On the structure of typical states of a disordered Richardson model and many-body localization

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Talk Outline

- What do we mean by many-body localization transition?
- Why an integrable model?
  - The Richardson model and an algorithm for its solution
- In quest of an order parameter
- The structure of the many-body quantum state
  - Is it delocalized or not?
- The role of integrability: Mazur's inequality and integrability breaking
- Conclusions

The many-body localization transition

- In 1958, P. Anderson came out with a groundbreaking paper: disorder gives localization → No conductivity
- What happens if we put interactions?
  - Basko, Aleiner & Altshuler showed (cond-mat/0506617v2) a transition is still there at finite temperature
  - Pal & Huse studied the XXZ model in random field: localization transition even at **infinite temperature**
- Anderson localization in the Hilbert space
- Physical interpretation

Will it spread after some time?

Local energy excitation

The Richardson model and its spin-representation

- The Richardson model can be seen as a fully-connected XY model with random field:

$$H=-\frac{g}{N}\sum_{\alpha,\beta=1}^{N} s_\alpha^+ s_\beta^- - \sum_{\alpha=1}^{N} h_\alpha s_\alpha^z$$

- •Fully-connected
- •Integrable
- •Disordered

- It belongs to the family of Gaudin models, solvable by the tecnique of: ***Algebraic Bethe Ansatz***
  - We look for quantum states of the form

$$|E[w]\rangle=\prod_{j=1}^{M} B(w_j)|\downarrow\ldots\downarrow\rangle, \qquad B(w)=\sum_{\alpha=1}^{N}\frac{S_\alpha^+}{w-h_\alpha}$$

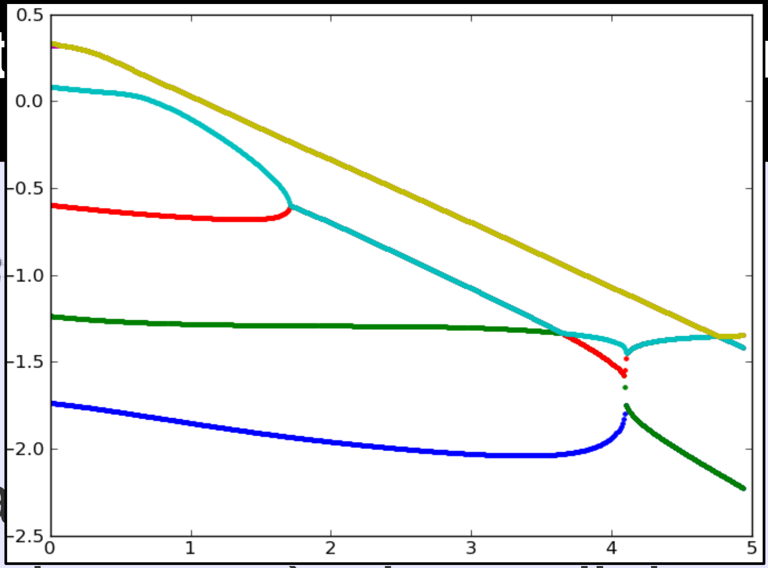
  - Imposing that they are eigenstates of the Richardson hamiltonian, we get the Bethe-Ansatz equations:

$$\frac{N}{g}+\sum_{\alpha=1}^{N}\frac{1}{w_j-h_\alpha}-\sum_{k=1,k\neq j}^{M}\frac{2}{w_j-w_k}=0$$

HOW TO DEAL WITH IT?

The standard approach to t... n

$$\frac{N}{g}+\sum_{\alpha=1}^{N}\frac{1}{w_j-h_\alpha}-\sum_{k=1,k}^{M}$$



- Analytic solutions? Only for M
- Numerical solutions? Known a
  - start from **g=0** (non-interacting case) where all the solutions are known as all the possible subsets of the set of fields
  - slowly increase the value of g using the Newton method to find the interacting solutions
  - change variables to “smooth” around the critical points
- This works extremely well for the groundstate and the low-excited states.

