High Energy Scattering in IFT

Bologna, September 2011

• Understanding behavior of the masses of particles in IFT as the functions of the parameters requires data about high-energy limit of scattering amplitudes.

• The high-energy limit of elastic amplitude at small magnetic field will be derived using relation of the zero-field IFT to classical integrable system. Based on recent work of I.Ziyatdinov & AZ

• The result leads to a puzzle about behavior of the resonance states in IFT.

Ising Field Theory = scaling limit of the 2D Ising model near its critical point $T = T_c$, $H = 0$. $\Rightarrow$ Euclidean quantum field theory

$$\mathcal{A}_{\text{IFT}} = \mathcal{A}_{\text{c=1/2 CFT}} - \frac{m}{2\pi}\int \varepsilon(x)\, d^2x + h\int \sigma(x)\, d^2x \ ,$$

$\varepsilon(x)$ with $(\Delta, \bar{\Delta}) = (1/2, 1/2)$ ("energy density");

$$m \sim T_c - T$$

$\sigma(x)$ with $(\Delta, \bar{\Delta}) = (1/16, 1/16)$ ("spin density");

$$h \sim H$$

• Statistical mechanics: IFT describes basic universality class in 2D phase transitions (e.g. critical point in liquid-vapor transition). Thermodynamics is expressed through the IFT vacuum energy density

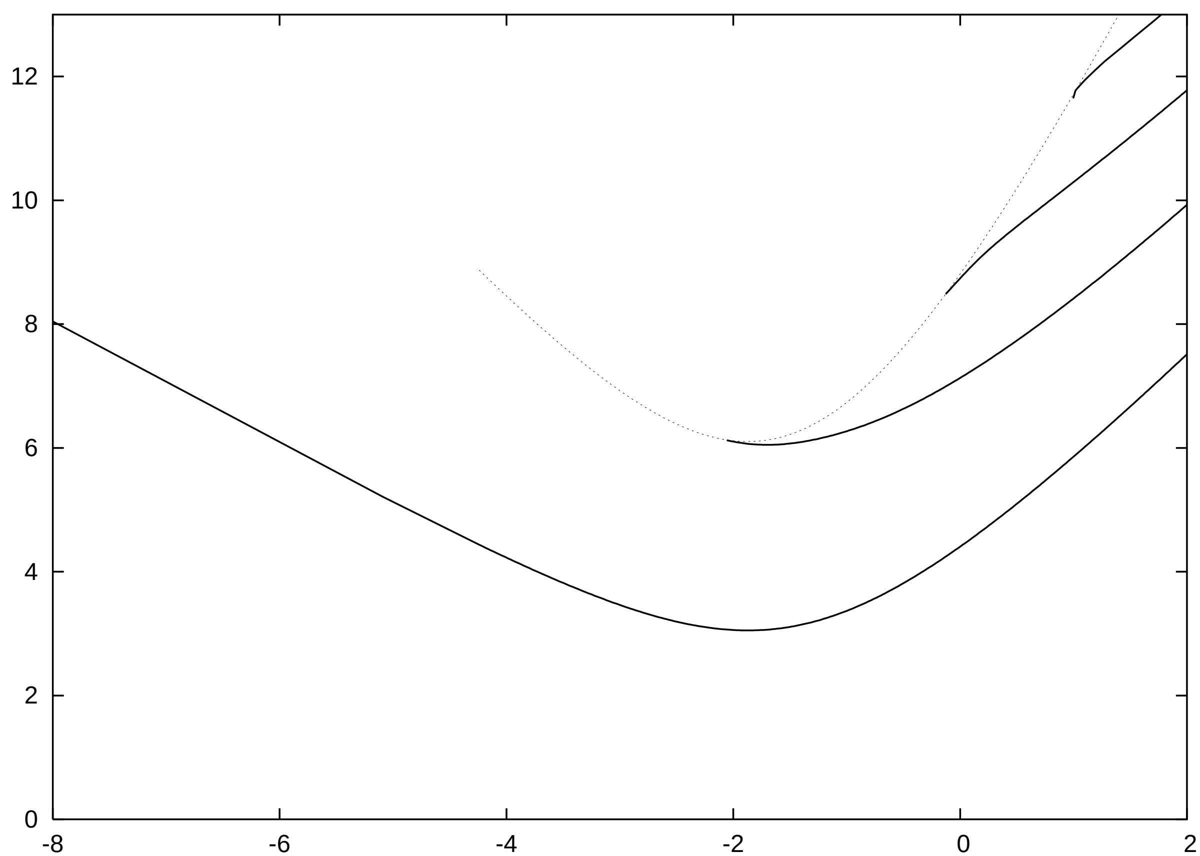
$$\mathcal{F}(m,h) = \frac{m^2}{8\pi}\ \log m^2 + h^{\frac{16}{15}}\,\Phi(\eta)\,, \qquad \eta = \frac{m}{|h|^{\frac{8}{15}}}$$

