On universal parts of OPE coefficients in N=4 SYM

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CFT AND INTEGRABLE MODELS

RJ, A. Wereszczyński: 1109.XXXX

Outline

Main questions for a CFT

- Find the spectrum of conformal weights
  $\equiv$ eigenvalues of the dilatation operator
  $\equiv$ (anomalous) dimensions of operators

$$\langle O(0) O(x) \rangle = \frac{1}{|x|^{2\Delta}}$$

- Find the OPE coefficients C_{ijk} defined through

$$\langle O_i(x_1) O_j(x_2) O_k(x_3) \rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_i + \Delta_j - \Delta_k} |x_1 - x_3|^{\Delta_i + \Delta_k - \Delta_j} |x_2 - x_3|^{\Delta_j + \Delta_k - \Delta_i}}$$

- Once Δ_i and C_{ijk} are known, all higher point correlation functions are, in principle, determined explicitly.

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- BPS operators correspond to supergravity modes
- Their anomalous dimensions are protected as well as OPE coefficients
- These states are (in this context) commonly called **light states**

- Operators which have nontrivial anomalous dimensions correspond to massive string states
- The lightest of these, corresponding to 'short strings' (e.g. the Konishi operator) can be called by analogy **medium states**

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- For *some specific* operators the OPE coefficient is known up to three loops! Eden, Heslop, Korchemsky, Sokhatchev
- At strong coupling, the case of 3-point correlation functions/OPE coefficients C_{HHL} for two **heavy** and one **light** is well studied Zarembo; Costa et.al. — use the known classical solution for a two-point function of the heavy operators and integrate with the propagator of the light supergravity mode see the talk by Ahn
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The AdS/CFT dictionary

Anomalous dimensions of operators	$\equiv$	Energies of corresponding string states in $AdS_5 \times S^5$

- This approach is very well developed using integrability
- Alternatively we might try to compute the anomalous dimensions directly by computing a 2-point correlation function

$$\langle O(0)O(x)\rangle = \frac{1}{|x|^{2\Delta}}$$

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What is involved?

- For two-point functions, we pass from a Green's function of the **light** state (supergravity field) to a classical string solution with the topology of a **cylinder**, approaching two given points on the boundary *(modulo caveats)*

- For a three-point function, we have to construct a classical solution with the topology of **a thrice punctured sphere**

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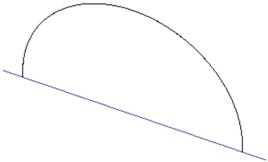

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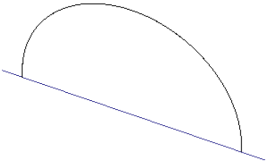

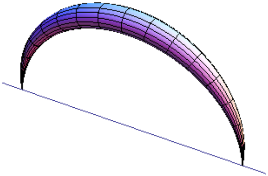

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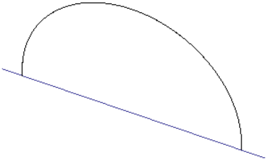

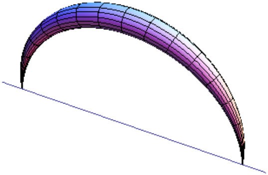

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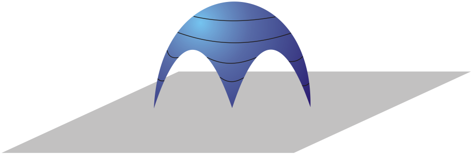

2-point correlation functions

- At the classical level one can consider an Euclidean worldsheet with the topology of a sphere with 2 punctures satisfying Virasoro constraints (saddle point of Minkowskian cylinder amplitude...)
- The solution is complex (i.e. *complexified*)
- The AdS part of the solution is a geodesic cut-off at $z = \mathcal{E}$
- The S^5 part of the solution is just the Wick rotated spinning string solution
  — here we have to include wavefunctions
  — the contribution of the S^5 part is the energy integral

$$\Psi^* \, e^{-\frac{\sqrt{\lambda}}{4\pi}\int_{cylinder} d\sigma d\tau \, \mathcal{L}_{S^5}} \Psi \longrightarrow e^{\frac{\sqrt{\lambda}}{4\pi}\int_{cylinder} d\sigma d\tau \, E^{S^5}_{euclidean}}$$

