3rd Bologna Workshop on

Conformal Field Theories and Integrable Models

Bologna, 1-4 October 1997

LIST OF PARTICIPANTS

1. **ANDREI** Natan (U. Rutgers - USA) Nat. USA
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7. **BEHREND** Roger (U. Bonn - Germany) Nat. Australia
8. **BERNARD** Denis (CEA Saclay - France) Nat. France
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19. **FEVERATI** Giovanni (U. Bologna - Italy) Nat. Italy
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22. **FORD** Christopher (U. Jena - Germany) Nat. UK
23. **FRAPPAT** Luc (ENSLAPP, Annecy - France) Nat. France
24. **GANNON** Terry (U. York, Canada) Nat. Canada
25. **GUIDA** Riccardo (CEA Saclay - France) Nat. Italy
26. **GURUSWAMY** Sathya (Sissa, Trieste - Italy) Nat. India
27. **HOLLOWOOD** Timothy (U. Swansea - UK) Nat. UK
28. **HONECKER** Andreas (Sissa, Trieste - Italy) Nat. Germany
29. **KONKOLI** Zoran (U. Goteborg - Sweden) Nat. Croatia
30. **LUDWIG** Andreas (U. California, S. Barbara - USA) Nat. Germany
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32. **MAGNOLI** Nicodemo (U. Genova - Italy) Nat. Italy
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37. **MUSSARDO** Giuseppe (Sissa, Trieste - Italy) Nat. Italy
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39. **NAKAYASHIKI** Atsushi (U. Kyushu - Japan) Nat. Japan
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59. **VALLERIANI** Angelo (U. Dresden - Germany) Nat. Italy
60. **VAN ELBURG** Ronald (U. Amsterdam - Netherlands) Nat. Netherlands
61. **VON GEHLEN** Guenter (U. Bonn - Germany) Nat. Germany
62. **WALTON** Mark (DAMTP, Cambridge - UK) Nat. Canada
63. **WATTS** Gerard (King's College, London - UK) Nat. UK
64. **WESTON** Robert (U. Durham - UK) Nat. UK
65. **ZAUGG** Philippe (ENSLAPP, Annecy - France) Nat. Switzerland
66. **ZUCCHINI** Roberto (U. Bologna - Italy) Nat. Italy

TENTATIVE TIME TABLE OF TALKS

WEDNESDAY MORNING

Chairman: **Prof.Werner NAHM**

9:00-9:20	Opening
9:20-10:10	**RITTENBERG Vladimir** *Phase Transitions in one-dimensional steady states*
10:20-11:10	**SORBA Paul** *Yangian realisations from finite W-algebras*
11:10-11:40	*Coffee Break*
11:40-12:05	**FRAPPAT Luc** *Poisson structures on the center of the elliptic algebra* $A_{p,q}(sl(2)_c)$
12:05-12:30	**PETKOVA Valentina** *Fusion rules for admissible representations of* $\widehat{sl}(3)$
12:30-13:30	**Lunch Time**
13:30-15:20	**Posters and free discussion**

WEDNESDAY AFTERNOON

Chairman: **Prof.Lazlo PALLA**

15:20-16:10	**LUKYANOV Sergei** *Angular quantization and form-factors in massive integrable models*
16:10-16:35	**BORDNER Andrew** *Quantum Commuting Charges of Generalized Boussinesq Theory*
16:35-16:55	*Tea Time*

16:55-17:20 **NAKAYASHIKI Atsushi**
On Relations of Three Integral Formulas for the Solutions of the qKZ Equation at Level 0

17:20-17:45 **WESTON Robert**
The monodromy matrices of the XXZ model in the infinite volume limit

17:55-18:20 **PILLIN Mathias**
Large Form factors in ADE affine Toda field theories

18:20-18:45 **VON GEHLEN Guenter**
Spectrum of the superintegrable Z_N-Chiral Potts quantum chain with twisted boundary conditions

18:45-19:10 **WATTS Gerard**
Conserved charges in the chiral 3-state Potts model

THURSDAY MORNING

Chairman: **Prof.Vladimir RITTENBERG**

9:20-10:10 **GANNON Terry**
A conjectured list of all possible WZNW partition functions

10:20-11:10 **SAGNOTTI Augusto**
Completeness Conditions for Boundary Operators

11:10-11:40 *Coffee Break*

11:40-12:05 **BEHREND Roger**
The Boundary Yang-Baxter Equation

12:05-12:30 **SCHILLING Anne**
The Bailey construction

12:30-13:30 **Lunch Time**

13:30-15:20 **Posters and free discussion**

THURSDAY AFTERNOON

Chairman: **Prof.Natan ANDREI**

15:20-16:10 **MUSSARDO Giuseppe**
Non-integrable Aspects of Quantum Field Theories

16:10-16:35 **EHOLZER Wolfgang**
Conformal Characters and Modular Units

16:35-16:55 *Tea Time*

16:55-17:20 **FORD Christopher**
Construction of the Liouville Theory via a Canonical Transformation

17:20-17:45 **TESHNER Joerg**
On the conformal bootstrap for the SL(2,R) WZNW model

17:55-18:20 **APYKIAN Samvel**
Liouville Theory in Z_N-Surface

18:20-18:45 **PRATA Joao N.G.N.**
The Super-Liouville-Equation on the Half-Line

20:15-24:00 *Crescentinata*

FRIDAY MORNING

Chairman: **Prof.Augusto SAGNOTTI**

9:20-10:10 **LUDWIG Andreas**
Impurity Problems in Condensed Matter Physics

10:20-11:10 **ANDREI Natan**
Integrable Models in Condensed Matther Physics

11:10-11:40 *Coffee Break*

11:40-12:05 **ARNAUDON Daniel**
New approach to the Hubbard model with pair hopping

