3-point functions of finite-size (dyonic) Giant magnons

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Holographic approach to

3-pt correlation functions (strong coupling limit)

*Related talk by Romuald Janik

Plan

- Introduction
- Formulation for 3-point function
- 3-pt for Giant Magnon

Ref: arXiv PLB702, PLB703; 1106.5656

Introduction

AdS / CFT

- Type IIB superstrings on $AdS_5 \times S^5$

dual to

$\mathcal{N} = 4 \quad SU(N_c)$ super-Yang-Mills theory

[Maldacena (1997)]

Parameter relations

$$\boxed{g_s = \frac{4\pi\lambda}{N_c} \quad \& \quad \frac{R^2}{\alpha'} = \sqrt{\lambda}}$$

$$\lambda = N_c g_{\mathrm{YM}}^2$$

- a planar limit of SYM = no quantum gravity limit

$$g_s \to 0 \;\equiv\; N_c \to \infty \text{ with fixed } \lambda$$

- Classical string theory

$$\alpha' \ll 1 \quad \Rightarrow \quad \lambda \gg 1$$

N=4 Super Yang-Mills theory

- $\mathcal{N} = 4\ \ SU(N_c)$ SYM

$$\boxed{S = \frac{\mathrm{Tr}}{g_{\mathrm{YM}}^2} \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu}^2 + (D_\mu \Phi^a)^2 + \left[\Phi^a, \Phi^b\right]^2 + \bar{\chi} \not{D} \chi - i \bar{\chi} \Gamma_a [\Phi^a, \chi] \right\}}$$

- R-symmetry : $\mathcal{N}$=4 SUSY so(6)

- Conformal field theory

- Irreducible rep. are given by eigenvalues of Cartan subalgebra

$$\Big(\overbrace{\quad\Delta\quad}^{D},\overbrace{S_1,S_2}^{L_{\mu\nu}}\,\Big|\,\overbrace{J_1,J_2,J_3}^{R}\Big)$$

- Scalar fields

$$Z\equiv\Phi_1+i\Phi_2,\quad Y\equiv\Phi_3+i\Phi_4,\quad X\equiv\Phi_5+i\Phi_6$$
$$\overline{Z}\equiv\Phi_1-i\Phi_2,\quad \overline{Y}\equiv\Phi_3-i\Phi_4,\quad \overline{X}\equiv\Phi_5-i\Phi_6$$

- General gauge-invariant composite operators

$$\tilde{O}(x)=\mathrm{Tr}\,[O_1(x)O_2(x)\ldots O_L(x)]$$

- ½ -BPS operator

$$\mathrm{Tr}\left[Z^L\right]\quad\rightarrow\quad(L,0,0|L,0,0)$$

Conformal Data

- CFT: all correlation functions are in principle decided by
  - Conformal dimensions : 2-pt functions
  - Structure constants : 3-pt functions

2-point function

$$\langle O_n(x) O_m(y) \rangle$$

Conformal dimensions

$$\langle \mathcal{O}_n(x)\mathcal{O}_m(y)\rangle = \frac{\delta_{mn}}{|x-y|^{2\Delta_n}}$$

- Conformal symmetry determines x,y dependence
- Need to know only conformal dim's $\Delta = \Delta_0 + \gamma$
- Exact dimension can be determined due to

INTEGRABILITY

Integrable spin chain

- Operator mixing: (ex) su(2) sector

$$\left\{\mathrm{Tr}\left[Z^L\right],\mathrm{Tr}\left[Z^{L-1}X\right],\mathrm{Tr}\left[Z^{L-n-1}XZ^{n-1}X\right],\ldots,\mathrm{Tr}\left[X^L\right]\right\}$$

- Maps to Heisenberg spin chain model $|\Uparrow\rangle\equiv|Z\rangle,\quad|\Downarrow\rangle\equiv|X\rangle$
  - Map: $|\Uparrow\Downarrow\Uparrow\Downarrow\Uparrow\Uparrow\ldots\rangle+\ldots\equiv\mathrm{Tr}\left[ZXZXZZ+\ldots\right]+\ldots$
  - Excited states: Bethe roots $e^{ip_jL}=\prod_{\substack{k=1\\k\neq j}}^{M}\left[\sigma^2(u_j,u_k)\frac{u_j-u_k+i}{u_j-u_k-i}\right]$

$$\Delta=M+\gamma=\sum_{j=1}^{M}\sqrt{1+16g^2\sin^2\frac{p_j}{2}}\xrightarrow{g\gg1\text{ limit}}4g\sin\frac{p_j}{2}\qquad\left(g\equiv\frac{\sqrt{\lambda}}{4\pi}\right)$$

