8th Bologna Workshop on: CFT AND INTEGRABLE MODELS and their applications to Quantum Field Theory, Statistical Mechanics and Condensed Matter Physics, September 12 - 15, 2011

TBA and NLO Lüscher corrections in integrably twisted theories

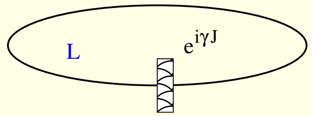
Z. Bajnok

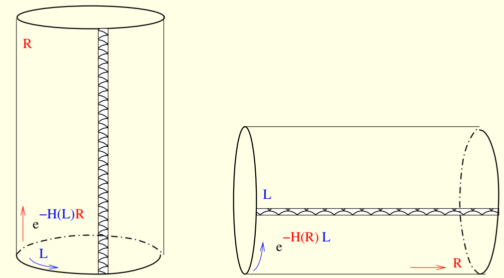
Theoretical Physics Research Group of the Hungarian Academy of Sciences

In memoriam:
Alyosha Zamolodchikov

Based on: arXiv:1108.4914 TBA, NLO Luscher correction, and double wrapping in twisted AdS/CFT, Changrim Ahn, Zoltan Bajnok, Diego Bombardelli, Rafael I. Nepomechie,
Double wrapping in twisted AdS/CFT? See next talk by Diego!

Plan

Ground-state energy 

the basic idea of TBA 

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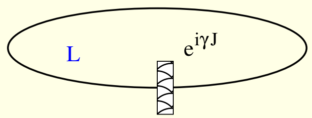
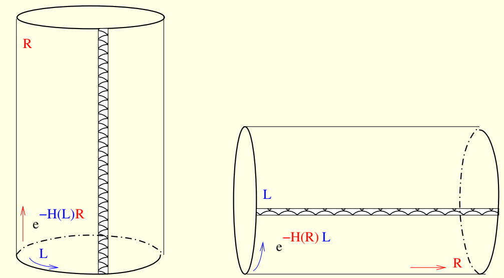
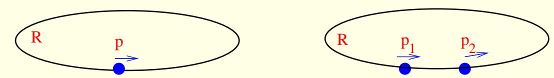
Ground-state energy 

the basic idea of TBA 

Cluster expansion: LO and NLO Lüscher corrections 

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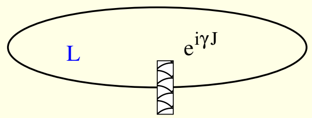
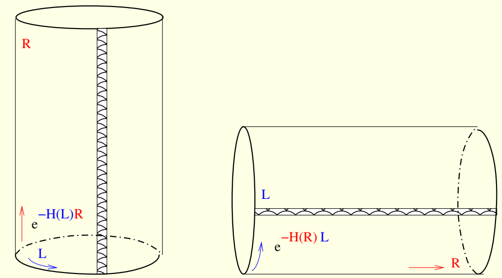
Ground-state energy  the basic idea of TBA



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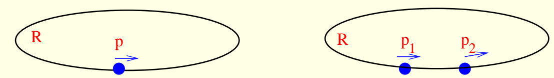
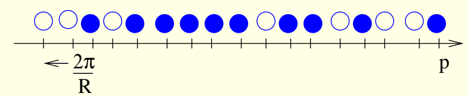


Twisted TBA equations, untwisted Y-system



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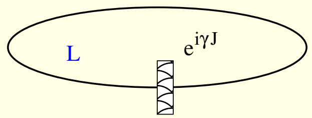
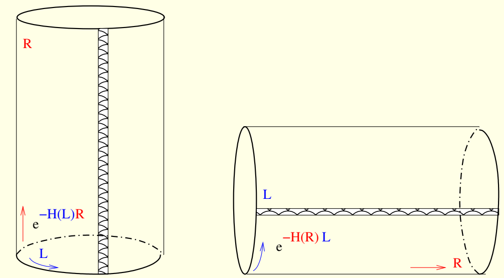
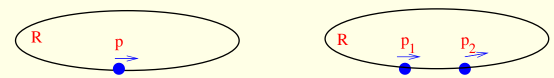
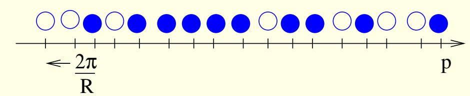
Ground-state energy 

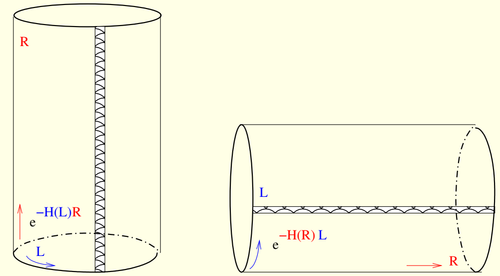
the basic idea of TBA 

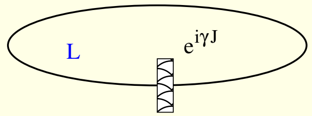
Cluster expansion: LO and NLO Lüscher corrections 