12:05-12:30 **VAN ELBURG Ronald**
Quasi-particles in Fractional Quantum Hall Effect Edge Theories

12:30-13:30 **Lunch Time**

13:30-15:20 **Posters and free discussion**

FRIDAY AFTERNOON

Chairman: **Prof.Paul SORBA**

15:20-16:10 **MIRAMONTES Luis**
New Integrable perturbations of CFT's from non-abelian Affine Toda Equations

16:10-16:35 **TAKACS Gabor**
Scattering amplitudes in imaginary coupled $g_2^{(1)}$ *and* $d_4^{(3)}$ *Toda field theories*

16:35-16:55 *Tea Time*

16:55-17:20 **MORICONI Marco**
Ising Model with Random Boundary Field

17:20-17:45 **PEARCE Paul**

TBA Equations, Classification of Excited States and Massless Flow from the Tricritical to Critical Ising Model

17:55-18:20 **TIRKKONEN Olav**
Excited States in the twisted XXZ spin chain

18:20-18:45 **MAGNOLI Nicodemo**
Correlation functions near a critical point

18:45-19:10 **CARACCIOLO Raffaele**
New geometrical properties of deformed conformal field theories

SATURDAY MORNING

Chairman: **Prof.Edward CORRIGAN**

9:20-10:10	**NAHM Werner** *Electric-magnetic duality and modular forms*
10:20-11:10	**HOLLOWOOD Timothy** *Integrable Systems, non-critical strings and the Dynamics of N=2 Gauge Theories*
11:10-11:40	*Coffee Break*
11:40-12:05	**PUJOL Pierre** *Magnetization Plateaux in Antiferromagnetic N-Leg Ladders*
12:05-12:30	**PALLA Lazlo** *Quantum equivalence of dually related sigma models*

End (12:30)

PLENARY TALKS

ANDREI Natan

U. Rutgers - USA

Integrable Models in Condensed Matther Physics

GANNON Terry

U. York, Canada

A conjectured list of all possible WZNW partition functions

The genus 1 partition function of a WZNW CFT will be a sesquilinear combination of Kac-Moody characters obeying certain properties such as modular invariance. Progress on the problem of classifying these has been slow but steady, and finally we can now reliably conjecture the final list, corresponding to any simple Lie algebra A_r, B_r, ..., G_2. We can realistically hope to prove the completeness of this list within the next couple years. In my talk I will review the problem, discuss its current status, describe the conjectured list, and introduce you to some of its curiousities (e.g. unexpected appearances of the A-D-E pattern, of Fermat curves, and of Grassmannians).

The following papers hint at what will be covered in the talk:

[1] Cappelli, Itzykson, Zuber: Commun. Math. Phys. 113 (1987), 1-26.
[2] Gannon: Invent. Math. 122 (1995), 341-357.
[3] Gannon, Ruelle, Walton: Commun. Math. Phys. 179 (1996), 121-156.
[4] Gannon: q-alg/9510026.
[5] Bauer, Coste, Itzykson, Ruelle: hep-th/9604104.

HOLLOWOOD Timothy

U. Swansea - UK

**Integrable Systems, non-critical strings
and the Dynamics of N=2 Gauge Theories**

I will discuss how the strong coupling dynamics of N=2 gauge theories with arbitrary gauge groups are described by Riemann surfaces derived from integrable systems. Particular attention will be paid to the physical mechanism which drives the coupling; namely massless dyons. I will then explain that, viewed as part of a string theory, the dyons arise as special wrappings of a non-critical string.

LUDWIG Andreas

U. California, S. Barbara - USA

Impurity Problems in Condensed Matter Physics

LUKYANOV Sergei

U. Cornell - USA

Angular quantization and form-factors in massive integrable models

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MIRAMONTES Luis

U. Santiago de Compostela - Spain

New Integrable perturbations of CFT's from non-abelian Affine Toda Equations

We summarize recent work aimed to the construction of new integrable quantum field theories whose equations of motion are related to the non-abelian version of the affine Toda equations, and which are described by a unitary action.

The non-abelian affine Toda (NAAT) equations are integrable generalizations of the sine-Gordon equations of motion for a bosonic field that takes values in some, generally non-abelian, Lie group. However, it is well known that not all these equations can be derived by extremizing an action with sensible properties like a positive-definite kinetic term and a real potential.

In the first part of the talk, we show that NAAT equations give rise to just two series of models referred to as the Homogeneous sine-Gordon (HSG) and Symmetric Space sine-Gordon (SSSG) theories [1], in addition to the well known abelian affine Toda theories with real coupling constant. Both of them are integrable, admit soliton solutions, exhibit a mass-gap, and have a real positive-definite action. These theories are of the form of a (gauged) Wess-Zumino-Witten (WZW) action plus a certain potential that deforms the theory away from the critical point along an integrable direction. This indicates their natural description as integrable perturbations of $c > 1$ CFT's associated with cosets of the form $G_k/U(1)^{\times p}$, where G is a semi-simple Lie group and p is an integer number ≥ 0 and $\leq \mathrm{rank}(G)$.

Both the HSG and the SSSG theories exhibit remarkable features as integrable field theories. For instance, they are integrable for any value of their continuous coupling constants, which makes them counter-examples to the generally held belief that integrability is a very special property which requires carefully tuned couplings. Moreover, their spectrum includes both stable and unstable particles; a peculiar feature shared with the spectrum of monopoles and dyons in $N = 2$ and $N = 4$ supersymmetric gauge theories.

References:

[1] C.R. Fernández-Pousa, M.V. Gallas, T.J. Hollowood, and J.L. Miramontes, Nucl. Phys. **B 484** (1997) 609.