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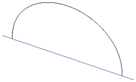

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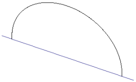
- The solution is complex (i.e. *complexified*)
- The *AdS* part of the solution is a geodesic cut-off at $z = \mathcal{E}$



- The S^5 part of the solution is just the Wick rotated spinning string solution
  — here we have to include wavefunctions
  — the contribution of the S^5 part is the energy integral

$$\Psi^* \, e^{-\frac{\sqrt{\lambda}}{4\pi}\int_{cylinder} d\sigma d\tau \, \mathcal{L}_{S^5}} \, \Psi \longrightarrow e^{\frac{\sqrt{\lambda}}{4\pi}\int_{cylinder} d\sigma d\tau \, E^{S^5}_{euclidean}}$$

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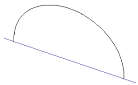

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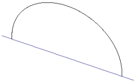

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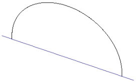

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3-point correlation functions

We are interested in classical solutions with the topology of a thrice-punctured sphere

- Close to the punctures, the solutions should approach the *known* solutions for 2-point functions
- For operators with nontrivial charges only on the S^5, the problem separates into two almost decoupled parts — the S^5 part and the AdS part, resulting in

$$C^{OPE} = \underbrace{C^{OPE}_{AdS}}_{universal} \cdot \underbrace{C^{OPE}_{S^5}}_{operator-dependent}$$

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$$T_{AdS}(w) + T_{S^5}(w) = 0$$

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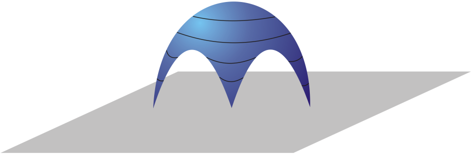

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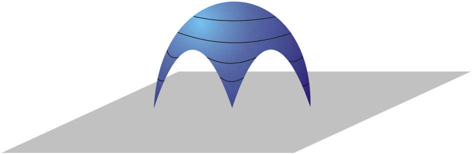

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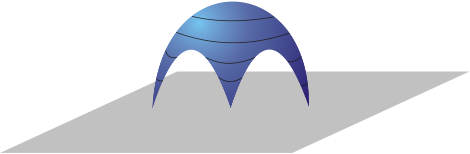

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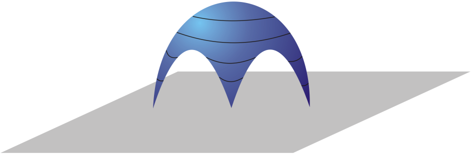

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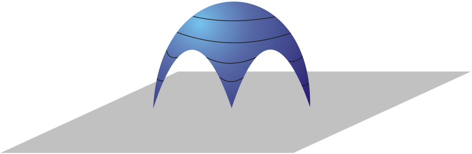

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$$\frac{\partial z \bar{\partial} z + \partial x \bar{\partial} x}{z^2}$$

subject to the constraint

$$\frac{(\partial z)^2 + (\partial x)^2}{z^2} = T(w)$$

- At the punctures, the classical solution should approach three given points x_1, x_2 and x_3 in a way similar to two-point functions of appropriate operators
- The punctures can be fixed e.g. to $w = \pm 1$ and $w = \infty$
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- $T(w)$ for a 2-point correlation function is

$$T_{2-point}(w) = \frac{\Delta^2/4}{w^2}$$

where the anomalous dimension $\boldsymbol{\Delta} \equiv \sqrt{\lambda}\,\Delta$

- Hence $T(w)$ should have poles of order at most two at each puncture with prescribed leading coefficients
- Taking into account the transformation property

$$T(w) \to \frac{1}{u^4} T\left(\frac{1}{u}\right)$$

$T(w)$ is fixed uniquely

$$T(w) = \frac{\Delta_\infty^2}{4} \frac{w^2 + a^2}{(1-w^2)^2} \qquad \text{where} \qquad a^2 = \frac{4\Delta^2}{\Delta_\infty^2} - 1$$