The general eigenstate case: the polynomial trick

- For high energy states such procedure fails!

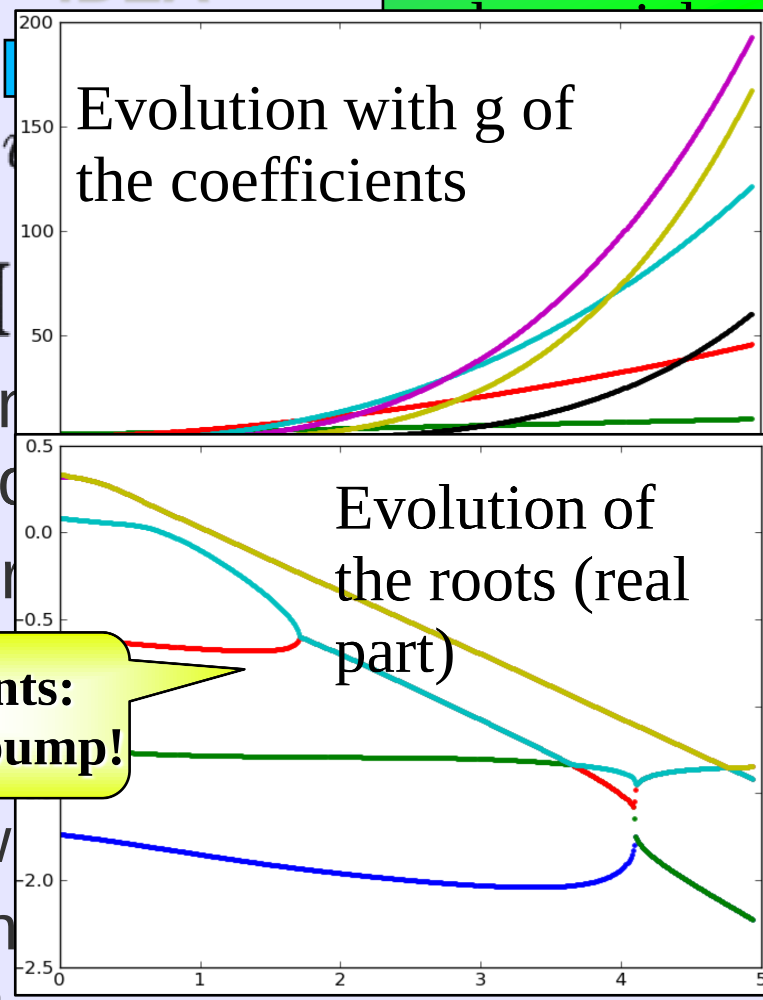
Solutions are always real or complex conjugate

IDEA

The associated

$P(z) \equiv \prod$

- The algebraic equation ... tial equation for the polyno...
- We used a mixed appr...
  - solve the
  - compute
  - extrapolate them tow...
  - use the roots of the n... r the Richardson equation

