$ MeV, which corresponds to approximately three standard deviations, and decrease $\sin^2 \theta_{eff}$ by $\delta \sin^2 \theta_{eff}$ up to about $-8 \times 10^{-4}$ ($\sim 4\,\sigma$). Note the marked anticorrelation between $\delta m_W$ and $\delta \sin^2 \theta_{eff}$. $\Gamma_\ell$ is moved upwards, but only by less than 90 keV, or about 1 $\sigma$.

To summarize, the request of an improved electroweak data fit is making precise demands on the supersymmetric mass spectrum. The left-handed charged sleptons have to be very close to their experimental bounds, the sneutrino mass is selected to be below about 80 GeV, the squarks are in the TeV range, and $\tan \beta \gtrsim 4$, while there is no information on right-handed slepton masses. The lightest chargino, preferably a gaugino state with mass below about 150 GeV, further improves the fit. This range of supersymmetric parameters is very adequate in explaining the alleged discrepancy between the experimental and theoretical values of the mu anomalous magnetic moment [4]. In practice, requiring the supersymmetric contribution to $g-2$ to be in the range indicated by the data amounts to determining a precise value of $\tan \beta$ and selecting a sign (positive in our conventions) of the parameter $\mu$. We recall that, for moderately large $\tan \beta$, the negative sign of $\mu$ is disfavoured by the present measurements of the $B \to X_s \gamma$ branching ratio.
Figure 2. Measured values (cross) of $\epsilon_3$ and $\epsilon_2$ (left) and of $\epsilon_1$ and $\epsilon_3$ (right), with their 1 $\sigma$ region (solid ellipses), corresponding to case a of fig. 1. The area inside the dashed curves represents the MSSM prediction for $m_{\tilde{\ell}_L}$ between 96 and 300 GeV, $m_{\tilde{\chi}^+}$ between 105 and 300 GeV, $-1000$ GeV < $\mu$ < $1000$ GeV, $\tan\beta = 10$, $m_{\tilde{\ell}_L} = 1$ TeV, and $m_A = 1$ TeV.

Figure 3. The area inside the dotted curves represents the shifts in the values of $\sin^2 \theta_{\text{eff}}$, $m_W$ and $\Gamma_t$ induced by supersymmetric corrections, for the same parameter region as in fig. 2. The shifts necessary to reproduce the central values of the data with $m_{\tilde{\chi}} = 174.3$ GeV and $m_H = 113$ GeV are also shown, together with the corresponding experimental errors. The dot-dashed lines are obtained by varying the left slepton masses, with all other supersymmetric particle decoupled. The solid curve is obtained analogously, but also keeping a gaugino-like chargino of 105 GeV. In each curve, the circles correspond to $m_{\tilde{\chi}} = 60, 70, 80$ GeV from left to right.
REFERENCES

2. LEP Electroweak Working Group, LEPEWWG/2001-01.