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Pohlmeyer reduction

- Pohlmeyer reduction for the AdS_2 σ-model with *prescribed* $T(w)$ yields the modified sinh-Gordon model

- This is **exactly** the same modified sinh-Gordon model as for *polygonal minimal surfaces* in AdS_3!
- However there are some key differences in the analytic structure and in the target-space picture (AdS_2 versus AdS_3)

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- The generalized sinh-Gordon model is integrable
- There exists a family of flat connections

$$J = \frac{1}{\xi}\,\Phi_w\,dw + A + \xi\,\Phi_{\overline{w}}\,d\overline{w}$$

- Flatness is equivalent to the compatibility of the linear system

$$\partial\Psi + J_w\Psi = 0 \qquad\qquad \overline{\partial}\Psi + J_{\overline{w}}\Psi = 0$$

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$$\int_{\Sigma\setminus\{\varepsilon_i\}} \frac{\partial x \bar{\partial} x + \partial z \bar{\partial} z}{z^2}$$

- $\Sigma \setminus \{\varepsilon_i\}$: 3-punctured sphere with worldsheet cut-offs at the punctures $|w - w_i| > \varepsilon_i$
- nontrivial: ε_i is fixed by a **target-space** cut-off $z = \mathcal{E}$
- The first term – regularized 'Pohlmeyer' action – depends on the solution of modified sinh-Gordon, but not on the worldsheet cut-off
- The second term has a known and explicit integrand but the domain of integration depends on the **target-space** cut-off $z = \mathcal{E}$ — so we need to have some information on reconstructing the target space solution from the Pohlmeyer reduction...

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AdS_2 **string action:**

$$\int_{\Sigma \setminus \{\varepsilon_i\}} \frac{1}{2} \left(e^{2\gamma(w,\overline{w})} + T(w)\overline{T}(\overline{w}) e^{-2\gamma(w,\overline{w})} \right)$$

- $\Sigma \setminus \{\varepsilon_i\}$: 3-punctured sphere with worldsheet cut-offs at the punctures $|w - w_i| > \varepsilon_i$
- nontrivial: ε_i is fixed by a **target-space** cut-off $z = \mathcal{E}$
- The first term – regularized 'Pohlmeyer' action – depends on the solution of modified sinh-Gordon, but not on the worldsheet cut-off
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$$\int_\Sigma \left(2\,\mathrm{tr}\,\Phi_w \Phi_{\overline{w}} - \sqrt{T(w)\overline{T}(\overline{w})}\, d^2 w \right) + \int_{\Sigma \setminus \{\varepsilon_i\}} \sqrt{T(w)\overline{T}(\overline{w})}\, d^2 w$$

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We can use the techniques of Alday, Maldacena, Sever, Vieira to evaluate the regularized ‘Pohlmeyer’ action (albeit with some twists...)

1. Go to a gauge where Φ_w is diagonal.

$$\Phi_w \longrightarrow \begin{pmatrix} -\frac{\sqrt{T}}{2} & 0 \\ 0 & \frac{\sqrt{T}}{2} \end{pmatrix}$$

2. Pass to the double cover $\widetilde{\Sigma}$ given by $y^2 = T(w)$
3. Then the regularized ‘Pohlmeyer’ contribution can be written as

$$\int_{\widetilde{\Sigma}} \omega \wedge \eta$$

where

$$d\omega = 0 \qquad\qquad d\eta = 0$$

Explicitly

$$\underbrace{\omega = \sqrt{T(w)}dw}_{\text{easy}} \qquad\qquad \underbrace{\eta = \frac{1}{2}\sqrt{\overline{T}(\overline{w})}\left(\cosh\tilde{\gamma} - 1\right)d\overline{w} + \frac{1}{4}\frac{1}{\sqrt{T(w)}}(\partial\tilde{\gamma})^2 dw}_{\text{hard}}$$

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Regularized 'Pohlmeyer' contribution

5. If $\widetilde{\Sigma}$ had genus g, one would use Riemann bilinear identity (or reciprocity) to reduce the integral to products of integrals over cycles

$$\int_{\Sigma_g} \omega \wedge \eta = \sum_{i=1}^{g} \int_{A_i} \omega \int_{B_i} \eta - \int_{A_i} \eta \int_{B_i} \omega$$

But here $\widetilde{\Sigma}$ has genus 0 and punctures...
Moreover the forms have singularities at the punctures *and* at the branch points of the double cover...