Twisted TBA equations, untwisted Y-system 

O(4) model: LO and NLO Lüscher corrections

Plan



Ground-state energy 

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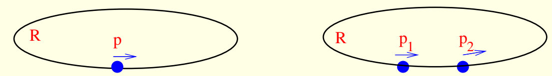
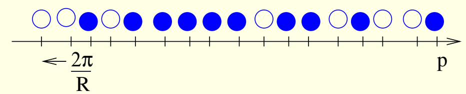
Cluster expansion: LO and NLO Lüscher corrections 

Twisted TBA equations, untwisted Y-system 

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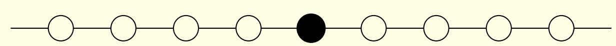
O(4) model: Twisted TBA and untwisted Y-system: 

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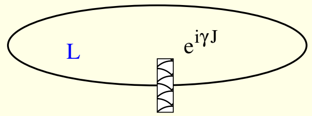
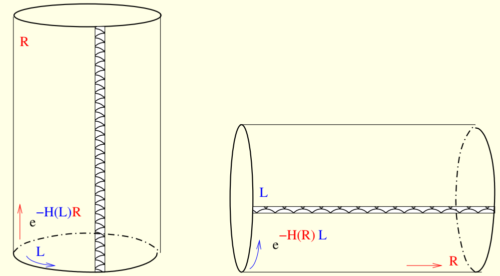
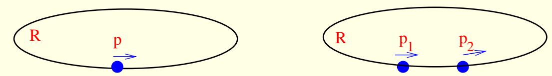
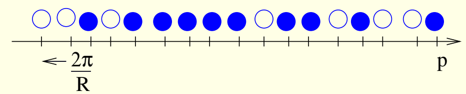
Ground-state energy 

the basic idea of TBA 

Cluster expansion: LO and NLO Lüscher corrections 

Twisted TBA equations, untwisted Y-system 

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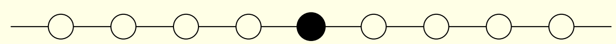
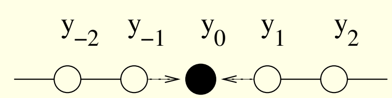
O(4) model: Twisted TBA and untwisted Y-system: 

O(4) model: Asymptotic expansion of TBA vs. Lüscher corrections 

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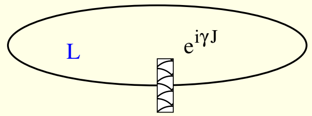
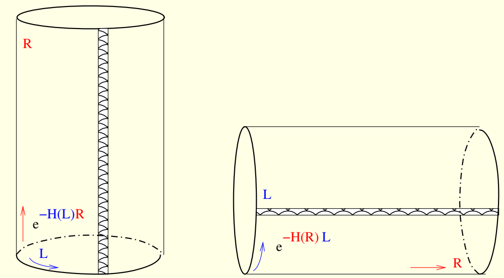
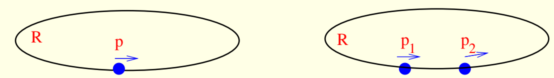
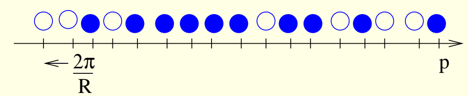
Ground-state energy 

the basic idea of TBA 

Cluster expansion: LO and NLO Lüscher corrections 

Twisted TBA equations, untwisted Y-system 

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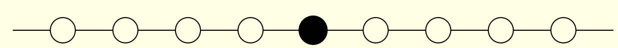
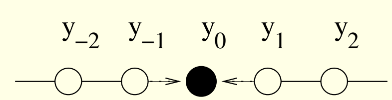
O(4) model: Twisted TBA and untwisted Y-system: 

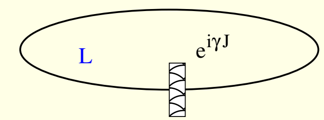
O(4) model: Asymptotic expansion of TBA vs. Lüscher corrections 

Conclusion, outlook

Ground-state energy in a finite volume with twisted BC

Ground-state energy with twisted BC ($e^{i\gamma J}$)

J conserved charge



Ground-state energy in a finite volume with twisted BC

Ground-state energy with twisted BC ($e^{i\gamma J}$)

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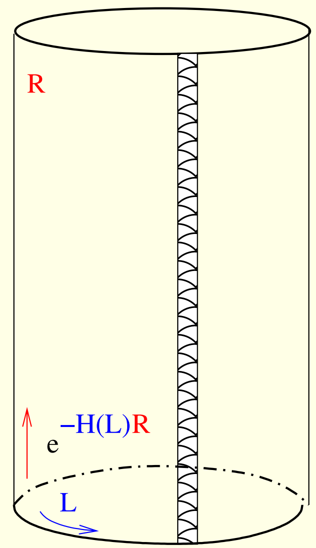
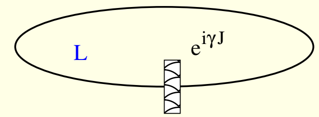
Euclidean twisted partition function:

$$Z^t(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H^t(L)R})$$
$$Z^t(L,R) =_{R\to\infty} e^{-E_0(L)R}(1+e^{-\Delta E\, R})$$