Main Problem

- All-loop BAE applies only to infinite size spin chain

- If it is finite size, the asymptotic BAE fails due to wrapping problem

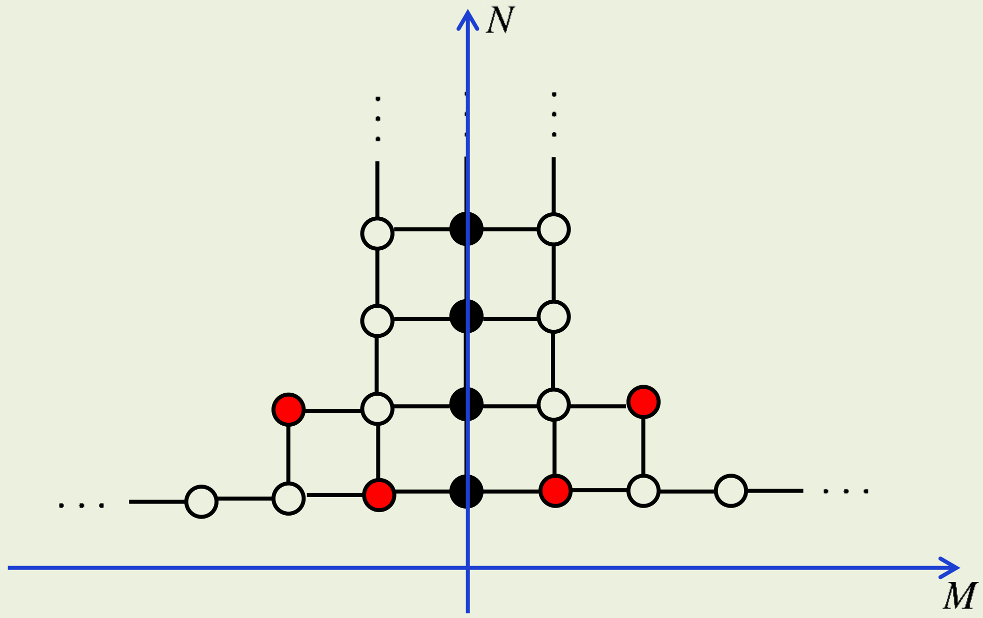
- Need a new approach based on

S-MATRIX

TBA, Y, NLIE systems

- Need to solve infinitely coupled nonlinear integral equations

Arutyunov,Frolov;
Bombardelli,Fioravanti,Tateo
Gromov,Kazakov,Kozak,Vieira
...



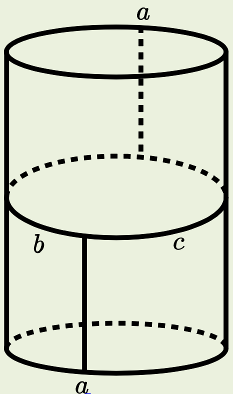
$$\ln Y_{N,M} = s\star\left[\ln(1+Y_{N,M+1})+\ln(1+Y_{N,M-1})\right]-s\star\left[\ln(1+Y^{-1}_{N+1,M})+\ln(1+Y^{-1}_{N-1,M})\right]$$

- Extremely complicated for strong coupling limit

Luscher correction for $\lambda \gg 1$

Ambjorn, Janik, Kristjansen; Janik, Lukowski

- Finite-size effect: "μ-term" Luscher correction
  - Interaction with a virtual particle



μ-term

- Conformal dimension for a magnon state $J \gg g \gg 1$

$$\Delta \approx 4g\sin\frac{p}{2} - 16g\sin^3\frac{p}{2}\,\exp\left[-\left(\frac{J}{2g\sin\frac{p}{2}}+2\right)\right]+\dots$$

3-point function

$$\langle O_l(x_1)O_m(x_2)O_n(x_3)\rangle$$

Structure constants

$$\langle O_l(x_1)O_m(x_2)O_n(x_3)\rangle = \frac{C_{lmn}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}|x_3-x_1|^{\Delta_3+\Delta_1-\Delta_2}}$$

- Conformal symmetry determines x_i dependence

- Exact results so far
  - Chiral primary operators dual to supergravity fields

    Freedman,Mathur,Matusis,Rastelli
    Lee,Minwalla,Rangamani,Seiberg
    Arutyunov,Frolov

  - Recent interesting developments from the string theory side

Marginal deformation

- Deformed CFT

Costa, Monteiro, Santos, Zoakos

$$S_{\text{New CFT}} = S_{\mathcal{N}=4} + u \int d^4y\, \mathcal{D}(y)$$

  - Two-point function of new CFT

$$\begin{aligned} \langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{\text{New}} &= \langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{\mathcal{N}=4} - u \int d^4y\, \langle \mathcal{D}(y)\mathcal{O}(x)\mathcal{O}(0)\rangle_{\mathcal{N}=4} + \text{renorm} \\ &= |x|^{-2\left[\Delta + 2\pi^2 u C_{\mathcal{DOO}} + \ldots\right]} \equiv |x|^{-2\Delta_{\text{New}}(u)} \end{aligned}$$

$$2\pi^2 C_{\mathcal{DOO}} = \frac{\partial}{\partial u}\Delta_{\text{New}}(u)|_{u=0}$$

- Special case: $g^2 \to g^2 = g^2(1-u) \quad : \quad \mathcal{D} = \mathcal{L}_{\mathcal{N}=4\ \text{SYM}}$

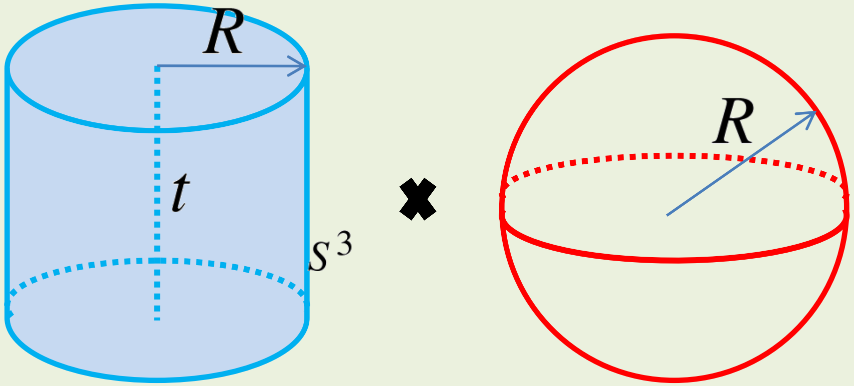
$$\boxed{2\pi^2 C_{\mathcal{LOO}} = -\lambda \frac{\partial}{\partial \lambda}\Delta(\lambda)}$$