6. Generalize the Riemann formula to the present case...
Two ways to proceed:
— redo the proof directly for $\widetilde{\Sigma} \setminus \{singularities\}$ **quite messy...**
— treat the punctures as infinitesimal cuts, use reciprocity for genus 3 and deal only with the singularitites of η at the branch points... **easier...**

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8. We use simplifying properties:
   — integrals of ω around punctures are proportional to Δ_i
   — integrals of ω around branch points vanish
   — integrals of η around punctures vanish
   — the contribution of the branch point is just $\pi/6$
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$$\int_\Sigma \left(2\,\mathrm{tr}\,\Phi_w \Phi_{\overline{w}} - \sqrt{T(w)\overline{T}(\overline{w})}\, d^2 w \right) = \frac{\pi}{6} - \frac{\pi}{2} \left((\Delta_\infty - 2\Delta) \int_{C_{-1\,1}} \eta - 2\Delta_\infty \int_{C_{1\,\infty}} \eta \right)$$

10. The integrals of η between the punctures are related to the WKB asymptotics of the products between local solutions of the linear system...

c.f. Alday, Maldacena, Sever, Vieira

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Overlaps between solutions of the linear system

- Around each puncture w_k we have two distinguished solutions (k and $\overline{k}$) of the linear system

$$(*) \qquad \partial\Psi + J_w \Psi = 0 \qquad\qquad \overline{\partial}\Psi + J_{\overline{w}}\Psi = 0$$

characterized by the mondromies

$$e^{\pm i\Delta_k \frac{\pi}{2}\left(\xi - \frac{1}{\xi}\right)} \equiv e^{\pm i \Delta_k \pi \sinh\theta} \equiv e^{\pm i p_k(\theta)} \qquad \text{where} \quad \xi = e^\theta$$

- Given two solutions of the linear system $(*)$, Ψ_1^a and Ψ_2^a, one can form the (skew) product

$$\langle \Psi_1 \Psi_2 \rangle \equiv \varepsilon_{ab} \Psi_1^a \Psi_1^b$$

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Alday, Maldacena, Sever, Vieira link the large $|\theta|$ WKB asymptotics of the solution of the linear system to the integral of ω (leading asymptotics) and η (subleading asymptotics).

In our context this becomes:

The integral of η along C_{kl} can be extracted from the $\theta \to -\infty$ asymptotics the overlap $\langle kl \rangle (\theta)$:

$$\langle kl \rangle (\theta) \sim \exp \left\{ e^{-\theta} \int_{C_{kl}} \frac{1}{2}\omega + const + e^{\theta} \int_{C_{kl}} \left(\eta + \frac{1}{2}\sqrt{\overline{T}} d\overline{w} \right) + \ldots \right\}_{reg.}$$

Question: How to find $\langle kl \rangle (\theta)$ without solving any differential equations?

Overlaps between solutions of the linear system

Alday, Maldacena, Sever, Vieira link the large $|\theta|$ WKB asymptotics of the solution of the linear system to the integral of ω (leading asymptotics) and η (subleading asymptotics).