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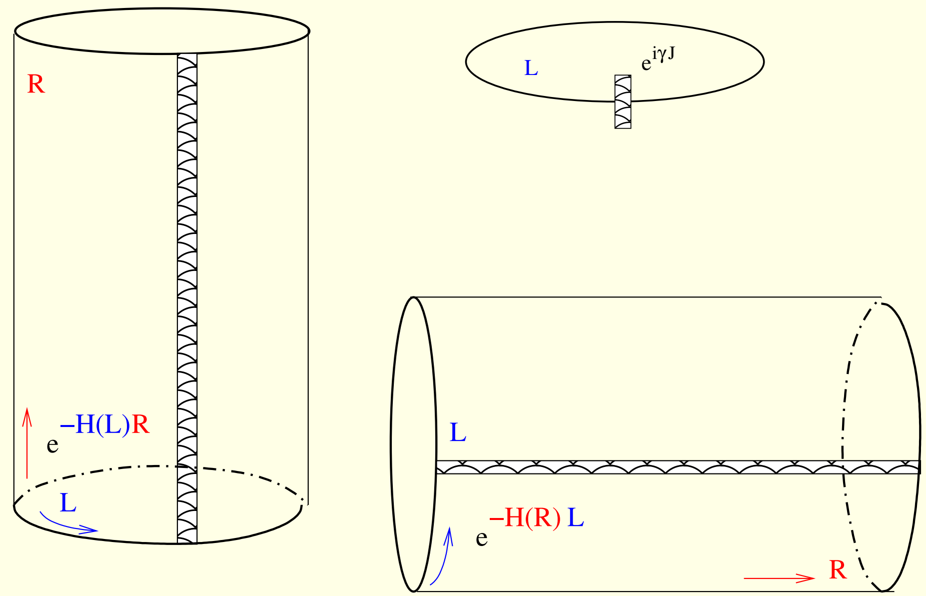
Exchange space and Euclidean time



$$Z^t(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H(R)L}D) =_{R\to\infty} \sum_n e^{-E_n(R)L+i\gamma J_n}$$

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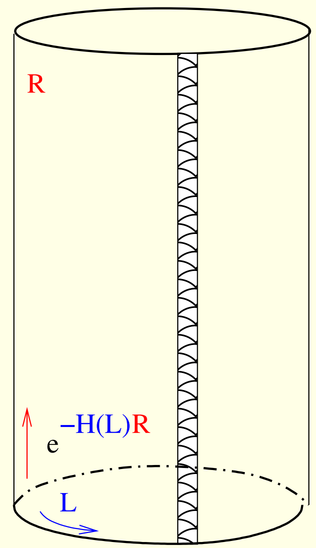

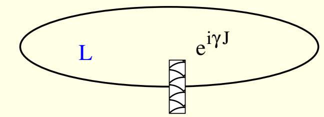

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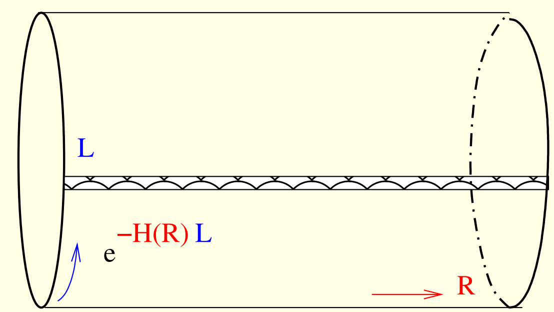

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Large (L) volume (cluster) expansion

$$\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}) = 1 + \text{one particle term} + \text{two particle term} + \ldots$$

$$\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}) = 1 + \sum_{k,\alpha} e^{i\gamma J_\alpha - e(p_k)L} + \sum_{k,l,(\alpha,\beta)} e^{i\gamma J_{(\alpha,\beta)}-(e(p_k)+e(p_l))L} + \ldots$$

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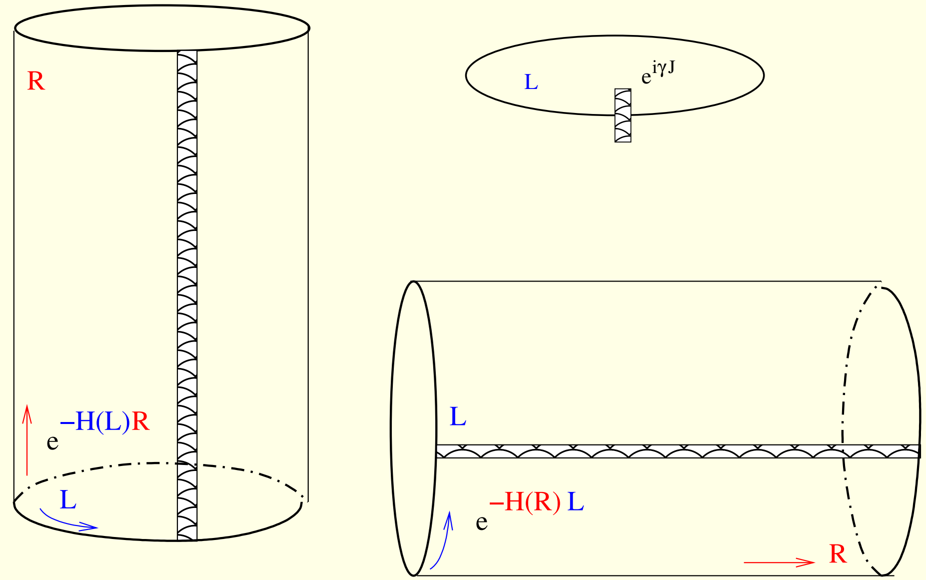