• Generally [i.e. except for $(m,h)=(0,0)$, and the Yang-Lee point $h/(-m)^{15/16} = \pm i\,(0.1893)$] IFT is massive $\Rightarrow$ Particle theory in $1+1$ (mass spectrum, scattering amplitudes, etc) $\Rightarrow$ Determines the "fine structure" of the scaling theory (amplitude ratios, correlation functions, finite-size effects, etc).

Qualitative understanding of the IFT particle theory - "McCoy-Wu scenario" (1978): The mass spectrum interpolates between the infinite tower of "mesons" at $\eta \to +\infty$ (low-T regime) and one stable particle at $\eta - \infty$ (high-T regime).

Numerical analysis (via TCSA of Al.Zamolodchikov, by Delfino, Mussardo, Simonetti (1996), Fonseca, AZ (2001))

Particle masses (in the units $|h|^{8/15}$) vs $\eta = m/|h|^{8/15}$



I will refer to the stable particle as A_n, and their masses (measured in the units of $|h|^{8/15}$) as $M_n = M_n(\eta)$,

$$n = 1, 2, ..., N$$

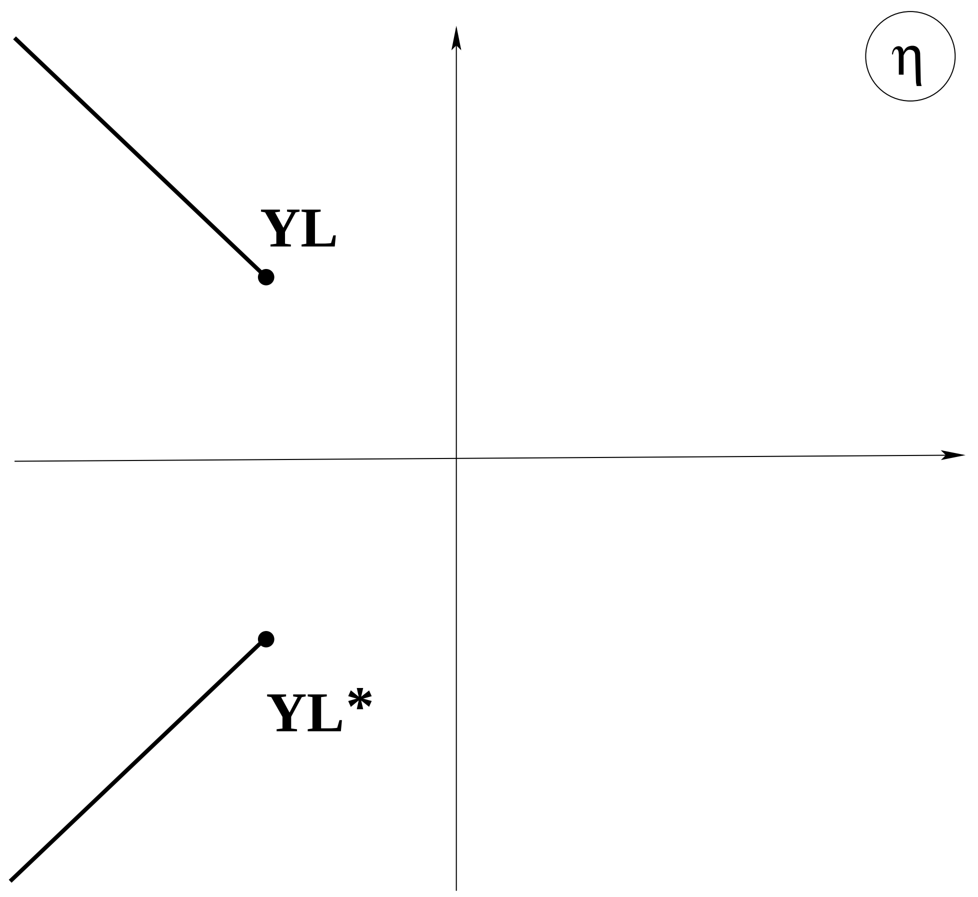
Questions:

What happens to the particle masses when they leave the spectrum of stable particles?