Classical string theory

Superstring on AdS background

- Type IIB superstrings on $AdS_5 \times S^5$ is described by

$$S = \frac{R^2}{\alpha'} \int d\tau d\sigma \left[G_{mn}^{(S^5)} \partial_a X^m \partial^a X^n + G_{mn}^{(AdS)} \partial_a Y^m \partial^a Y^n + \text{fermions} \right]$$



Metsaev, Tseytlin (1998)

Bena, Polchinski, Roiban (2003)

- Virasoro constraints

$$\dot{X}^m X'_m + \dot{Y}^n Y'_n = 0, \quad \dot{X}^m \dot{X}_m + \dot{Y}^n \dot{Y}_n + X'^m X'_m + Y'^n Y'_n = 0$$

- Classically integrable nonlinear sigma model

- $AdS_5 \times S^5$ space embedding coordinates

$$X_1^2 + \cdots + X_6^2 = 1, \qquad Y_0^2 - Y_1^2 - \cdots - Y_4^2 + Y_5^2 = 1$$

- Global coordinates

$$Y_1 + iY_2 = \sinh\rho\cos\psi e^{i\phi_1}, \qquad Y_3 + iY_4 = \sinh\rho\sin\psi e^{i\phi_2},$$
$$Y_5 + iY_0 = \cosh\rho e^{it}, \qquad X_5 + iX_6 = \cos\gamma e^{i\varphi_3},$$
$$X_1 + iX_2 = \sin\gamma\cos\theta e^{i\varphi_1}, \qquad X_3 + iX_4 = \sin\gamma\sin\theta e^{i\varphi_2}$$
$$\begin{aligned}(ds^2)_{AdS_5} &= R^2\left[d\rho^2 - \cosh^2\rho dt^2 + \sinh^2\rho(d\psi^2 + \cos^2\psi d\phi_1^2 + \sin^2\psi d\phi_2^2)\right]\\ (ds^2)_{S^5} &= R^2\left[d\gamma^2 + \cos^2\gamma d\varphi_3^2 + \sin^2\gamma(d\theta^2 + \cos^2\theta d\varphi_1^2 + \sin^2\theta d\varphi_2^2)\right]\end{aligned}$$

- 6 isometry coordinates $t,\ \phi_1,\ \phi_2,\ \varphi_1,\ \varphi_2,\ \varphi_3$
- Conserved charges

$$S_{pq} = \sqrt{\lambda}\int_0^{2\pi}\frac{d\sigma}{2\pi}(Y_p\dot{Y}_q - Y_q\dot{Y}_p), \quad J_{mn} = \sqrt{\lambda}\int_0^{2\pi}\frac{d\sigma}{2\pi}(X_m\dot{X}_n - X_n\dot{X}_m)$$

E

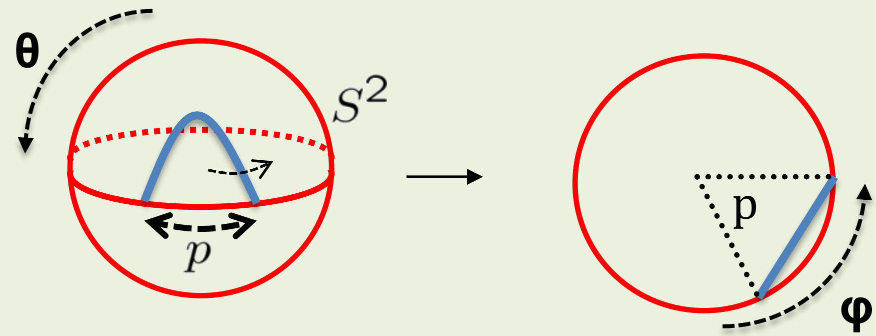
$$\boxed{(S_{50}, S_{12}, S_{34}|J_{12}, J_{34}, J_{56}) \quad \leftrightarrow \quad (\Delta, S_1, S_2|J_1, J_2, J_3)}$$

- AdS/CFT : Energy of string configuration = Conformal dim.

Giant magnon

- Classical string configuration in R x S^2 Hofman, Maldacena

$$Y_5 + iY_0 = e^{it},$$
$$X_1 + iX_2 = \cos\theta e^{i\varphi},$$
$$X_3 = \sin\theta$$



$$\cos\theta = \frac{\sin\frac{p}{2}}{\cosh\xi}, \quad \tan\varphi = \tan\frac{p}{2}\tanh\xi, \quad \xi \equiv \frac{\sigma - \cos\frac{p}{2}\tau}{\sin\frac{p}{2}}$$

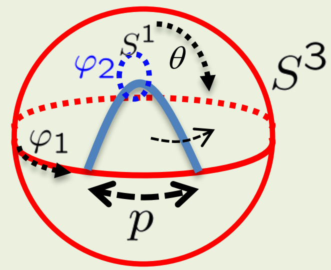
- Energy of the string $E = 4g\sin\frac{p}{2}$
- Sine-Gordon solition after Pohlmeyer reduction
- Dual to magnons in the SYM spin chain $\cdots \Uparrow\Uparrow \Downarrow \Uparrow\Uparrow \cdots$