[2] C.R. Fernández-Pousa and J.L. Miramontes, *Semi-classical spectrum of the Homogeneous sine-Gordon theories*, hep-th/9706203.

[3] C.R. Fernández-Pousa, M.V. Gallas, T.J. Hollowood, and J.L. Miramontes, *Solitonic Integrable Perturbations of Parafermionic Theories*, hep-th/9701109, to appear in Nucl. Phys. **B**.

MUSSARDO Giuseppe

Sissa, Trieste - Italy

Non-integrable Aspects of Quantum Field Theories

Many interesting statistical models in the vicinity of their fixed point may be described by non-integrable QFT. These are theories with striking phenomena of resonances, multi-particle production and a non-trivial spectrum of excitations. Aim of the talk is to illustrate the rich variety of non-integrable models and the way to understand it.

The QFT of a scalar field self-interacting via two periodic terms of frequencies α and β will be discussed. Looking at the theory as a perturbed Sine-Gordon model, Form Factor Perturbation Theory can be used to analyse the evolution of the spectrum of particle excitations. Within this formalism, the non-locality of the perturbation with respect to the solitons results responsible for their confinement in the non-integrable theory. The effects of the frequency ratio α/β being a rational or irrational number and the occurrence of massless flows from the gaussian to the Ising fixed point are also discussed. A two-layer Ising model, alias a generalisation of the Ashkin-Teller model, and the massive Schwinger model are presented as explicit examples of non-integrable theories.

References:

[1] G. Delfino, G. Mussardo and P. Simonetti, *Non-integrable Quantum Field Theories as Perturbations of Certain Integrable Models*, Nucl. Phys. B **473** (1996), 469.

[2] G. Delfino and G. Mussardo, *Non–Integrable Aspects of the Multi-frequency Sine-Gordon Model*, hep-th/9709028.

NAHM Werner

U. Bonn - Germany

Electric-magnetic duality and modular forms

RITTENBERG Vladimir

U. Bonn - Germany

Phase Transitions in one-dimensional steady states

First we give an example of CP spontaneous symmetry breaking. In the broken phase the free energy functional is not a convex function of the order parameter but looks like a standard Ginsburg-Landau picture. If a symmetry breaking term is introduced, the Ginsburg-Landau picture remains and one obtains spinoidal points. The second example deals with the spontaneous breaking of translational invariance. The results described are obtained by Monte-Carlo simulations and by using quadratic algebras. We explain the latter technique.

SAGNOTTI Augusto

U. Roma II - Italy

Completeness Conditions for Boundary Operators in 2D Conformal Field Theory

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SORBA Paul

ENSLAPP, Annecy - France

Yangian realisations from finite W-algebras

It is possible to show that a large class of finite W algebras satisfy the defining relations of the Yangian algebras Y (sl(n)). Such a connection between two,a priori different,types of symmetry occuring in integrable systems deserves (in my opinion) some interest. Technically,it allows to construct,when it exists,product of representations,and to classify finite dimensional irreducible representations of W algebras.

COMMUNICATIONS

APIKYAN Samvel

U. Yerevan - Armenia

Boundary Quantum Liouville Field Theory

BLFT is considered. Boundary three-point correlation function are calculated for the Liouville vertex operators. Based on these results explicit expressions are found for accessory parameters. Also studied uniformization problem of Riemann surface with boundary.

ARNAUDON Daniel

ENSLAPP, Annecy - France

New approach to the Hubbard model with pair hopping

We construct two (non-trivially) equivalent quantum spin chains Hamiltonians with quantum $sl(1|2)$ invariance. These spin chains are generalized Hubbard models which describe electron model with pair hoppings. An algebra with less relations than the Birman–Wenzl–Murakami algebra allows exact solvability of the periodic chain. Reflection equations are solved to get exact solvability on open chains.

BEHREND Roger

U. Bonn - Germany

The Boundary Yang-Baxter Equation

A two-dimensional model in statistical mechanics can be considered as solvable with periodic boundary conditions if its Boltzmann weights satisfy the Yang-Baxter equation. In this talk, I shall show that such a model can also be considered as solvable with certain non-periodic boundary conditions if the so-called boundary Yang-Baxter equation is satisfied.

References:

[1] R. E. Behrend, P. A. Pearce and D. L. O'Brien 'Interaction-Round-a-Face Models with Fixed Boundary Conditions: The ABF Fusion Hierarchy' J. Stat. Phys. 84 (1996) 1-48

[2] R. E. Behrend and P. A. Pearce 'A Construction of Solutions to Reflection Equations for Interaction-Round-a-Face Models' J. Phys. A 29 (1996) 7827-7835

[3] R. E. Behrend and P. A. Pearce 'Boundary Weights for Temperley-Lieb and Dilute Temperley-Lieb Models' To appear in Int. J. Mod. Phys. B (1997)

BORDNER Andrew

U. Kyoto - Japan

Quantum Commuting Charges of Generalized Boussinesq Theory at $c = -2$ from Reduction of W_∞.

Integrability of the quantum Boussinesq equation for $c = -2$ is demonstrated by giving a recursive algorithm for generating explicit expressions for the infinite number of commuting charges. These charges exist for all spins $s \geq 2$. This same method may also be used to find the commuting charges for quantum field theories whose fields are currents of W_N for $N \geq 3$. The quantum commuting charges for the associated $A_N^{(1)}$ affine Toda field theory are also presented.