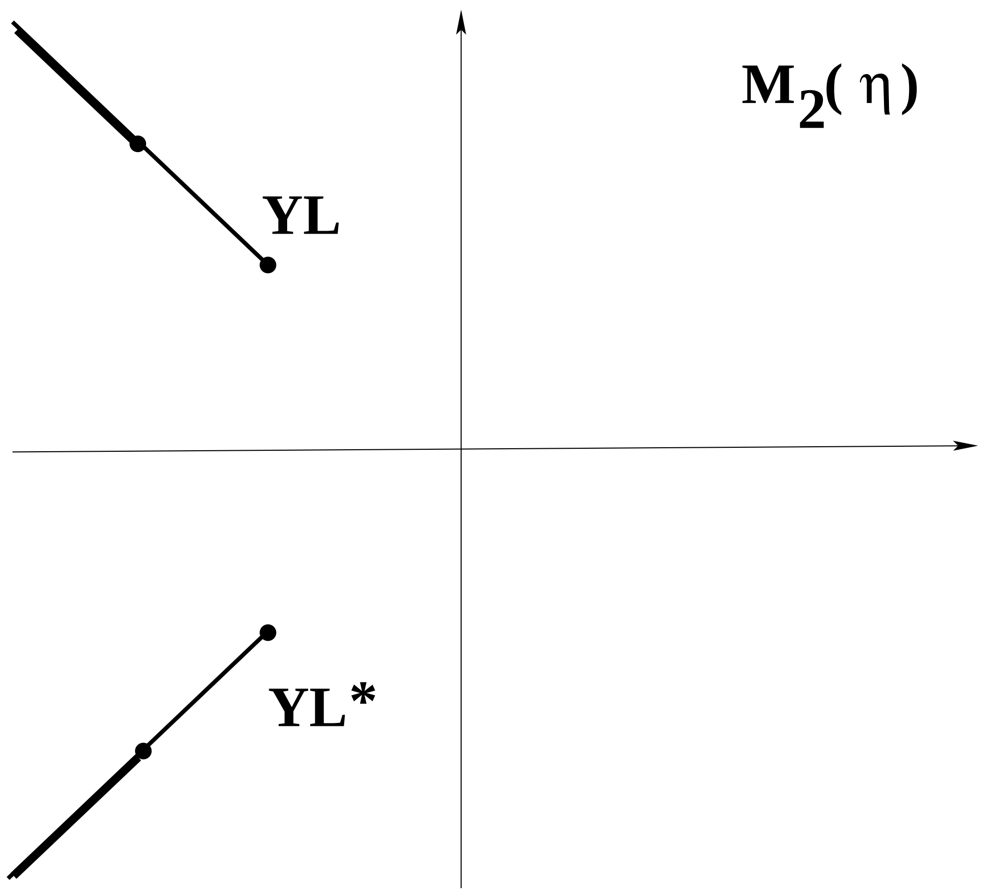
The resonance states may also disappear. How this happens?