Dyonic giant magnon

- GM in R x S^3

Chen, Dorey, Okamura

$$Y_5 + iY_0 = e^{it},$$
$$X_1 + iX_2 = \cos\theta e^{i\varphi_1},$$
$$X_3 + iX_4 = \sin\theta e^{i\varphi_2}$$

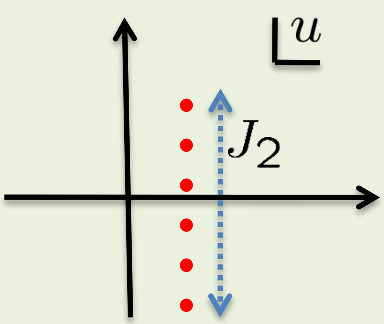


– Energy-charge relation:

$$E - J_1 = \sqrt{J_2^2 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}, \qquad J_2 \sim \sqrt{\lambda} >> 1$$

– Complex sine-Gordon solition

– Dual to magnon bound states "Bethe string"



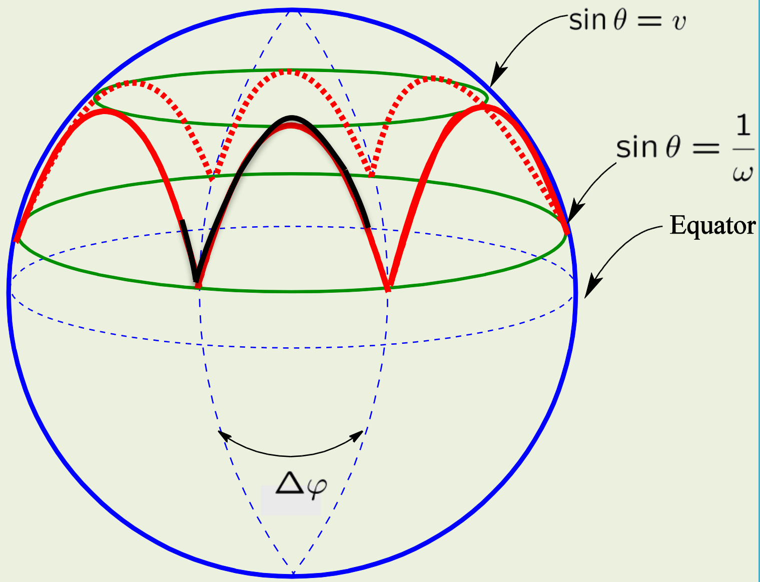
Finite-size GM in R x S^2

- Elliptic solution

Arutyunov, Frolov, Zamaklar; Klose, McLoughlin

$$\cos\theta = \sqrt{1-v^2}\,\mathrm{dn}\left(\frac{1}{\sqrt{\eta}}\frac{\sigma - v\omega\tau}{\sqrt{1-v^2\omega^2}}, \eta\right), \qquad \eta = \frac{1-\omega^2 v^2}{\omega^2(1-v^2)}$$

$$\approx \frac{\sin\frac{p}{2}}{\cosh\xi}, \qquad \left[\eta \to 1, \quad v \to \cos\frac{p}{2}, \quad \omega \to 1\right]$$