In our context this becomes:

The integral of η along C_{kl} can be extracted from the $\theta \to -\infty$ asymptotics the overlap $\langle kl \rangle (\theta)$:

$$\langle kl \rangle (\theta) \sim \exp \left\{ e^{-\theta} \int_{C_{kl}} \frac{1}{2}\omega + const + e^{\theta} \int_{C_{kl}} \left(\eta + \frac{1}{2}\sqrt{\overline{T}} d\overline{w} \right) + \dots \right\}_{reg.}$$

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1. Collect all overlaps into a connection matrix between solutions at two punctures

$$M_{kl} = \begin{pmatrix} -\langle \bar{k} l \rangle & -\langle \bar{k}\bar{l} \rangle \\ \langle kl \rangle & \langle k\bar{l} \rangle \end{pmatrix}$$

2. Write all obvious compatibility relations

$$M_{km} = M_{kl} M_{lm} \qquad\qquad M_{kl} M_{lk} = id$$

and the condition of zero monodromy outside the punctures

$$\Omega_1 M_{13} \Omega_3 M_{32} \Omega_2 M_{21} = id$$

3. **Key point:** One can show that

$$\langle \bar{k}\bar{l} \rangle (\theta) = - \langle kl \rangle (\theta + i\pi) \qquad\qquad \langle \bar{k} l \rangle (\theta) = - \langle k\bar{l} \rangle (\theta + i\pi)$$

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Overlaps between solutions of the linear system

6. We obtain in particular

$$\langle 32\rangle \langle 32\rangle^{++} = \frac{\sin \frac{p_1(\theta)-p_2(\theta)-p_3(\theta)}{2} \sin \frac{p_1(\theta)+p_2(\theta)+p_3(\theta)}{2}}{\sin p_2(\theta) \sin p_3(\theta)}$$

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7. The basic functional equation to solve is

$$f^+ f^- = 1 - e^{-a\pi \cosh\theta}$$

with the solution

$$f(\theta) = \underbrace{e^{Me^{\theta}+\bar{M}e^{-\theta}}}_{\text{zero mode part}} \cdot \exp \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \frac{\log\left(1 - e^{-a\pi\cosh\theta'}\right)}{\cosh(\theta-\theta')}$$

8. The zero-mode constants can be obtained from integrals of $\sqrt{T}$ and $\sqrt{\bar{T}}$

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Regularized 'Pohlmeyer' contribution

Recall:

$$\int_\Sigma \left(2\,\mathrm{tr}\,\Phi_w \Phi_{\overline{w}} - \sqrt{T(w)\overline{T}(\overline{w})}\, d^2 w \right) = \frac{\pi}{6} - \frac{\pi}{2} \left((\Delta_\infty - 2\Delta) \int_{C_{-1\,1}} \eta - 2\Delta_\infty \int_{C_{1\,\infty}} \eta \right)$$

From the solution of the functional equations we get

$$\int_{C_{-1\,1}} \eta = h(2\Delta - \Delta_\infty) + h(2\Delta + \Delta_\infty) - 2h(2\Delta)$$

$$\int_{C_{1\,\infty}} \eta = h(\Delta_\infty) + h(2\Delta + \Delta_\infty) - h(2\Delta) - h(2\Delta_\infty)$$

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$$h(a) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^\theta \log\left(1 - e^{-a\pi \cosh\theta}\right) d\theta$$

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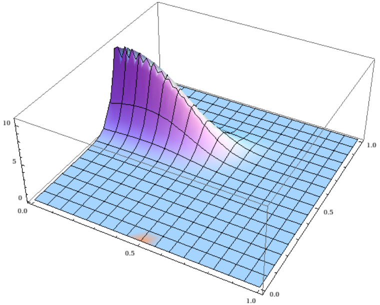
Numerical check

We have solved the modified sinh-Gordon equations numerically and evaluated the resulting regularized 'Pohlmeyer' contribution in order to compare with our analytical formula.

Regularized action density



$\Delta = \Delta_\infty = 4.0$

Δ	Δ_∞	numerics	our formula
0.2	0.3	0.04536	0.0450779
0.5	0.9	0.107649	0.107622
1.	1.	0.426311	0.426166
1.	1.05	0.429572	0.429503
2.	2.	0.517689	0.517688
2.	3.	0.488985	0.488985
4.	4.	0.523584	0.523584
4.	7.99	0.0152435	0.0152435

Numerics become more difficult and less reliable for small Δ's

$$AdS_2 \text{ action} = \underbrace{\int_\Sigma \left(2\,\mathrm{tr}\,\Phi_w \Phi_{\overline{w}} - \sqrt{T(w)\overline{T}(\overline{w})}\, d^2w \right)}_{\textbf{Done!}} + \int_{\Sigma \setminus \{\varepsilon_i\}} \sqrt{T(w)\overline{T}(\overline{w})}\, d^2w$$