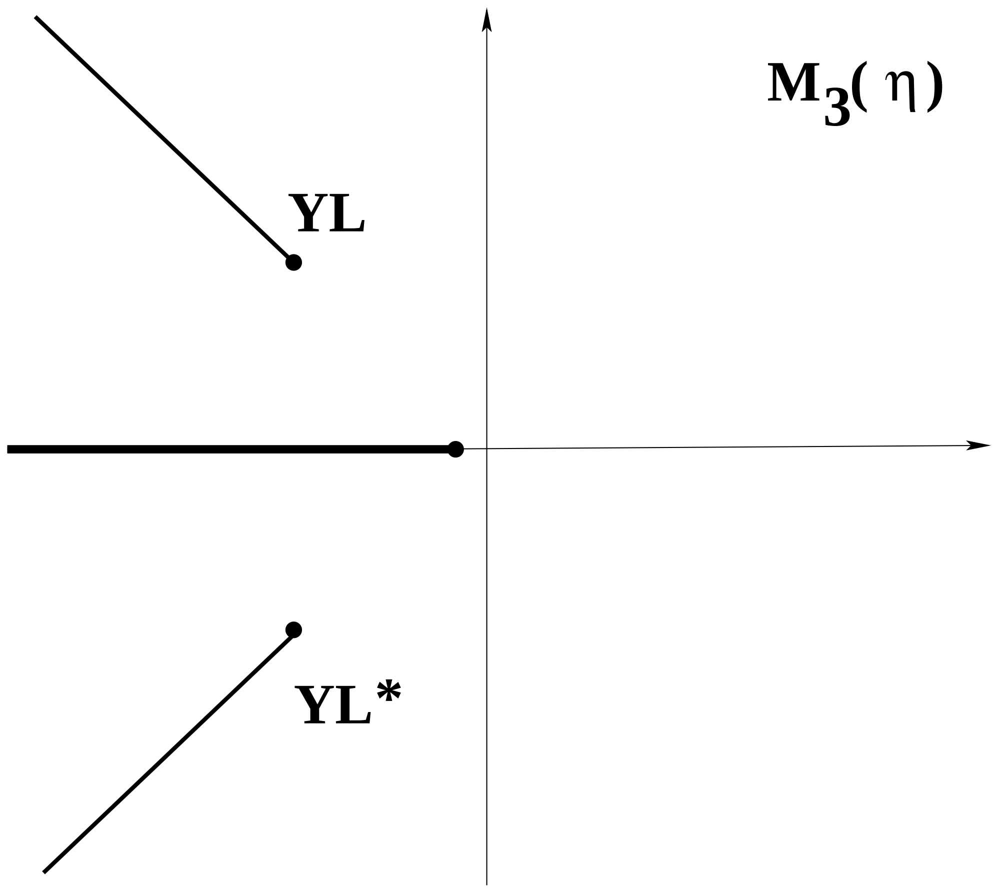
What are the analytic properties of $M_n(\eta)$ as the functions of η?

Analyticity is (partly) understood for the free energy, i.e. $\Phi(\eta)$ [P.Fonseca, AZ (2001)], and for $M_1(\eta)$.



For the higher masses the situation becomes somewhat more complicated: Algebraic singularities (akin to "level crossings") emerge. E.g.





$M_3(\eta)$
YL
YL*

It is useful to discuss in terms of the elastic $A_1 + A_1 \to A_1 + A_1$ scattering amplitude $S(\theta)$, defined as usual

$$\begin{aligned} \mid A_1(\theta_1) A_1(\theta_2)\rangle_{in} &= S(\theta_1 - \theta_2) \mid A_1(\theta_1) A_1(\theta_2)\rangle_{out} + \\ &+ \text{ inelastic terms} \end{aligned}$$

$S(\theta)$:

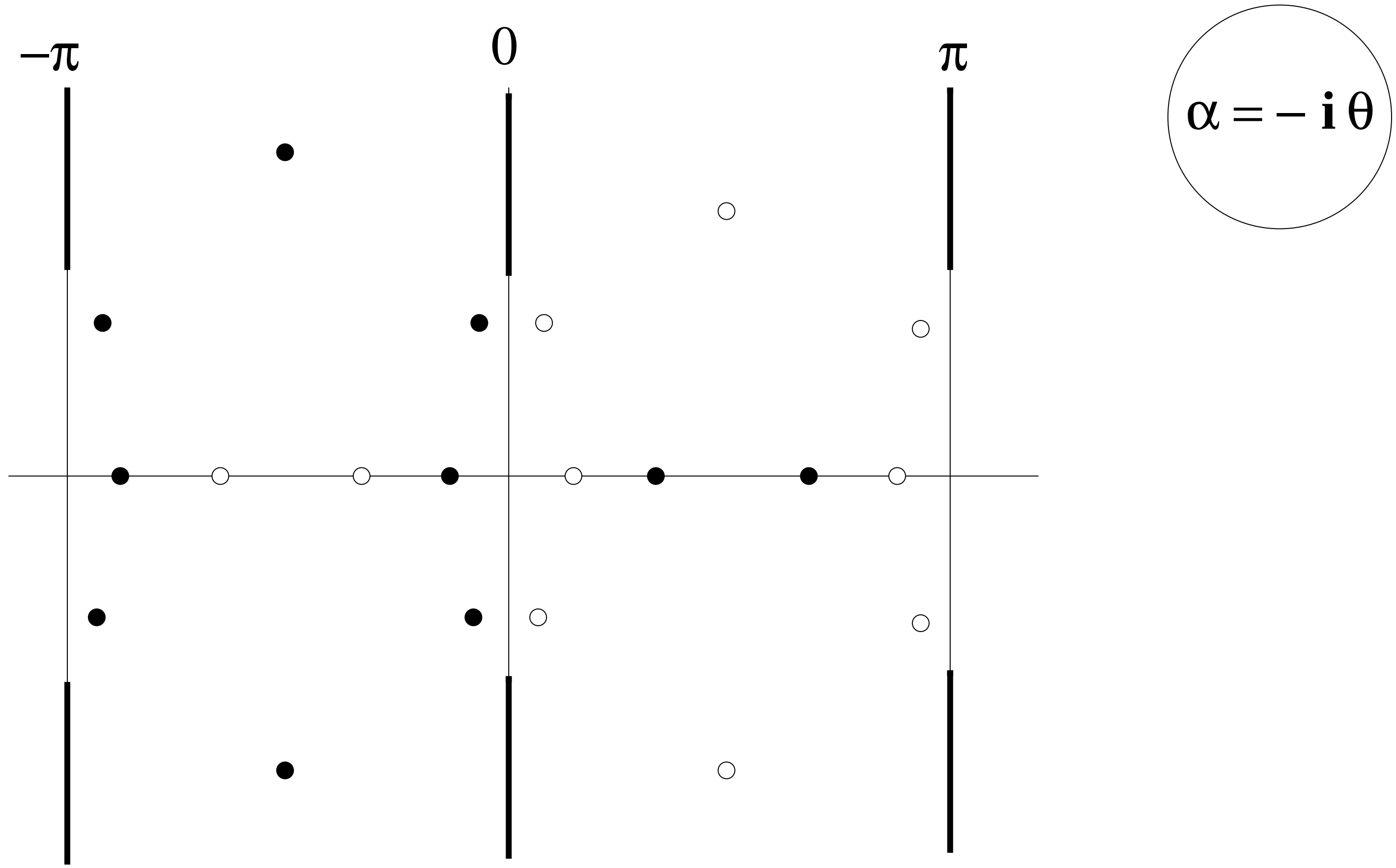
• Is analytic in the θ-plane with the branching singularities at $\theta = \pm\theta_X + i\pi\mathbb{Z}$, associated with the inelastic thresholds $A_1 + A_2 \to X$

• Satisfies (on the principle sheet)

$$S(\theta)S(-\theta) = 1\,, \qquad S(\theta) = S(i\pi - \theta)$$

and hence it is periodic, $S(\theta) = S(2\pi i + \theta)$.

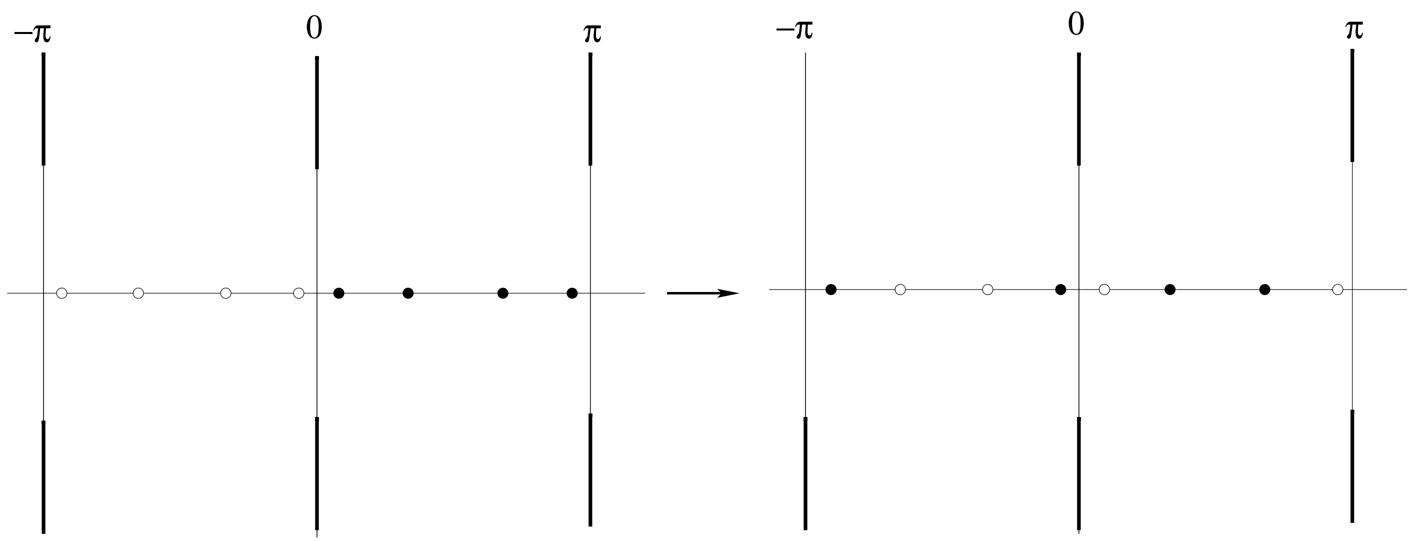
• Has poles associated with bound states, virtual and resonance states