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Groundstate energy:

$$-\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J})) = E_0(L) = \text{LO Lüscher} + \text{NLO Lüscher} + \ldots = TBA$$

LO and NLO Lüscher corrections

Large (L) volume (cluster) expansion of the twisted partition function

$\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}) = 1 + \sum_{k,\alpha} e^{i\gamma J_\alpha - e(p_k)L} + \sum_{k>l,(\alpha,\beta)} e^{i\gamma J_{(\alpha,\beta)} - (e(p_k)+e(p_l))L} + \dots$

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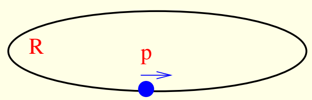

mom. quant.: $e^{ip_kR} = 1 \to \frac{R}{2\pi}p_k = k \in \mathbb{Z}$

Expanding the log $\log(1+x) = x - \frac{x^2}{2} + \dots$

change $\sum_k \to R\int \frac{dp}{2\pi}$

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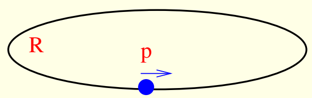
One particle term: 

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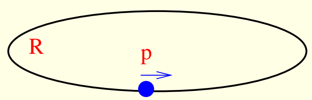

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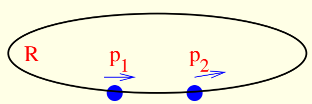
Two particle term:



$$e^{ip_1R}S(p_1,p_2) = 1 \quad \to \quad p_1R + \delta(1,2) = 2\pi k_1$$
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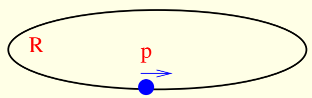

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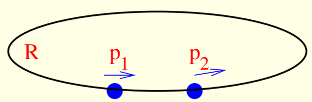

$$\begin{array}{lcl} e^{ip_1R}S(p_1,p_2) = 1 & \to & p_1R + \delta(1,2) = 2\pi k_1 \\ e^{ip_2R}S(p_2,p_1) = 1 & \to & p_2R - \delta(1,2) = 2\pi k_2 \end{array}$$

Rewriting the sum: $\sum_{k>l} = \frac{1}{2}\sum_{k,l} - \frac{1}{2}\sum_k$ change $\sum_{k_1,k_2} \to \int \frac{dp_1}{2\pi}\int\frac{dp_2}{2\pi} \begin{bmatrix} R+\partial_1\delta & \partial_2\delta \\ -\partial_1\delta & R-\partial_2\delta \end{bmatrix}$

.

$$R^2 + R(\partial_1\delta - \partial_2\delta)$$

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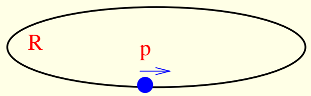
$$\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}) = 1 + \sum_{k,\alpha} e^{i\gamma J_\alpha - e(p_k)L} + \sum_{k>l,(\alpha,\beta)} e^{i\gamma J_{(\alpha,\beta)}-(e(p_k)+e(p_l))L} + \dots$$

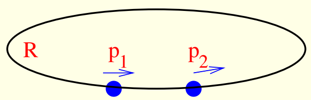
Groundstate energy: $E_0(L) = -\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}))$

One particle term: 

mom. quant.: $e^{ip_kR} = 1 \to \frac{R}{2\pi}p_k = k \in \mathbb{Z}$

Expanding the log $\log(1+x) = x - \frac{x^2}{2} + \dots$ change $\sum_k \to R\int\frac{dp}{2\pi}$

LO Lüscher correction: $\boxed{E_0^{(1)}(L) = -\mathrm{Tr}_1(e^{i\gamma J})\int\frac{dp}{2\pi}e^{-e(p)L} + \dots}$

Two particle term: 

$$e^{ip_1R}S(p_1,p_2) = 1 \quad\to\quad p_1R + \delta(1,2) = 2\pi k_1$$
$$e^{ip_2R}S(p_2,p_1) = 1 \quad\to\quad p_2R - \delta(1,2) = 2\pi k_2$$

Rewriting the sum: $\sum_{k>l} = \frac{1}{2}\sum_{k,l} - \frac{1}{2}\sum_k$ change $\sum_{k_1,k_2} \to \int\frac{dp_1}{2\pi}\int\frac{dp_2}{2\pi}\begin{bmatrix} R+\partial_1\delta & \partial_2\delta \\ -\partial_1\delta & R-\partial_2\delta \end{bmatrix}$