To this we have to add the (unknown) S^5 contribution

$$\underbrace{\int_\Sigma \left(S^5 \text{ contribution} - \sqrt{T(w)\overline{T}(\overline{w})}\, d^2w \right)}_{\textbf{Unknown but finite!}} + \int_{\Sigma \setminus \{\varepsilon_i\}} \sqrt{T(w)\overline{T}(\overline{w})}\, d^2w$$

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The cut-off integral

- We have to calculate

$$\int_{\Sigma\setminus\{\varepsilon_i\}} \sqrt{T(w)\overline{T}(\overline{w})}\, d^2w$$

- This integral can be done analytically

$$\int_{\Sigma\setminus\{\varepsilon_i\}} \sqrt{T(w)\overline{T}(\overline{w})}\, d^2w = \mathit{Finite} + \Delta^2 \log\varepsilon_1 + \Delta^2 \log\varepsilon_{-1} + \Delta_\infty^2 \log\varepsilon_\infty$$

where

$$\mathit{Finite} = \pi\left(-\left(\Delta + \frac{\Delta_\infty}{2}\right)^2 \log(2\Delta+\Delta_\infty) - 2\Delta^2\log\Delta + \ldots\right)$$

- We have to link the worldsheet cut-off's $\varepsilon_1, \varepsilon_{-1}, \varepsilon_\infty$ to the target space cut-off $z = \mathcal{E}$

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Reconstruction formula

The classical solution in AdS_2 is uniquely specified by a choice of **two** specific solutions Ψ_A, Ψ_B of the linear system at $\theta = 0$

$$\partial\Psi + J_w\Psi = 0 \qquad \overline{\partial}\Psi + J_{\overline{w}}\Psi = 0$$

normalized by $\langle \Psi_A \Psi_B \rangle = 1$

Then the coordinates of the punctures are encoded in the products at $\theta = 0$

$$x_k = \frac{\langle k \Psi_B \rangle}{\langle k \Psi_A \rangle}$$

and the target-space ($z = \mathcal{E}$) and worldsheet cut-offs are linked by

$$\Delta_k \log \varepsilon_k = \log \left(\mathcal{E} |\langle k \Psi_A \rangle|^2 \right)$$

These expressions are enough to evaluate

$$\Delta^2 \log \varepsilon_1 + \Delta^2 \log \varepsilon_{-1} + \Delta^2_\infty \log \varepsilon_\infty$$

in terms of i) the target-space data $x_k, \mathcal{E}$ and ii) overlaps $\langle kl \rangle \, (\theta = 0)$

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normalized by $\langle \Psi_A \Psi_B \rangle = 1$

Then the coordinates of the punctures are encoded in the products at $\theta = 0$

$$x_k = \frac{\langle k \Psi_B \rangle}{\langle k \Psi_A \rangle}$$

and the target-space ($z = \mathcal{E}$) and worldsheet cut-offs are linked by

$$\Delta_k \log \varepsilon_k = \log \left(\mathcal{E} |\langle k \Psi_A \rangle|^2 \right)$$

These expressions are enough to evaluate

$$\Delta^2 \log \varepsilon_1 + \Delta^2 \log \varepsilon_{-1} + \Delta^2_\infty \log \varepsilon_\infty$$

in terms of i) the target-space data $x_k, \mathcal{E}$ and ii) overlaps $\langle kl \rangle$ $(\theta = 0)$

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1. The standard spacetime dependence ($\boldsymbol{\Delta}_i \equiv \sqrt{\lambda}\Delta_i$)

$$\frac{1}{\left(\frac{x_{23}}{\mathcal{E}}\right)^{\boldsymbol{\Delta}_2+\boldsymbol{\Delta}_3-\boldsymbol{\Delta}_1} \left(\frac{x_{13}}{\mathcal{E}}\right)^{\boldsymbol{\Delta}_1+\boldsymbol{\Delta}_3-\boldsymbol{\Delta}_2} \left(\frac{x_{12}}{\mathcal{E}}\right)^{\boldsymbol{\Delta}_1+\boldsymbol{\Delta}_2-\boldsymbol{\Delta}_3}}$$