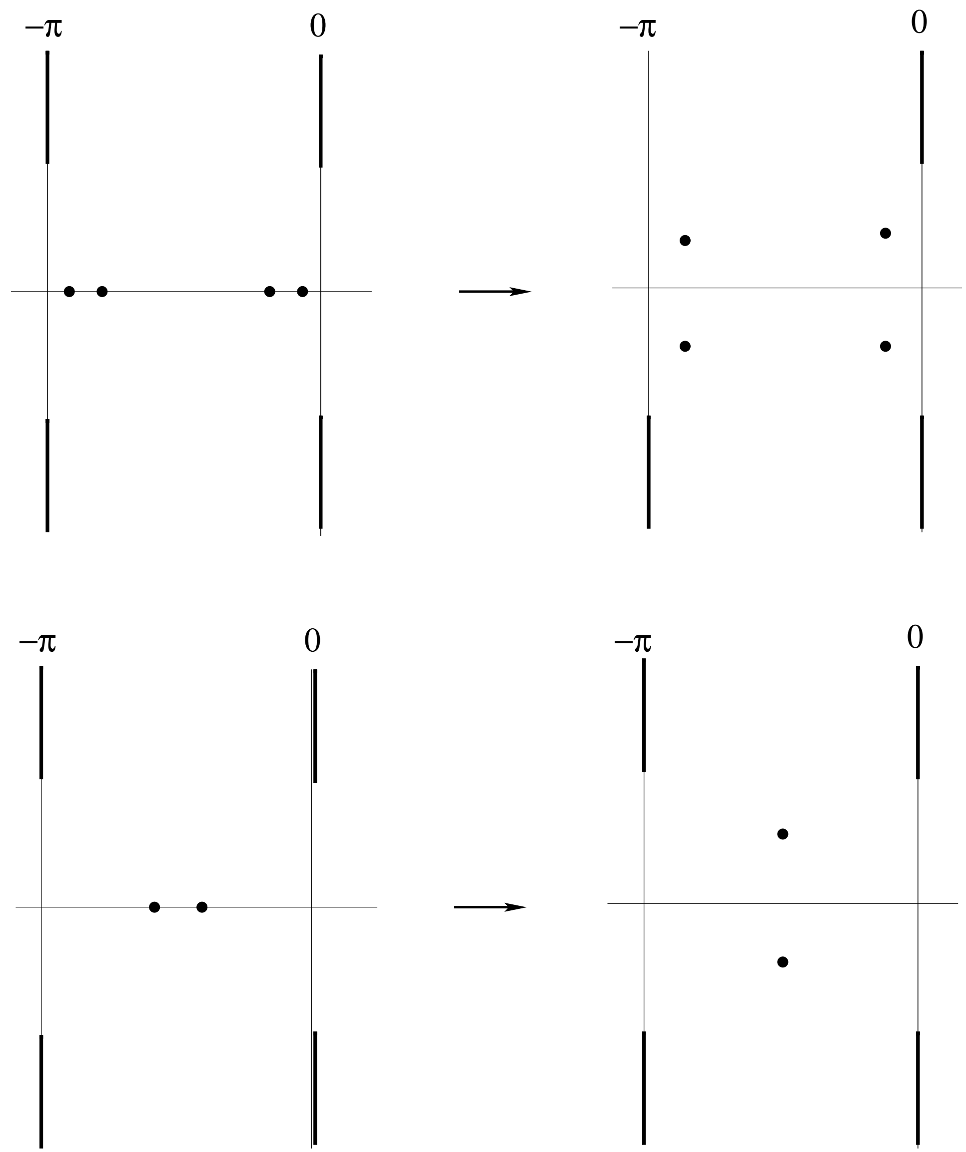
"Physical Strip": $0 < \Im m\,\theta < \pi$; Poles here are associated with bound states.