.

$$R^2 + R(\partial_1\delta - \partial_2\delta)$$

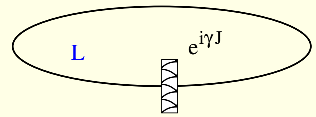
NLO Lüscher correction: $\boxed{E_0^{(2,1)}(L) = \frac{1}{2}\mathrm{Tr}_1(e^{i\gamma J})^2\int\frac{dp}{2\pi}e^{-2e(p)L}}$

$$\boxed{E_0^{(2,2)}(L) = \int\frac{dp_1}{2\pi}e^{-e(p_1)L}\int\frac{dp_2}{2\pi}e^{-e(p_2)L}i\partial_{p_1}\mathrm{Tr}_2(e^{i\gamma J}\log S(p_1,p_2))\dots}$$

NLO: [Dashen,Ma,Bernstein '69] Cluster expansion: [Dorey, Fioravanti, Rim,Tateo '04], [Bajnok, Palla,Takacs '05]

Twisted TBA equations

Ground-state energy exactly



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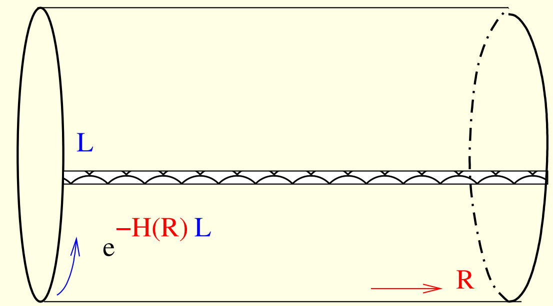

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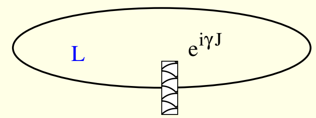

Euclidean twisted partition function, rotated:

$$Z^t(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H(R)L} D) =_{R\to\infty} \sum_n e^{-E_n(R)L + i\gamma J_n}$$

Twisted TBA equations

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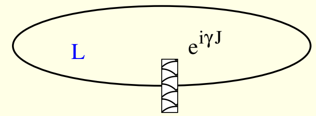

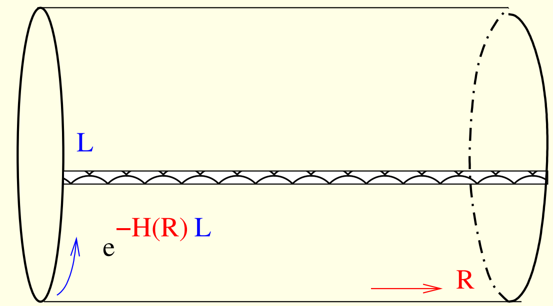

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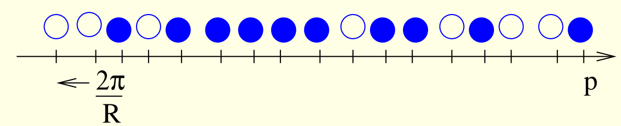

Dominant contribution: finite particle/hole density ρ, ρ_h:

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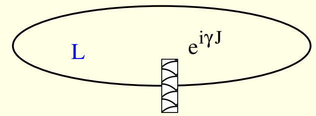

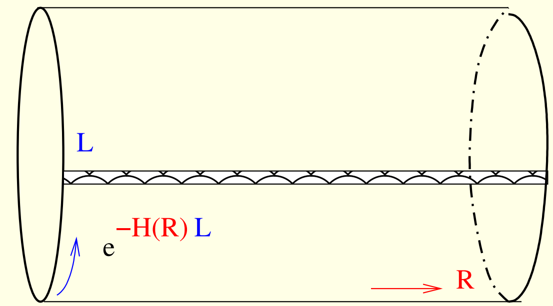

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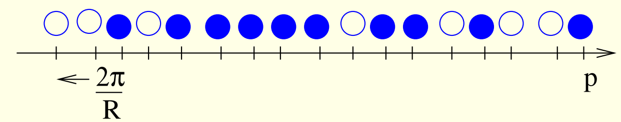

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momentum quantization:

$$p_j^{Q_j}R + \sum_k \frac{1}{i}\log S^{Q_jQ_k}(p_j,p_k) = (2n+1)i\pi \quad \longrightarrow R + \int(-id_p \log S^{QQ'}(p,p'))\rho^{Q'}(p')dp' = 2\pi(\rho^Q + \rho_h^Q)$$

Twisted TBA equations

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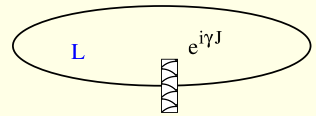