CARACCIOLO Raffaele

U. Torino - Italy

New geometrical properties of deformed conformal field theories

In the framework of the thermodynamic Bethe Ansatz approach to the renormalization group flow of integrable (1+1)-dimensional quantum field theories it is possible to encode all dynamical information in the so called Y-system, a set of coupled algebraic functional equations. The solution of this system admits a beautiful geometric interpretation in terms of triangulations of certain 3D manifolds. By using the properties of these triangulations one is able to prove the Rogers' dilogarithm identities that lead to the central charge of the related stable fixed points.

EHOLZER Wolfgang

DAMTP, Cambridge - UK

Conformal Characters and Modular Units.

It is a well-known phenomenon that occasionally the conformal characters of rational conformal field theories (RCFTs) have simple product expansions. We investigate this systematically and explain why such product expansions, if they exist at all, always have a certain simple form.

The main observation for explaining the existence of such product expansions is that in many cases the conformal characters of RCFTs in two dimensions are not only modular functions, but even modular units, and that modular units with integral Fourier coefficients always have product expansions of a very simple type.

A modular unit is a modular function on some congruence subgroup of $\mathrm{SL}(2, \mathbf{Z})$ which has no poles or zeros in the upper half plane. Thus it takes on all its poles and zeros in the cusps. The set of all modular units is obviously a group with respect to the usual multiplication of modular functions.

It is believed that the conformal characters of rational models are modular functions on some congruence subgroup. If this were true, then every conformal character having no zeros or poles in the upper half plane would be a modular unit.

In the context of conformal characters, it is natural to look at the subgroup E of modular units whose Fourier expansions are of the form $q^s\mathbf{Z}[\![q]\!]$ with rational numbers s. It can be shown that a modular unit contained in E is always of the form $\alpha(1+q\mathbf{Z}[\![q]\!])$ for some $\alpha \in \mathbf{Q}$ [2]. Thus conformal characters which are modular units are, up to multiplication by a positive integer, elements of the semigroup E^+ of all units in E which have non-negative Fourier coefficients.

Special modular units in E^+ are the functions

$$[r]_l = q^{\frac{-l\mathbf{B}_2}{(rl)/2}}, \quad n \equiv +r \bmod l, \qquad n > 0$$

$$(1-q^n)^{-1} \qquad n \equiv -r \bmod l, \qquad n > 0$$

$$(1-q^n)^{-1} \qquad (l \geq 1, (r,l) = 1)$$

where $\mathbf{B}_2(x) = y^2 - y + 1/6$ with $y = x - \lfloor x \rfloor$.

In [2] it was proved that the group E is generated by the functions $[r]_l$ (up to multiplication by constants in $\mathbf{Z}$). In other words, a conformal character χ is a modular unit, if and only if χ can be written as a product of integral powers of the functions $[r]_l$ (times a constant). This explains the extremely simple form of the product expansions of conformal characters which are modular units.

A closer look shows that in many cases even all conformal characters of a RCFT are modular units, we call such RCFTs *modular*. Several examples of modular RCFTs have been described in [1].

Even more, in all but one of such examples which are known to us, the conformal characters can be written as products of positive integral powers of functions $[r]_l$. Let E_* be the semigroup of all such products, i.e. the semigroup generated by the functions $[r]_l$.

Modular sets, *i.e.* finite subsets of $\mathbf{Z}_{>0} \cdot E^+$ which span an $\mathrm{SL}(2,\mathbf{Z})$-module, contained in E_* have been studied in [2] where a certain finiteness property has been shown. Furthermore, an algorithm was presented in *loc.cit.* which allows to compute all modular sets in $E_n(l)$, the set of all modular units in E_* consisting of at most n-fold products of the functions $[1]_l, \ldots, [\lfloor l/2 \rfloor]_l$. For $n \leq 10$ we give a complete list of all modular subsets of $E_n(l)$ [3]. In all but three cases these modular sets turn out to be related to modular RCFTs.

breferences:

[1] W.Eholzer, N.-P.Skoruppa, *Product Expansions of Conformal Characters*, Phys. Lett. B388 (1996), 82-89 (**hep-th/9607165**)

[2] W.Eholzer, N.-P.Skoruppa, $\mathrm{SL}(2,\mathbf{Z})$ – *Invariant Spaces Spanned by Modular Units*, preprint DAMTP-96-62 (**q-alg/9704004**)

[3] W.Eholzer, N.-P. Skoruppa, in preparation.

FORD Christopher

U. Jena - Germany

Construction of the Liouville Theory via a Canonical Transformation

We give an alternative generating functional for a canonical transformation mapping the Liouville theory to a free field. This new generating functional is more complicated than the standard form, but is parity-invariant and generalizes to arbitrary dimensions. The qunatization of this transformation is discussed briefly. It is well known that the two-dimensional Liouville field theory defined by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 - m^2 e^{\beta\Phi} \tag{1}$$

is equivalent to a massless free theory at the classical and quantum level. In ref [1] Braaten, Curtright and Thorne gave a time-independent canonical transformation (CT) which exchanges the Liouville and free theories. It was demonstrated that this transformation may be derived from the following remarkably simple generating functional

$$\mathcal{F} = \int_{-L}^{L} dx \left[\phi'(x)\Phi(x) - \frac{2m}{\beta^2} e^{\beta\Phi(x)} \sinh \beta\phi(x)\right], \tag{2}$$

x being the spatial coordinate and $-L \leq x \leq L$. A strange feature of [1] is the parity violating nature of the CT. The generating functional is not invariant under parity, and this manifests itself as a pseudoscalar conformal improvement term in the free theory Hamiltonian density. An alternative generating functional to (2) is presented which is invariant under parity, and which immediately generalizes to higher dimensions. The two dimensional case generates a canonical transformation first given by D'Hoker and Jackiw [2].

References:

[1] E. Braaten, T. Curtright and C. Thorn, Annals of Physics (1982) 147 365.