"Mirror strip": $-\pi < \Im m\,\theta < 0$; Here the virtual state and resonance poles are located.

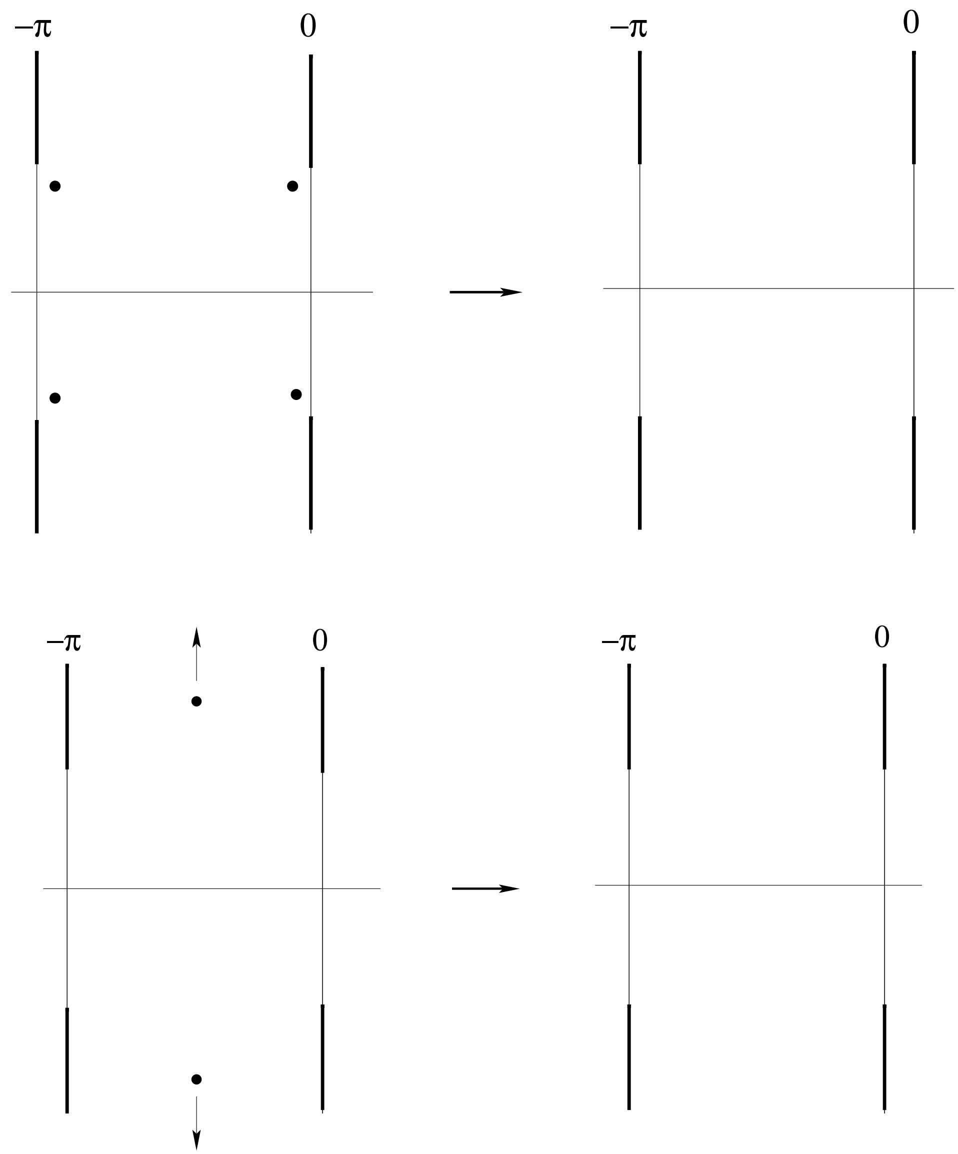
A particle leaves the spectrum in a familiar way - by becoming a virtual state (the poles moves from PS to MS)



Virtual states can turn to resonances (and vice versa) when poles collide in the MS. This leads to "level-crossing" square-root singularities of $M_n(\eta)$



In turn, resonances can disappear, in two ways:



The later scenario - departing of the resonance poles to infinity - can be observed by looking at high-energy behavior of $S(\theta)$.