2. An additional contribution to the OPE coefficients (all products at $\theta = 0$)

$$-(\Delta_2+\Delta_3-\Delta_1)\log\langle 32\rangle - (\Delta_1+\Delta_3-\Delta_2)\log\langle 13\rangle - (\Delta_1+\Delta_2-\Delta_3)\log\langle 12\rangle$$

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3. The zero-mode part of $\langle kl\rangle$ **exactly cancels** the remaining *Finite* part of $\int_{\Sigma\setminus\{\varepsilon_i\}}\sqrt{T(w)\overline{T}(\overline{w})}$
4. We obtain, however, a nonzero contribution from the nontrivial part of $\langle kl\rangle$

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Final answer

The final answer for the OPE coefficient is a product of the currently unknown regularized S^5 contribution

$$\underbrace{\exp^{\frac{\sqrt{\lambda}}{2}\int_\Sigma \left(S^5 \text{ contribution} - \sqrt{T(w)\overline{T}(\overline{w})}\, d^2 w\right)}}_{\textbf{Unknown but finite!}}$$

and the explicit AdS answer

$$\underbrace{\exp\left\{-\frac{\sqrt{\lambda}}{6} - \sqrt{\lambda}\left[(2\Delta - \Delta_\infty)\tilde{P}_{-11} + 2\Delta_\infty \tilde{P}_{1\infty}\right]\right\}}_{\textbf{Our main result}}$$

where

$$\begin{aligned} \tilde{P}_{-11} &= \tilde{h}(2\Delta - \Delta_\infty) + \tilde{h}(2\Delta + \Delta_\infty) - 2\tilde{h}(2\Delta) \\ \tilde{P}_{1\infty} &= \tilde{h}(\Delta_\infty) + \tilde{h}(2\Delta + \Delta_\infty) - \tilde{h}(2\Delta) - \tilde{h}(2\Delta_\infty) \end{aligned}$$

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Comments:

- In the extremal limit $\Delta_\infty = 2\Delta$ we obtain 1 (the $1/6$ is crucial here)
- For large Δ, $\tilde{h}(a) \propto e^{-\pi a}$ and the AdS contribution approaches a constant

$$e^{-\frac{\sqrt{\lambda}}{6}}$$

- For small Δ, $\tilde{h}(a) \sim -\frac{1}{6a} - \frac{1}{2}\log a$. Then we get ($\boldsymbol{\Delta} \equiv \sqrt{\lambda}\Delta$)

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The large Δ limit and the Painleve transcendent

- It is possible to understand the universal $\exp(-\sqrt{\lambda}/6)$ large Δ limit directly
- The Pohlmeyer equation reads

$$\partial\bar{\partial}\tilde{\gamma} = \sqrt{T\overline{T}}\sinh\tilde{\gamma}$$

with $\sqrt{T\overline{T}} \propto \Delta_\infty^2 |w^2 + a^2| \cdot \ldots$

- The solution vanishes at the punctures, but has a logarithmic singularity at $w = \pm ia$
- For large Δ, the solution is almost everywhere zero with logarithmic spikes around $w = \pm ia$
- The Pohlmeyer equation reduces there to radially symmetric Euclidean sinh-Gordon equation for the Painleve III transcendent Zamolodchikov '94

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- Using the asymptotics Zamolodchikov '94

$$F_c(R)\sim -R/4 \quad \text{for } R\to\infty \qquad\qquad R\,F_c(R)\to 1/18 \quad \text{for } R\to 0$$

the integral may be evaluated to

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the integral may be evaluated to

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The large Δ limit and the Painleve transcendent

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- We have evaluated the (universal) AdS part of the OPE coefficients which depends only on the dimensions of the operators

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- It would be interesting to compare with the C_{HHL} computations with $\Delta_{light} \to \infty$. One should determine backreaction and vertex operator contributions in the C_{HHL} calculations...
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