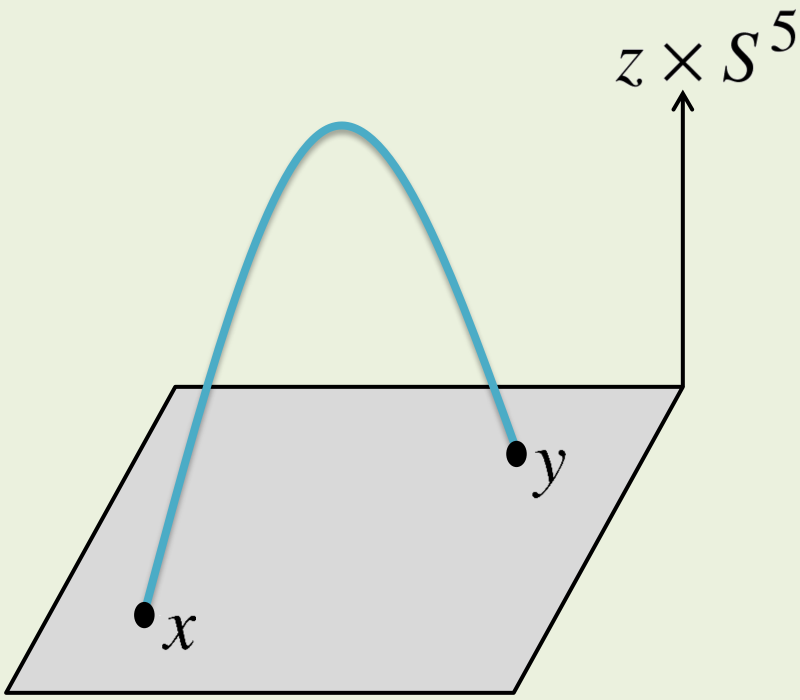
- Correction to E-J relation

$$J_1 \quad \gg \quad \sqrt{\lambda} \quad \gg \quad 1$$

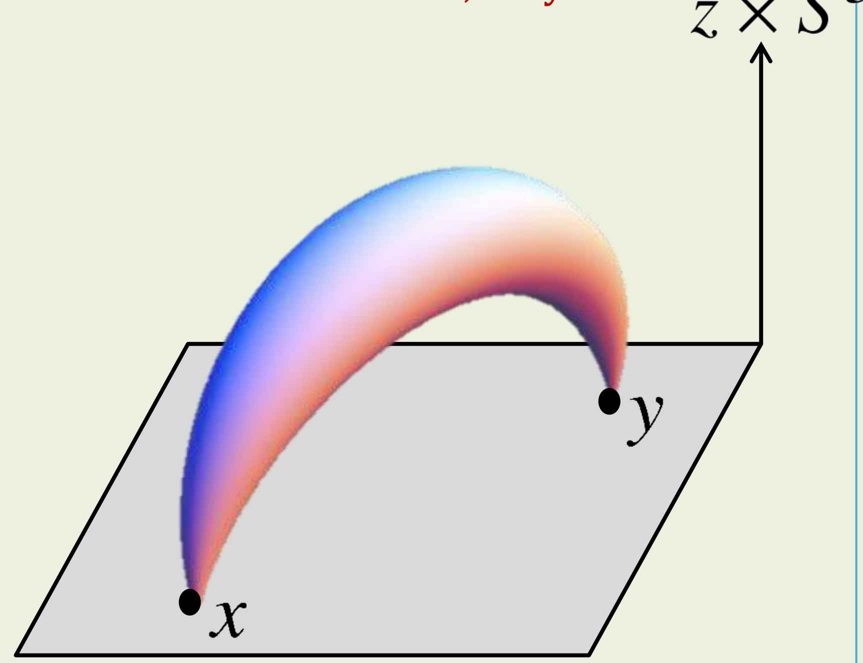
$$E - J \approx 4g\sin\frac{p}{2} \underbrace{- 16g\sin^3\frac{p}{2}\,\exp\left[-\left(\frac{J}{2g\sin\frac{p}{2}} + 2\right)\right]}_{\text{Finite-size effect}} + \ldots$$