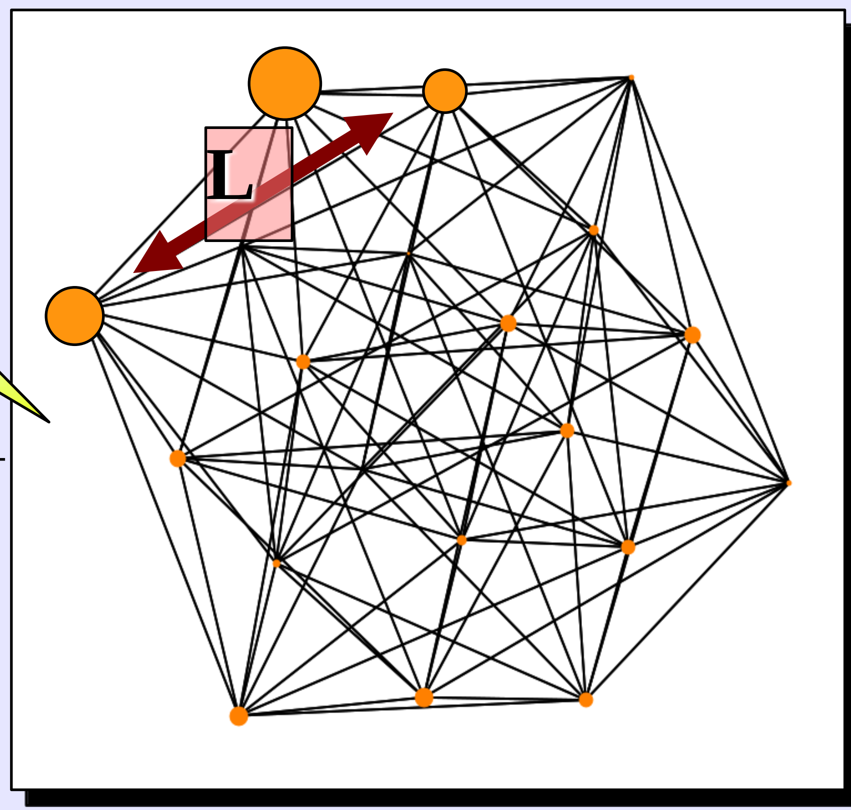


Critical points: where roots bump!

In quest of an order parameter

- Now we have the eigenstates: we managed to work till 40 spins. But...
- How to define delocalization?
  - Non-interacting states are localized
  - How to quantify if interaction is able to make them delocalize?

Hilbert state structure for N = 6



IPR is the number of significative states

L is the average Hamming distance

The q Edward-Anderson order parameter

N determinant of N/2xN/2 matrices: computable!

$$\Big(\sum_{s_1\ldots s_N,\ \sum_\alpha s_\alpha = 0} |\langle s_1 \ldots s_N | E\rangle|^4\Big)^{-1}$$

$$q(E) = \frac{4}{N}\sum_{\alpha=1}^{N} |\langle E | s^z_\alpha | E\rangle|^2$$

Hardly computable: it involves exponential number of terms.
We tried important sampling through Montecarlo and it worked!

$$L \equiv \sum_{s,s'} p_s p_{s'} d(s,s') = \frac{N}{2}(1-q).$$

Other interpretation of the qEA order parameter:

$$\rho_0 = (1+\epsilon s^z_\alpha)/Z \qquad \langle s^z_\alpha\rangle_\infty = \lim_{t\to\infty} \mathrm{Tr}(e^{-iHt}\rho_0 e^{iHt} s^z_\alpha)$$

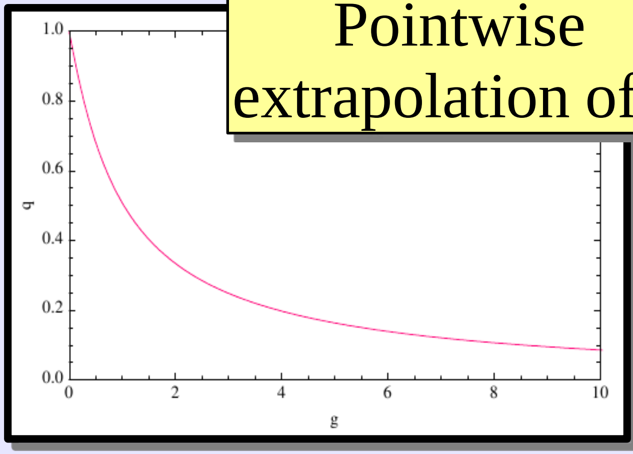
it is **the average survival fraction of the initial magnetization after very long times**.

$$q = \frac{1}{Z}\sum_E q(E) = \frac{1}{N}\sum_\alpha \frac{\langle s^z_\alpha\rangle_\infty}{\langle s^z_\alpha\rangle_0}$$