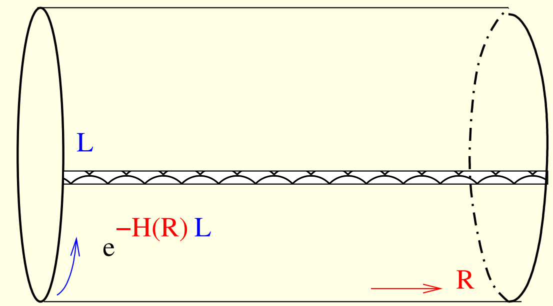

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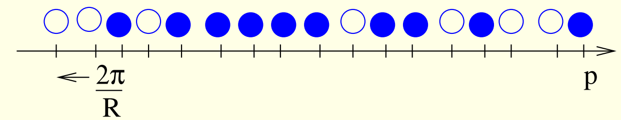

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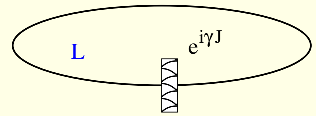
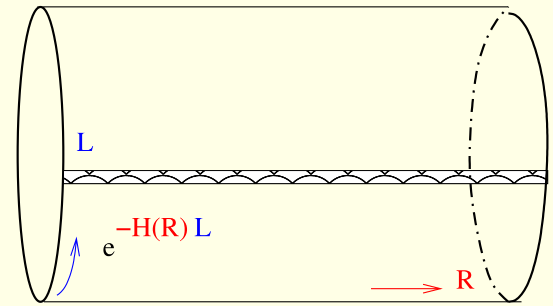
Twisted TBA equations





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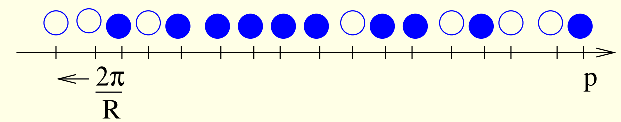

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Saddle point for pseudo energy: $\epsilon^Q(p) = \ln\frac{\rho_h^Q(p)}{\rho^Q(p)}$

$$\boxed{\epsilon^Q(p) + \mu^Q = e^Q(p)L + \int\frac{dp}{2\pi}id_p\log S^{Q'Q}(p',p)\log(1+e^{-\epsilon^{Q'}(p')})}$$

Ground state energy exactly: $\boxed{E_0(L) = -\sum_Q\int\frac{dp}{2\pi}\log(1+e^{-\epsilon^Q(p)})}$ [Alyosha '90 '91]

O(4) model: LO and NLO Lüscher correction

Relativistic theory, one multiplet of mass m: $e(\theta) = m \cosh \pi\theta \quad ; \; p(\theta) = m \sinh \pi\theta$

Factorized scattering, $su(2) \otimes su(2)$ invariance: $(\dot{\uparrow}, \dot{\downarrow}) \otimes (\uparrow, \downarrow)$ [Alyosha & Sasha '79]

$$S(\theta) = \frac{S_0^2(\theta)}{(\theta - i)^2} \hat{S}(\theta) \otimes \hat{S}(\theta) \qquad \hat{S}(\theta) = \theta\, \mathbb{I} - i\, \mathbb{P} \qquad S_0(\theta) = i \frac{\Gamma(\frac{1}{2} - \frac{i\theta}{2}) \Gamma(\frac{i\theta}{2})}{\Gamma(\frac{1}{2} + \frac{i\theta}{2}) \Gamma(-\frac{i\theta}{2})}$$

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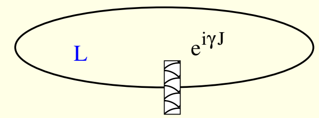

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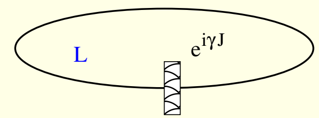

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$$[n]_q = \frac{q^n - q^{-n}}{q - q^{-1}}$$

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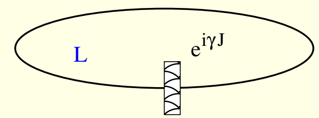

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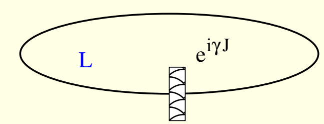

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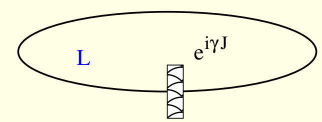

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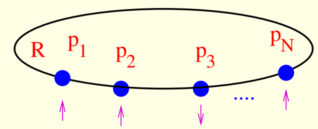
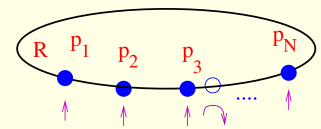
$$\mathrm{Tr}_2(\mathrm{D}\log\mathrm{S}) = \mathrm{Tr}_2(e^{i\gamma J}(-i\partial_\theta) \log S) = -i[2]_q^2 [2]_{\dot q}^2 \partial_\theta \log S_0^2(\theta) - i\left([2]_q^2 + [2]_{\dot q}^2\right) \partial_\theta \log \frac{\theta + i}{\theta - i}$$

O(4) model: twisted TBA and Y-system

Particle content in the thermodynamic limit: 