[2] E. D'Hoker and R. Jackiw, Phys. Rev. D, Phys. Rev. D26 (1982) 3517.

FRAPPAT Luc

ENSLAPP, Annecy - France

Poisson structures on the center of the elliptic algebra $A_{p,q}(sl(2)_c)$

It is shown that the elliptic algebra $A_{p,q}(sl(2)_c)$ has a non trivial center at the critical level $c = -2$, generalizing the result of Reshetikhin and Semenov-Tian-Shansky for trigonometric algebras. A family of Poisson structures indexed by a nonnegative integer k is constructed on this center. We show also that $t(z)$ close an exchange algebra when $p^m = q^{c+2}$ for $m \in Z$, they commute when in addition $p = q^{2k}$ for k non-zero integer, and they belong to the center of $\mathcal{A}_{q,p}(\widehat{sl}(2)_c)$ when k is odd. The Poisson structures obtained for $t(z)$ in these classical limits contain the q-deformed Virasoro Poisson algebra, characterizing the structures at $p \neq q^{2k}$ as new $\mathcal{W}_{q,p}(sl(2))$ algebras.

[1] *Poisson structures on the center of the elliptic algebra $\mathcal{A}_{q,p}(sl(2)_c)$*,, J. Avan, L. Frappat, M. Rossi, P. Sorba, preprint ENSLAPP-AL-644/97, CRM-2474, PAR-LPTHE 97-17, q-alg/9705012.

[2] *New $\mathcal{W}_{q,p}(sl(2))$ algebras from the elliptic algebra $\mathcal{A}_{q,p}(\widehat{sl}(2)_c)$*,, J. Avan, L. Frappat, M. Rossi, P. Sorba, preprint ENSLAPP-AL-649/97, CRM-2482, PAR-LPTHE 97-22, q-alg/9706013.

MAGNOLI Nicodemo

U. Genova - Italy

Correlation functions near a critical point

We consider the short distance behavior of correlators near a critical point. It is well known that expansions in relevant parameters near a critical point are plagued by infrared divergences. We show how it is possible to reformulate the perturbation expansion in such a way to avoid this problem. The main idea of the method is based on the OPE. This approach leaves unfixed some constants related to the vacuum expectation values of some operators. We propose to determine numerically these Vevs by using a variational approach known as Truncated Conformal Approach.

MORICONI Marco

ICTP - Trieste - Italy

Ising Model with Random Boundary Field

We study the 2d Ising Model with random boundary magnetic field via replica trick. After arguing its integrability, we use the tools of boundary integrable models to find its reflection matrices and then compute some correlation functions. We also perform a thermodynamic Bethe ansatz computation for its boundary entropy.

NAKAYASHIKI Atsushi

U. Kyushu - Japan

On Relations of Three Integral Formulas for the Solutions of the qKZ Equation at Level 0

Form factors in 2-dim integrable QFT or SLM (solvable lattice models) satisfy qKZ equations of Frenkel-Reshetikhin or its some variants. In QFT case it is a consequence of the axioms of locality for form factors formulated by F.Smirnov. A special feature of the qKZ equations appeared in this context is that it is of level 0.

Up to now three kinds of contour integral solutions of the qKZ equation at level 0 are known. They are listed as

[1] Smirnov type formula (F.Smirnov, Jimbo-Kojima-Miwa-Quano)

[2] Tarasov-Varchenko's (TV) formula

[3] the formula for the trace of vertex operators calculated by bosonization (Jimbo-Miwa, Lukyanov, Khoroshkhin-Lebedev-Pakuiak). par

Each type of the integral formulas has its merit and demerit in the study so far. So it is quite a natural desire and a curious problem to give a relation among these three integral formulas. It will help us to understand the structure of the space of local operators and to understand the correlation functions.

In this talk we treat the case corresponding to the $SU(2)$ invariant Thirring model. This means that we consider the sl_2 rational qKZ equation at level 0 taking the value in the tensor product of the spin 1/2 representation of sl_2. In this case we give a complete answer on the relation of the formulas 1 and 2. Using this results the relation between 1 and 3 is given in some interesting cases. The key idea is to notice that TV singlet solutions have the total difference form as their integrand. Thus, in order to obtain a non-trivial result, we need to integrate along "paths" not along "cycles". We put the quotaion marks since they are not actual paths or cycles but the defomed objects, which are some functions. Our results shows that if we use some kind of "paths" (or "domain with a boundary") in the integration of the TV formula then we obtain Smirnov's formula.

We remark that TV formula tends to Schechtman-Varchenko's formula in the quasi-classical limit. The Smirnov's integral formula also has the quasi-classical limit. Our relation is valid and new even in this limit.

This is a joint work with S. Pakuliak and V. Tarasov. The paper is now in preparation.

PALLA Laszlo

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Quantum equivalence of dually related sigma models

Coupling constant renormalization is compared in sigma models related by Abelian and non Abelian duality transformations. Classically these models are related by a canonical transformation, thus they are equivalent. It is pointed out however, that quantum mechanically, to acheive their equivalence, beyond the one loop order of perturbation theory, one has to modify the duality transformation rules for renormalized quantities. A difference in this respect between Abelian and non Abelian dualities is also noted. Comments on recent works of P.E. Haagensen and K. Olsen (hep-th/9704157) and N. Kaloper and K. Meissner (hep-th/9705193) are also made.

References:

[1] J. Balog, P. Forgács, Z. Horváth and L. Palla: *Nucl. Phys. Proc. Suppl.* **B 49** (1996) 16.

[2] J. Balog, P. Forgács, Z. Horváth and L. Palla: *Phys. Lett.* **B388** (1996) 121.