Direct computation of 2-point function

Buchbinder
Janik, Surowka, Wereszczynski
Buchbinder, Tseytlin



$$\langle V_L(x) V_L(y) \rangle$$

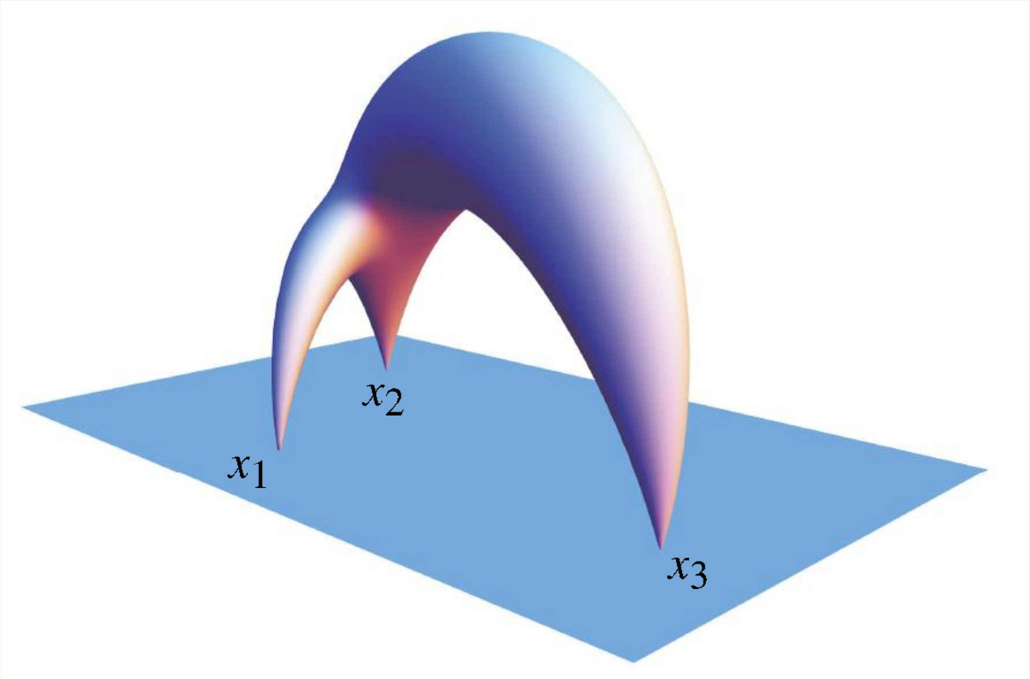


$$\langle V_H(x) V_H(y) \rangle$$

3-point function

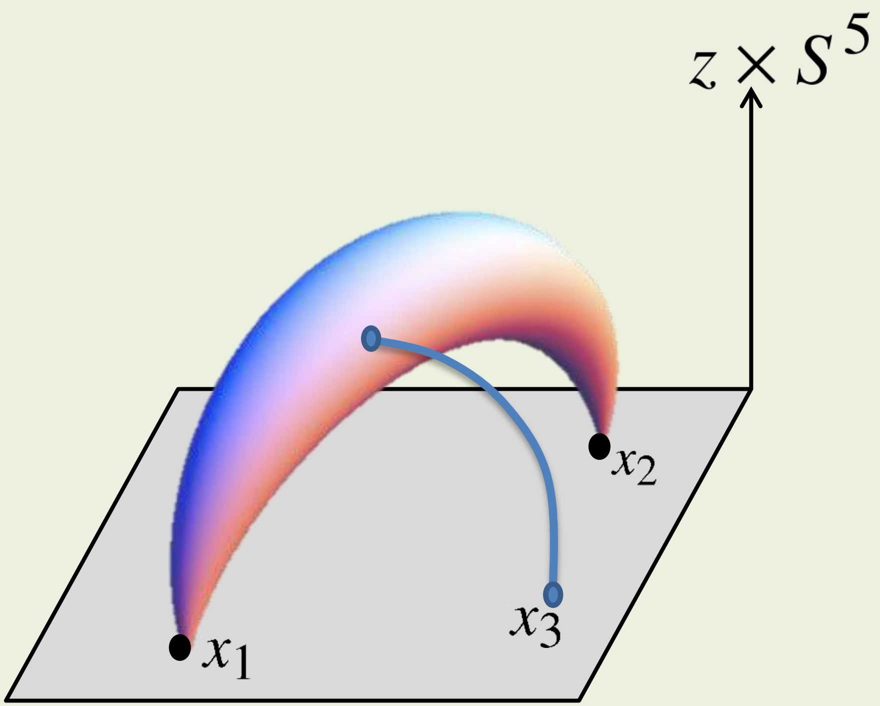
- General 3-point function

$$\langle V_H(x_1)V_H(x_2)V_H(x_3)\rangle$$



Simpler 3-point function

- One light mode $\langle V_H(x_1) V_H(x_2) V_L(x_3) \rangle$



Zarembo
Costa,Monteiro,Santos,Zoakos
Roiban, Tseytlin
Hernandez
Arnaudov, Rashkov
Georgiou
Park, Lee
Buchbinder, Tseytlin
Bak, Chen, Wu
Bissi, Kristjansen, Young, Zoubos
...

$$\frac{J_{1,2}}{\sqrt{\lambda}} \to \infty$$

Formulation

Zarembo
Roiban, Tseytlin

- Insertion of a light vertex does not change H background

$$C_{LHH} = \frac{\langle V_H(x_1) V_H(x_2) V_L(0)\rangle}{\langle V_H(x_1) V_H(x_2)\rangle} \qquad \propto \qquad V_L(0)[\text{H background}]$$

- L= dilaton ← dual to → SYM Lagrangian

$$V_L(0) = c_d \int_{-\infty}^{\infty} d\tau_e \int_{-L}^{L} d\sigma \underbrace{(Y_4+Y_5)^{-4}\left[z^{-2}(\partial_+ x_m \partial_- x^m + \partial_+ z \partial_- z) + \partial_+ X_k \partial_- X_k\right]}_{\text{Classical background}}$$

- Our problem: H = finite-size (D)GM

$$\frac{J_{1,2}}{\sqrt{\lambda}} = \text{arbitray}, \qquad \sqrt{\lambda} \to \infty$$

Neumann-Rosochatius reduction

Arutyunov, Russo, Tseytlin

- A string in $R_t \times S^3 \;\to\; \rho = 0,\ \gamma = \frac{\pi}{2},\ \varphi_3 = 0$

$$t = \kappa\tau,\ \cos\theta(\sigma,\tau) = r_1(\xi),\ \sin\theta(\sigma,\tau) = r_2(\xi),\ \varphi_j(\sigma,\tau) = \omega_j\tau + f_j(\xi),\ \xi = \alpha\sigma + \beta\tau$$

- Effective 1d Lagrangian (Neumann-Rosochatius):

$$f_j' = \frac{1}{(\alpha^2-\beta^2)^2}\left(\frac{C_j^2}{r_j^2} + \beta\omega_j\right)$$

$$L_{NR} = (\alpha^2-\beta^2)\sum_{j=1}^{2}\left[r_j'^2 - \frac{1}{(\alpha^2-\beta^2)^2}\left(\frac{C_j^2}{r_j^2} + \alpha^2\omega_j^2 r_j^2\right)\right] + \Lambda\left(\sum_{j=1}^{2} r_j^2 - 1\right)$$

- Conserved charges and Virasoro constraints

$$E = \frac{\lambda}{2\pi}\frac{\kappa}{\alpha}\int d\xi, \quad J_j = \frac{\lambda}{2\pi}\frac{1}{\alpha^2-\beta^2}\int d\xi\left(\frac{\beta}{\alpha}C_j + \alpha\omega_j r_j^2\right), \qquad \sum_{j=1}^{2} C_j\omega_j + \beta\kappa^2 = 0$$

- Eq. of motion

$$\theta'(\xi) = \pm\frac{1}{\alpha^2-\beta^2}\left[\kappa^2\left(\alpha^2-\beta^2\right) - \frac{C_1^2}{\sin^2\theta} - \frac{C_2^2}{\cos^2\theta} - \alpha^2\left(\omega_1^2\sin^2\theta + \omega_2^2\cos^2\theta\right)\right]^{1/2}$$

  - GM: $C_2 = 0, \quad \omega_2 = 0$
  - DGM: $C_2 = 0, \quad \omega_2 \neq 0$
  - Gamma-deformed (Lunin-Maldacena) $C_2 \neq 0, \quad \omega_2 \neq 0$

- Change of integration variable

$$\int_{-L}^{L} d\sigma \quad \to \quad 2\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\theta'}$$