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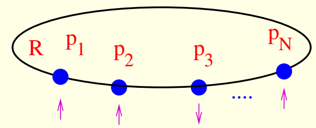

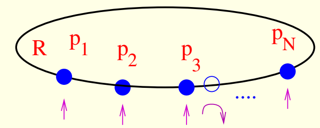

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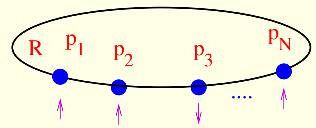
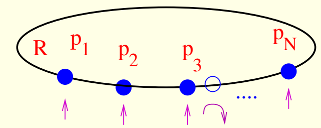
Particle content in the thermodynamic limit: 



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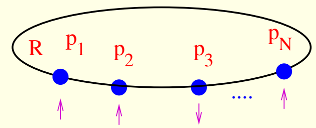
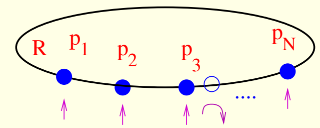
O(4) model: twisted TBA and Y-system

Particle content in the thermodynamic limit:





0. massive particle: $|\dot{\downarrow}\rangle \otimes |\downarrow\rangle$ $\begin{matrix} e_0(\theta) = m\cosh\pi\theta \\ p_0(\theta) = m\sinh\pi\theta \end{matrix}$ $\quad S_{00}(\theta) = S_0^2(\theta) \quad \mu_0 = -i(\gamma_- + \gamma_+)$

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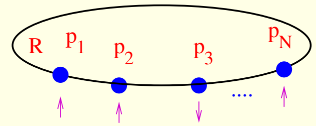

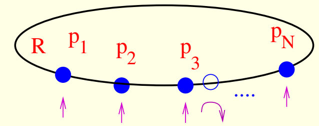

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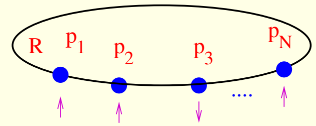

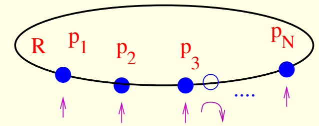

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O(4) model: twisted TBA and Y-system

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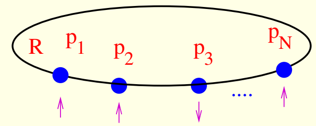

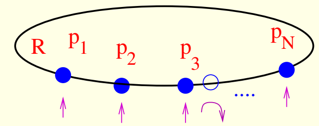

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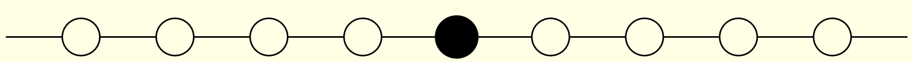
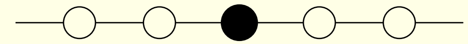
Universal TBA equations and $Y_{M\neq 0} = e^{\epsilon_M}$ Y-system: $\frac{1}{2}(\mu_{N-1} + \mu_{N+1}) - \mu_N = 0$

$$\log Y_n + \delta_{n,0} Lm \cosh \pi\theta = \log\Big((1+Y_{n-1})(1+Y_{n+1})\Big) \star s \qquad s(\theta) = \frac{1}{2\cosh \pi\theta}$$

$\lim_{M\to\infty} \log Y_M = 2iM\gamma_+$ 

O(4) model: TBA asymptotic expansion

Groundstate energy: $E_0(L) = -\frac{m}{2}\int d\theta \cosh \pi\theta \, \log(1+Y_0)$



$\log Y_n = -\delta_{n,0} L m \cosh \pi\theta + \log\Big((1+Y_{n-1})(1+Y_{n+1})\Big) \star s$; $\frac{\log Y_M}{M} \to 2i\gamma_+$

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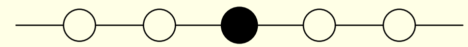

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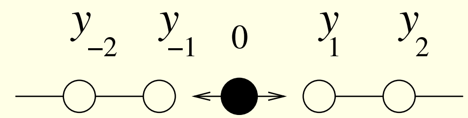

Asymptotic expansion: $Y_M = \mathcal{Y}_M(1+y_M) + \ldots$

$(\mathcal{Y}_M)^2 = (1+\mathcal{Y}_{M-1})(1+\mathcal{Y}_{M+1}) \longrightarrow \mathcal{Y}_M = [M]_q[M+2]_q \quad ; \quad \mathcal{Y}_{-M} = \mathcal{Y}_M(q \leftrightarrow \dot{q})$

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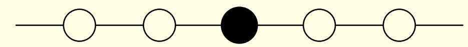

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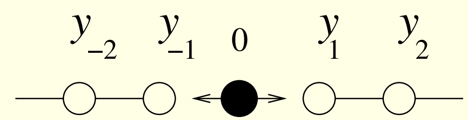

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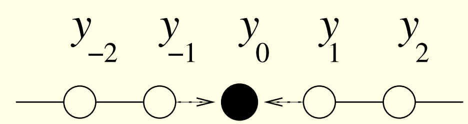