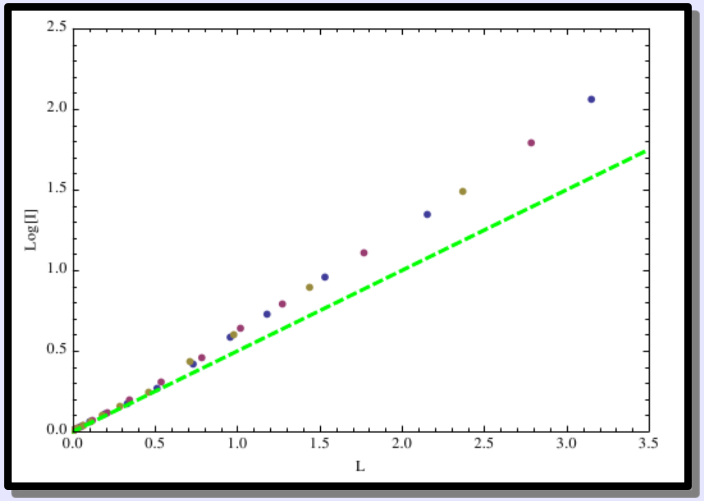
Numerical results and an almost sure guess

$$\langle q \rangle = \frac{1+3\times10^{-8}g}{1+1.003g+0.009g^2} \simeq \frac{1}{1+g}$$

Extremely accurate fit! We could guess the analytic expression



Pointwise extrapolation of q



Linear relation with **LogI**: states are omogeneously spreading

To be or not to be localized?

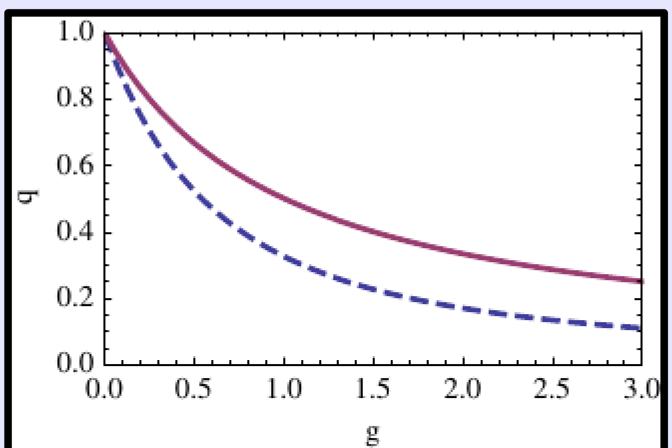
- With a finite coupling each eigenstate is spread over an exponential number of classical states
- But just an infinitesimal fraction of the whole system is covered (the entropy is always smaller than log2, actually close to its half)
- A local excitation will remain localized forever (q != 0)
- The Montecarlo dynamics recalls the one obtained by random percolation on the hypercube

The system remains localized on a larger and larger region, uniformly spread in a much larger hilbert space

- The strange behavior we found may be due to:
  - The space-structure of the model: fully-connected!
  - Integrability
- To quantify the role of the second, we use the Mazur's inequality:

$$\bar{q} = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt \langle s^z_\alpha(0) s^z_\alpha(t)\rangle \geq \sum_{\beta=1}^{N} \frac{|\langle \tau_\beta s^z_\alpha\rangle|^2}{\langle \tau_\beta^2\rangle}$$

$$q = \sqrt{3\pi} g \int_{-\infty}^{\infty} dx\, \mathcal{P}(x)^2 e^{3\pi\mathcal{P}(x)^2 g^2} \Phi(\sqrt{3\pi}\mathcal{P}(x)g)$$



If the conserved charges of the integrable model are local, local observables will be localized forever

Summary and conclusions

- We developed a new algorithm to numerically deal with the Richardson equation for general fields and arbitrary energy states
- Through a Montecarlo procedure we were able to perform a random walk in a quantum state
- We obtained such an extraordinary fit that we could guess the analytic expression for the qEA, but the proof is missing
- The Richardson model seems to be always localized: we analyzed the role played by integrability using the Mazur's inequality
- We considered through exact diagonalization the effect of integrability breaking: the existence of a transition is however questionable