Finite-size GM

- Energy and charges

$$
\begin{aligned}
E &= \frac{\sqrt{\lambda}}{\pi}\sqrt{(1-v^2)(1-\epsilon)}\mathbf{K}(1-\epsilon),\\
J_1 &= \frac{\sqrt{\lambda}}{\pi}\sqrt{\frac{1-v^2}{1-v^2\epsilon}}\left[\mathbf{K}(1-\epsilon)-\mathbf{E}(1-\epsilon)\right],\\
p &= 2v\sqrt{\frac{1-v^2\epsilon}{1-v^2}}\left[\frac{1}{v^2}\Pi\left(1-\frac{1}{v^2}|1-\epsilon\right)-\mathbf{K}(1-\epsilon)\right]
\end{aligned}
$$

$$
E-J_1 \equiv \Delta = \frac{\sqrt{\lambda}}{\pi}\sqrt{\frac{1-v^2}{1-v^2\epsilon}}\left[\mathbf{E}(1-\epsilon)-\left(1-\sqrt{(1-v^2\epsilon)(1-\epsilon)}\right)\mathbf{K}(1-\epsilon)\right]
$$

$$
v,\ \epsilon \quad \leftrightarrow \quad J_1,\ p
$$

- Structure constant derived from above formula for general J_1

$$
\boxed{C_{LHH} = \frac{16}{3}c_d\sqrt{\frac{1-v^2}{1-\epsilon}}\left[\mathbf{E}(1-\epsilon)-\epsilon\mathbf{K}(1-\epsilon)\right]}
$$

- Consistency with SYM side

  – Take λ-derivative

$$\frac{dJ_1}{d\lambda} = \frac{dp}{d\lambda} = 0 \quad \to \quad \frac{dv}{d\lambda} = -\frac{v(1-v^2)\epsilon\left[\mathbf{E}(1-\epsilon)-\mathbf{K}(1-\epsilon)\right]^2}{2\lambda(1-\epsilon)\left[\mathbf{E}(1-\epsilon)^2 - v^2\epsilon\mathbf{K}(1-\epsilon)^2\right]},$$

$$\frac{d\epsilon}{d\lambda} = -\frac{\epsilon\left[\mathbf{E}(1-\epsilon)-\mathbf{K}(1-\epsilon)\right]\left[\mathbf{E}(1-\epsilon) - v^2\epsilon\mathbf{K}(1-\epsilon)\right]}{\lambda\left[\mathbf{E}(1-\epsilon)^2 - v^2\epsilon\mathbf{K}(1-\epsilon)^2\right]}$$

  – Mathematica computation yields

$$\lambda\frac{\partial(E-J_1)}{\partial\lambda} = \frac{\sqrt{\lambda}}{2\pi}\sqrt{\frac{1-v^2}{1-\epsilon}}\left[\mathbf{E}(1-\epsilon)-\epsilon\mathbf{K}(1-\epsilon)\right]$$

$$2\pi^2 C_{LHH} = -\lambda\frac{\partial}{\partial\lambda}\Delta(\lambda)$$

Finite-size DGM

- Energy and charges

$$
\begin{aligned}
E &= \frac{\sqrt{\lambda}}{\pi}\frac{\kappa(1-v^2)}{\sqrt{1-u^2}\,\sqrt{\chi_p}}\mathbf{K}(1-\epsilon)\,,\\
J_1 &= \frac{\sqrt{\lambda}}{\pi}\frac{\sqrt{\chi_p}}{\sqrt{1-u^2}}\left[\frac{1-v^2\kappa^2}{\chi_p}\mathbf{K}(1-\epsilon)-\mathbf{E}(1-\epsilon)\right],\\
J_2 &= \frac{\sqrt{\lambda}}{\pi}\frac{u\sqrt{\chi_p}}{\sqrt{1-u^2}}\mathbf{E}(1-\epsilon)\\
p &= \frac{2v}{\sqrt{1-u^2}\,\sqrt{\chi_p}}\left[\kappa^2 1-\chi_p\Pi\left(-\frac{\chi_p}{1-\chi_p}(1-\epsilon)|1-\epsilon\right)-\mathbf{K}(1-\epsilon)\right]
\end{aligned}
$$

$$
\begin{aligned}
\epsilon &= \chi_p/\chi_m,\\
\chi_p,\,\chi_m &= \frac{1}{2(1-u^2)}\left\{q_1+q_2-u^2\pm\sqrt{(q_1-q_2)^2-\left[2(q_1+q_2-2q_1q_2)-u^2\right]u^2}\right\},\\
&\quad (q_1=1-\kappa^2,\quad q_2=1-v^2\kappa^2)
\end{aligned}
$$

$$
v,\ u,\ \kappa \quad \leftrightarrow \quad J_1,\ J_2,\ p
$$

- Structure constant

$$
\begin{aligned}
C_{LHH} = \frac{8}{3}c_d\frac{1}{\sqrt{(1-u^2)\kappa^2\chi_p(1-\chi_p)}}\Big\{&(1-\chi_p)\Big[2(1-u^2)\chi_p\mathbf{E}(1-\epsilon) \\
&-\left(u^2-\left(1-v^2\right)\kappa^2+(1-u^2)(1+\epsilon)\chi_p\right)\mathbf{K}(1-\epsilon)\Big] \\
&-\Big[1-v^2\kappa^4-\chi_p-(1-\chi_p)\left(\epsilon\chi_p+u^2(1-\epsilon\chi_p)\right)\Big]\Pi\left(-\frac{\chi_p}{1-\chi_p}(1-\epsilon)\,|1-\epsilon\right)\Big\}
\end{aligned}
$$