[3] Z. Horváth, R.L. Karp and L. Palla: *Nucl. Phys.* **B 490** (1997) 435.

[4] L.K. Balázs and L. Palla: hep-th/9704137

PEARCE Paul

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TBA Equations, Classification of Excited States and Massless Flow from the Tricritical to Critical Ising Model

We study the scaling limit of the off-critical A_4 Andrews, Baxter and Forrester model in Regime IV. This integrable quantum field theory is given by the massless perturbation of the unitary minimal theory $\mathcal{M}(4,5)$ with respect to the operator $\Phi_{1,3}$. We present TBA equations for all excited states for fixed boundary conditions labelled by the Kac labels (r,s). We completely classify the eigenvalues of the transfer matrices in the scaling limit and identify three mechanisms by which the zeros and analyticity of the eigenvalues change under perturbation. This enables us to map, eigenvalue by eigenvalue, the renormalization group flow from the tricritical Ising to the critical Ising conformal fixed points and leads to new finitized characters.

PETKOVA Valentina

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Fusion rules for admissible representations of $\widehat{sl}(3)$

The fusion rules (FR) are basic ingredient in any 2–dimensional conformal field theory. They are well understood in the most studied such models, namely, the ones based on the set of integrable representations of affine Lie algebras $\hat{g}_k$. Much less established is the larger class of WZNW models based on the admissible representations of $\hat{g}_k$. The latter are characterised by rational values of the level k and the highest weights. The importance of this class comes from the fact that it gives rise to a large class of W - algebra models, obtained via quantum Hamiltonian reduction. The FR in this case are known only for the simplest $\widehat{sl}(2)$ algebra (Awata and Yamada). The aim of the reported work [1] is to extend this result to higher rank algebras, starting with the simplest nontrivial example $\widehat{sl}(3)$ which is studied in more detail.

The difficulty in extending the $\widehat{sl}(2)$ result comes in particular from the complexity of the Verma modules singular vectors and the decoupling equations they imply in higher rank cases. More deeply the problem is complicated by the lack of knowledge of what is the finite dimensional algebra and its quantum counterpart, whose representation theory is "behind" the fusion rules of admissible representations. Recall that the integrable FR formula admits a form (Kac, Walton, [2]), which involves the weight multiplicities of finite dimensional irreps of the finite dimensional subalgebra g of $\hat{g}$: in the classical formula for the multiplicity of an irreducible representation in the tensor product of two such representations the Weyl group of g is replaced by the affine Weyl group of $\hat{g}$.

We overcome the first of these difficulties by reducing the problem to the solution of a basic set of decoupling equations sufficient to describe the fusion of some "fundamental" representations with an arbitrary admissible representation. The resulting formulae can be interpreted as defining a fundamental "fusion" graph. Studies of analogous graphs related to the nondiagonal modular invariants of integrable WZNW models (Pasquer, Di Francesco and Zuber) suggest a formula for the general FR multiplicities. It generalises the Verlinde formula replacing the symmetric modular matrix there with the normalised eigenvectors of the adjacency matrix defining the graph. We then observe that the basic fusion formulae admit an interpretation in terms of the affine group introduced by Kac and Wakimoto in the study of admissible irreps characters. This group replaces the ordinary affine Weyl group in a general formula for the FR multiplicities analogous to the integrable case formula. To achieve this we introduce a class of generalised (reducible) Verma modules (or, rather describe the "supports" of such modules, i.e., the spectrum of weights and their multiplicities). Their resolutions result in some finite dimensional irreps described by generalised weight diagrams. The latter are closely related to the weight diagrams of the $sl(3)$ finite dimensional representations. All these structures which come naturally in the study of the FR for admissible representations suggest that there is some "hidden" graded finite dimensional algebra behind. (In the $\widehat{sl}(2)$ case the relevant (quantum) algebra was identified (Malikov and Feigin) to be the deformed $osp(1|2)$.)

The results may find also some application in the study of the FR for representations of W - algebras obtained by (non principal) DS reduction from the affine algebras.

The report is based on a joint work with P. Furlan and A.Ch. Ganchev [1]; for related previous work of the same authors see [2-5].

[1] P. Furlan, A.Ch. Ganchev and V.B. Petkova, *Fusion rules for admissible representations of affine algebras: the case of $A_2^{(1)}$*, to appear.

[2] P. Furlan, A.Ch. Ganchev and V.B. Petkova, *Nucl. Phys.* **B 343** (1990) 205.

[3] P. Furlan, R. Paunov, A.Ch. Ganchev and V.B. Petkova, *Nucl. Phys.* **B 394** (1993) 665.

[4] P. Furlan, A.Ch. Ganchev and V.B. Petkova, *Nucl. Phys.* **B 431** (1994) 622.

[5] P. Furlan, A.Ch. Ganchev and V.B. Petkova, *Nucl. Phys.* **B 491** no. 3 [PM] (1997) 635.

PILLIN Mathias

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Large Form factors in ADE affine Toda field theories

The affine Toda field theories of type ADE provide interesting examples of factorizing diagonal scattering theories for which the exact S-matrices have been conjectured. It is known that these theories are classically integrable but due to the structure of the interaction it is elusive to successfully treat the corresponding quantum field theories within the standard perturbative framework.

An important ingredient in order to understand the quantum structure of these field theories is provided by the knowledge of form factors, i.e. the matrix elements of the local operators in our models.

Smirnov has shown that the form factors in two-dimensional integrable massive models are subject to a set of four axioms.

We apply these axioms to the case of real coupling ADE Toda field theories. We prove that once the minimal form factors are known the calculation of form factors can be reduced the the problem of solving recursive relations for symmetric polynomials, which are exact and do not rely on any perturbative treatment. This analysis shows in addition how Smirnov's axioms apply to models with higher order forward channel fusing poles and even order poles of the S-matrices, and that these polynomials are invariant under a weak-strong coupling duality.