Energy: $\log(1+\mathcal{Y}_0) = \mathcal{Y}_0 - \frac{1}{2}\mathcal{Y}_0^2 \to$ LO and NL0 $E_0^{(1)}, E_0^{(2,1)}$

LO contribution $\mathcal{Y}_0 = \sqrt{(1+\mathcal{Y}_1)(1+\mathcal{Y}_{-1})}e^{-mL\cosh\pi\theta} = [2]_q[2]_{\dot{q}}e^{-mL\cosh\pi\theta}$

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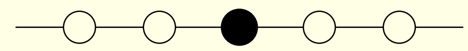

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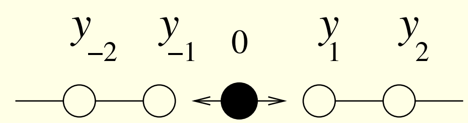

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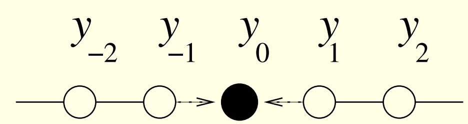

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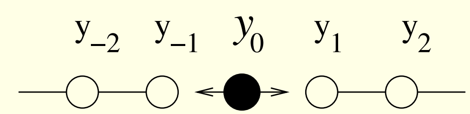
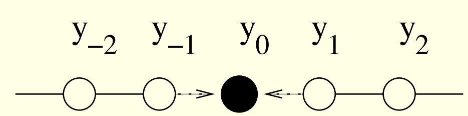


NLO: lin. diff. eq.: $y_k = s\star\Big[\mathcal{A}_{k+1}y_{k+1} + \mathcal{A}_{k-1}y_{k-1}\Big] \quad ; \mathcal{A}_k = \frac{1+\mathcal{Y}_k}{\mathcal{Y}_k}$



Solution: $\tilde{y}_k = e^{-\frac{|\omega|}{2}k}\frac{[k+1]_q}{[2]_q[k]_q[k+2]_q}([k+2]_q - [k]_q e^{-|\omega|})\tilde{\mathcal{Y}}_0$

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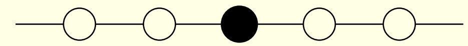

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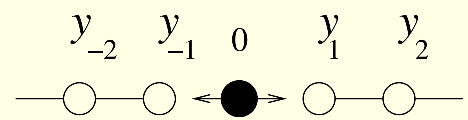

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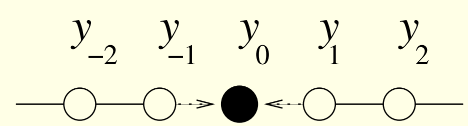

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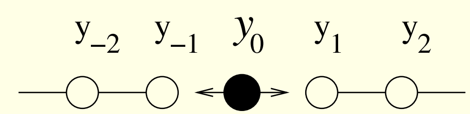

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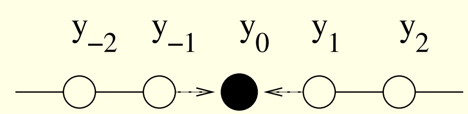

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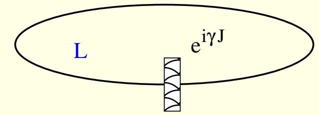
NLO contribution: $Y_0 = \mathcal{Y}_0 y_0 \left(1 + s\star[\mathcal{A}_1 y_1 + \mathcal{A}_1 y_{-1}]\right)$ Agrees with NLO Lüscher!

Conclusion

Ground-state energy $E_0(L)$ with twisted BC ($e^{i\gamma J}$)



LO Lüscher correction: $\boxed{E_0^{(1)}(L) = -\mathrm{Tr}_1(e^{i\gamma J}) \int \frac{dp}{2\pi} e^{-e(p)L} + \ldots}$ spectrum

NLO Lüscher correction: $\boxed{E_0^{(2,1)}(L) = \frac{1}{2}\mathrm{Tr}_1(e^{i\gamma J})^2 \int \frac{dp}{2\pi} e^{-2e(p)L}}$ scattering matrix

$$\boxed{E_0^{(2,2)}(L) = \int \frac{dp_1}{2\pi} e^{-e(p_1)L} \int \frac{dp_2}{2\pi} e^{-e(p_2)L} i\partial_{p_1} \mathrm{Tr}_2(e^{i\gamma J} \log S(p_1, p_2))\ldots}$$

Twisted TBA equation: $\epsilon \to \epsilon + \mu$

$$\boxed{\epsilon^Q(p) + \mu^Q = e^Q(p)L + \int \frac{dp}{2\pi} i d_p \log S^{Q'Q}(p', p) \log(1 + e^{-\epsilon^{Q'}(p')})}$$

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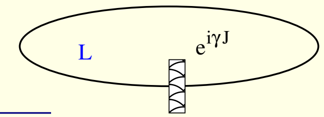

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Outlook

Double wrapping in twisted AdS/CFT, see next talk

Test NLIEs by comparing to Lüscher corrections

O(N) [Balog, Hegedus '01] SS [Hegedus '04]