- Too complicated for Mathematica

$$
\frac{dJ_1}{d\lambda}=\frac{dJ_2}{d\lambda}=\frac{dp}{d\lambda}=0 \quad \to \quad \frac{dv}{d\lambda},\ \frac{du}{d\lambda},\ \frac{d\kappa}{d\lambda} \quad \to \quad \frac{d}{d\lambda}(E-J_1)
$$

- Instead, small ε-expansion for $J_1 \gg J_2 \sim \sqrt{\lambda} \gg 1$

$$
E-J_1=\Delta_{dyonic} = \Delta_\infty(p)-\frac{8\lambda\sin^4(p/2)}{\pi\Delta_\infty(p)}\exp\left[-\frac{2\,(J_1+\Delta_\infty(p))\,\Delta_\infty(p)\ \sin^2(p/2)}{J_2^2+\frac{\lambda}{\pi^2}\sin^4(p/2)}\right]
\qquad \Delta_\infty(p)=\sqrt{J_2^2+\frac{\lambda}{\pi^2}\sin^2(p/2)}
$$

Hatsuda, Suzuki

$$
2\pi^2 C_{LHH} = -\lambda\frac{\partial}{\partial\lambda}(E-J_1)
$$

Yes

Deformed (N=1) SYM

- AdS/CFT duality seems to hold in the deformed SYM
  - Conformal symmetry
  - Integrability: Exact S-matrix CA, Bajnok, Bombardelli, Nepomechie

- String theory: target space is deformed by TsT map "Lunin-Maldacena" background

$$S^5 \quad \rightarrow \quad S^5_\gamma$$

- NR reduction method also works here

Finite-size GM in Lunin-Maldacena

- Energy and charges

$$\begin{aligned}
E &= \frac{\sqrt{\lambda}}{\pi}\frac{(1-v^2)\sqrt{W}}{\sqrt{1-u^2}}\frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}},\\
J_1 &= \frac{\sqrt{\lambda}}{\pi\sqrt{1-u^2}}\left[\frac{1-\chi_n-v(vW-uK)}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon)-\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right],\\
J_2 &= \frac{\sqrt{\lambda}}{\pi\sqrt{1-u^2}}\left[\frac{u\chi_n-vK}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon)+u\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right]
\end{aligned}$$

$$\begin{aligned}
2\pi n &= \frac{2}{\sqrt{1-u^2}}\left\{\frac{vW-uK}{(1-\chi_p)\sqrt{\chi_p-\chi_n}}\Pi\left(-\frac{\chi_p-\chi_m}{1-\chi_p}|1-\epsilon\right)-\left[v(1-\tilde{\gamma}K)+\tilde{\gamma}u\chi_n\right]\frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}}-\tilde{\gamma}u\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right\},\\
p &= \frac{2}{\sqrt{1-u^2}}\left\{\frac{K}{\chi_p\sqrt{\chi_p-\chi_n}}\Pi\left(1-\frac{\chi_m}{\chi_p}|1-\epsilon\right)-\left[uv+\tilde{\gamma}v(vW-uK)-\tilde{\gamma}(1-\chi_n)\right]\frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}}-\tilde{\gamma}\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right\}
\end{aligned}$$

- small ε-expansion for one spin

$$E-J_1=\frac{\sqrt{\lambda}}{\pi}\sin\frac{p}{2}-\frac{4\sqrt{\lambda}}{\pi}\sin^3\frac{p}{2}\cos\underbrace{\left(2\pi n-\frac{2\pi\tilde{\gamma}}{\sqrt{\lambda}}J_1\right)}_{\Phi}\exp\left(-2-\frac{2\pi J_1}{\sqrt{\lambda}\sin(p/2)}\right)$$

CA, Bozhilov; (cf) Bykov, Frolov

- Structure constant

$$C^{\tilde{\gamma}}_{LHH} = \frac{16}{3} c_d \frac{1}{\sqrt{(1-u^2)W(\chi_p-\chi_n)}} \Big[\Big((1-u^2)(1-\tilde{\gamma}K) - \tilde{\gamma}uvW \Big) \sqrt{\chi_p-\chi_n}\,\mathbf{E}(1-\epsilon)$$
$$+ \Big(\Big(W(1-\tilde{\gamma}uv\chi_n) - (1-\tilde{\gamma}K)\Big(1-(1-u^2)\chi_n\Big)\Big)\mathbf{K}(1-\epsilon)\Big)\Big]$$

- small ε-expansion for

$$\frac{\sqrt{\lambda}}{\pi}\sin\frac{p}{2} - 4\sin^3\frac{p}{2}\left(\frac{\sqrt{\lambda}}{\pi}\cos\Phi + J_1\csc\frac{p}{2}\cos\Phi - \tilde{\gamma}J_1\sin\Phi\right)e^{-2-\frac{\pi J_1}{\sqrt{\lambda}\sin\frac{p}{2}}}$$

$$2\pi^2 C_{LHH} = -\lambda\frac{\partial}{\partial\lambda}(E-J_1)$$ Yes

Concluding remarks

- We compute exact (any J's) three-point correlation functions of a dilaton and two string states which corresponds to strong coupling SYM elementary excitatons
  - ✓ Giant magnon
  - ✓ Dyonic giant magnon
  - ✓ Both for N=4 and N=1 SYM

- Still early stage
  - ✓ Quantum corrections ?
  - ✓ <HHH> ? Janik, Wereszczynski
  - ✓ <MMM> Klose,McLoughlin
  - ✓ Integrability ? (perturbative) Escobedo, Gromov, Sever, Vieira

- Relation to scattering amplitudes ?