In the general case we give a characterization of the form factor polynomials in terms of an interesting relation among the weight lattice of the underlying Lie algebra and the conjugacy classes of representations of the symmetric group.

We will report on the explicit solutions in the $A_2^{(1)}$ and $D_4^{(1)}$ theories.

[1] M. Pillin: Polynomial Recursion Equations in Form Factors of ADE -Toda Field Theories, preprint KCL-MTH-97-31, hep-th/9705142, to appear in *Lett. Math. Phys.*.

[2] M. Pillin: Form factors in $D_n^{1)}$ affine Toda field theories, *Nucl. Phys. B* **497** (1997) 569.

GARCIA NOBRE PRATA Joao Nuno

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The Super-Liouville-Equation on the Half-Line

A recursive formula for an infinity of integrals of motion for the super-Liouville theory is derived. The integrable boundary interactions for this theory are computed. We find that they are unambiguosly determined by supersymmetry.

PUJOL Pierre

Sissa, Trieste - Italy

Magnetization Plateaux in Antiferromagnetic N-Leg Ladders

We show that the magnetization curves for antiferromagnetic N-leg spin 1/2 quantum spin ladders present plateaux for fractional values of the magnetization. These results are obtained using strong coupling expansion, exact diagonalization on finite systems and Abelian Bosonization. These systems provide an interesting of perturbed conformal field theories in condensed matter.

SCHILLING Anne

U. Amsterdam - Netherlands

The Bailey construction

The original Bailey lemma was introduced by Bailey[1] in 1949 as a method to prove Rogers-Ramanujan type identities. These identities were first studied in the theory of partitions and hypergeometric functions. From the work of ref. 2, it however became clear that Rogers-Ramanujan type identities can also be interpreted as Bose-Fermi identities of conformal field theory characters, exhibiting different bases of conformal field theory Hilbert spaces.

Bailey's lemma provides a constructive method to build up q-series identities from simpler identities and hence, in hindsight, yields a constructive way to build up conformal field theory character identities. The resulting relations between the characters match in many cases renormalization group flows. We will demonstrate that Bailey's original lemma gives relations between minimal models and $N = 1$ and $N = 2$ supersymmetric models[3], whereas the recently discovered higher-level Bailey lemma[4] yields relations between minimal models and higher-level A_1 coset models[5].

References:

[1] W. N. Bailey, Proc. London Math. Soc. (2) **50** (1949), 1.

[2] R. Kedem, T. R. Klassen, B. M. McCoy and E. Melzer, Phys. Lett. B **304** (1993), 263;
R. Kedem, T. R. Klassen, B. M. McCoy and E. Melzer, Phys. Lett. B **307** (1993), 68.

[3] A. Berkovich, B. M. McCoy and A. Schilling, Physica A **228** (1996), 33.

[4] A. Schilling and S. O. Warnaar, Int. J. Mod. Phys. B **11** (1997), 189; A. Schilling and S. O. Warnaar, *A higher-level Bailey lemma: Proof and application*, to appear in the Ramanujan Journal (q-alg/9607014).

[5] A. Berkovich, B. M. McCoy, A. Schilling and S. O. Warnaar, *Bailey flows and Bose-Fermi identities for the conformal coset models* $(A_1^{(1)})_N \times (A_1^{(1)})_{N'}/(A_1^{(1)})_{N+N'}$, tp appear in Nucl. Phys. B (hep-th/9702026).

TAKACS Gabor

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Scattering amplitudes in imaginary coupled $g_2^{(1)}$ and $d_4^{(3)}$ Toda field theories

Drawing on two recent papers, the fundamental soliton S-matrices of the imaginary coupled $d_4^{(3)}$ and $g_2^{(1)}$ Toda field theories are presented and their structure is discussed. The former allows the application of the tensor product graph method, while in the latter case the appearance of multiplicities in the tensor product decomposition makes the derivation more complicated. The hitherto unknown fundamental R-matrix of the $U_q(d_4^{(3)})$ quantum affine algebra is derived as a side result. It is shown that the results are consistent with the real coupling scattering amplitudes via the familiar breather-particle correspondence. The relevance of the results for integrable perturbations of minimal models of the WA_2 algebra is outlined.

References:

[1] The R matrix of the $U_q\left(d_4^{(3)}\right)$ algebra and $G_2^{(1)}$ Affine Toda Field Theory. By Gabor Takacs (SISSA, Trieste). SISSA-28-97-EP, Feb 1997. 17pp. e-Print Archive: hep-th/97021196 Nuclear Physics B, in press.

[2] Quantum Affine Symmetry and Scattering Amplitudes of the Imaginary Coupled $D_4^{(3)}$ Affine Toda Field Theory. By Gabor Takacs (Eotvos U.). ITP-BUDAPEST-525, Jan 1997. 20pp. e-Print Archive: hep-th/9701118 Nuclear Physics B, in press.

TESCHNER Joerg

U. Montpellier - France

On the conformal bootstrap for the SL(2,R) WZNW model

The SL(2,R) WZNW model is besides Liouville theory expected to be among the simplest *noncompact* interacting CFT's. In contrast to the much studied rational and quasi-rational conformal field theories one now has to deal with continuous sets of representations appearing in the spectrum and the factorization of correlation functions over intermediate states. For this reason one finds important qualitative differences between these theories and (quasi-)RCFT's: One example is that the identity representation does not occur in the spectrum. Related to this is the fact that the correspondence between states and operators, which is a fundamental axiom in many approaches to (quasi-)RCFT's, is much more subtle now.

Until recently there was only very few exact information available. The aim of my talk will be to outline a program aimed at the construction and solution of such theories based on the "conformal bootstrap" or representation theoretic approach to CFT's, and to present the progress that was achieved in this direction.

Within the conformal bootstrap approach to CFT's one needs three pieces of information to be able to construct multipoint correlation functions of primary operators:

The conformal blocks. Their power series can be constructed from the representation theory of the relevant chiral algebra. However, the representations considered are in contrast to the usually considered representation of Kac-Moody algebras neither of highest nor lowest weight type. A generalization of the KZ equations can be derived which now has continuous sets of relevant solutions.

Three-point functions resp. operator product coefficients. The explicit expression for the three point function of general primary fields can be derived by a generalization of the technique introduced in [1].

Knowledge of the set of intermediate representations that appear in the factorization resp. fusion rules. Based on semiclassical consideration a conjecture on the set of intermediate representations that contribute in four point functions is motivated. Further evidence for that conjecture comes from the nontrivial consistency with the result on the three point functions.

A beautiful structure appears to be emerging putting noncompact CFT's in analogy with the representation theory of noncompact Lie groups just as (quasi-) RCFT's were found to have structures analogous to representation theory of compact Lie groups.

References:
[1] J. Teschner: On the Liouville three-point function, *Phys. Lett.* B**363**(1995)65
[2] J. Teschner: On the conformal bootstrap for SL(2,R) WZNW models, hep-th/9705214

TIRKKONEN Olav

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Excited States in the twisted XXZ spin chain

The finite size spectrum for the spin 1/2 XXZ chain with twisted boundary conditions is computed, for an anisotropy $0 < \gamma < \pi/2$. Excited states are described in terms of strings, for which coupled Destri-de Vega equations are derived. ¿From these, finite size energies are extracted in the thermodynamic limit. In a restricted Hilbert space, corresponding to unitary conformal field theory, all primary states in the Kac table are recovered in terms of states with specific twist and strings. Talk based on a paper by Nassar and Tirkkonen, hep-th/9707098.

VAN ELBURG Ronald

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Quasi-particles in Fractional Quantum Hall Effect Edge Theories

Recently a quasi-particle approach to conformal field theory was formulated [Sch]. We apply these ideas to effective edge theories for fractional quantum Hall effect states [Elb]. For the Laughlin state with $\nu = \frac{1}{m}$, the fundamental edge quasi-particles are the edge electrons of charge $-e$ and edge quasi-holes of charge $+\frac{e}{m}$. These quasi-particles satisfy exclusion statistics in the sense of Haldane [Hal,Wu]. To describe transport between a $\nu = \frac{1}{m}$ edge and a normal metal we propose a kinetic (Boltzman) equation for these quasi-particles.

References:
[1] [Hal] F.D.M. Haldane, *"Fractional Statistics" in arbitrary dimensions: A Generalization of the Pauli Principle*, **PRL 67**,937 (1991)
[2] [Sch] K. Schoutens, *Exclusion Statistics in Conformal Field Theory Spectra*, cond-mat/9706166, PRL in print
[3] [Wu] Y.S.Wu, *Statistical Distribution for Generalized Ideal Gas of Fractional-Statistics Particles*,**PRL 73** ,922 (1994)
[4] [Elb] R.A.J. van Elburg and K. schoutens, *Quasi-particles in Fractional Quantum Hall Efect Edge Theories*, in preparation

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Spectrum of the superintegrable Z_N-Chiral Potts quantum chain with twisted boundary conditions

We consider Baxter's [1] analytic formulae for the transfer matrix spectrum of the superintegrable Z_N-symmetrical Chiral Potts model. We solve the Bethe-type equations for $N = 3, 4, 5$ and $m_P = 0, \ldots, 4$ -excitations numerically and then by comparison to finite-size diagonalization results we deduce rules for the energy levels of the quantum chain with Z_N-symmetrical twisted boundary conditions. We find that the sectors with twist R equal to the Z_N-charge Q receive only contributions from Bethe-equations with $m_P = R = Q \mod N$, which simplifies the counting of levels considerably. At three special momenta we obtain all solutions of the $m_P = 2$ Bethe-equations analytically. For general N, and any chain length L, we derive a formula for the limited momentum range of the appearance of the two-string solution ("second quasiparticle" [2,3]).

References:

[1] R.J.Baxter, J.Phys.A, Math.Gen. **27**, 1837 (1994);
[2] S.Dasmahapatra, R.Kedem and B.M.McCoy, Nucl.Phys. **B396**, 506 (1993);
[3] A.Honecker and G.v.Gehlen, Nucl.Phys. **B435**, 505 (1995)

WATTS Gerard

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Conserved charges in the chiral 3-state Potts model

We consider the perturbations of the 3-state Potts model introduced by Cardy as a description of the 3-state chiral Potts model. By generalising Zamolodchikov's counting argument we are able to show the existence of inhomogeneous conserved charges in the double perturbation by the thermal field and a descendent. These complement the homogeneous charges found by Cardy. By explicit calculation we find an appealing pattern suggesting the existence of an infinite set of such charges containing arbitrarily many conformal fields of different spins. We discuss the relevance of this result to the general perturbation of the conformal model and its description of the chiral Potts model.

WESTON Robert

U. Durham - UK

The monodromy matrices of the XXZ model in the infinite volume limit

I will consider the XXZ model in the infinite volume limit with spin half quantum space and higher spin auxiliary space. I will show how the half infinite transfer matrices of this class of models are related to certain $U_q(\widehat{sl_2})$ intertwiners introduced by Nakayashiki. I will construct the monodromy matrices, and show that the one with spin 1 auxiliary space gives rise to the L operator.