This was the motivation of the high-energy amplitude via perturbation theory in h^2 (in the high-T regime) below.

High-energy scattering in IFT by perturbation theory in h

(I. Ziyatdinov & AZ, 2011)

$2 \to 2$ S-matrix element $S(\theta_{12}) \equiv S_{2\to 2}(\theta_1 - \theta_2)$ in

$$\begin{aligned} \mid \theta_1, \theta_2 \rangle_{in} \quad &= \quad S_{2\to 2}(\theta_1, \theta_2) \mid \theta_1, \theta_2 \rangle_{out} + \sum_{n=3}^{\infty} \int \frac{d\beta_1}{2\pi} \cdots \frac{d\beta_n}{2\pi} \times \\ & \frac{(2\pi)^2}{n!} \delta^{(2)}(P_{in} - P_{out}) S_{2\to n}(\theta_1, \theta_2 | \beta_1, \cdots, \beta_n) \mid \beta_1, \cdots, \beta_n \rangle_{out} \, , \end{aligned}$$

At $h = 0$: $S(\theta_{12}) = -1$, $S_{2\to n} = 0$.

At $h \neq 0$

$$S(\theta_{12}) \, \delta(\theta_1 - \theta_1') \delta(\theta_2 - \theta_2') = \\ - \langle \theta_1, \theta_2 \mid T \exp\Big\{ - ih \int \sigma(x) \, d^2 x \Big\} \mid \theta_1', \theta_2' \rangle_{\text{conn}}$$

Complication: $\sigma(x)$ is not local with respect to free field; no manifestly covariant perturbation theory (i.e. Feynman diagrams) is known.

Leading correction

$$S(\theta) = -\left[1 + h^2 \frac{iA(\theta)}{\sinh\theta} + O(h^4)\right],$$

$$i\,A(\theta_{12}) = -\frac{1}{2}\int d^2x\, \underbrace{\langle\theta_1,\theta_2 \mid T\sigma(x)\sigma(0) \mid \theta_1,\theta_2\rangle_{\text{conn}}}_{\Uparrow}$$

The matrix element here is expressed in terms of solutions of the linear problem associated with classical sinh-Gordon equation

$$\partial_{\mathsf{z}}\partial_{\bar{\mathsf{z}}}\varphi = \frac{1}{8}\sinh(2\varphi),$$

($\mathsf{z} = \mathsf{x} - \mathsf{t}$ and $\bar{\mathsf{z}} = \mathsf{x} + \mathsf{t}$ are the light-cone coordinates) The relevant solution is Lorentz invariant, i.e.

$$\varphi = \varphi(\rho), \qquad \rho = \mathsf{z}\bar{\mathsf{z}} = \mathsf{x}^2 - \mathsf{t}^2.$$

$$\varphi(\rho) \to -\frac{1}{2}\log\frac{\rho}{4} - \log\left(-\frac{1}{2}\log\left(\frac{\rho\, e^{2\gamma_E}}{64}\right)\right) \qquad \rho\to 0,$$

$$\varphi(\rho) \to \frac{2}{\pi}K_0\left(\sqrt{\rho}\right) \qquad \rho\to\infty$$

Wu, McCoy, Tracy, Barouch (1976):

$$G \equiv \langle 0 \mid T\sigma(\mathsf{z},\bar{\mathsf{z}})\sigma(0) \mid 0\rangle = e^{\chi/2}\,\sinh\Big(\varphi/2\Big)\,,$$
$$\tilde{G} \equiv \langle 0 \mid T\mu(\mathsf{z},\bar{\mathsf{z}})\mu(0) \mid 0\rangle = e^{\chi/2}\,\cosh\Big(\varphi/2\Big)\,.$$

$$\partial_{\mathsf{z}}\partial_{\bar{\mathsf{z}}}\chi = \frac{1}{8}\left[1 - \cosh(2\varphi)\right].$$

The matrix elements are expressed in terms of $\Psi_{\pm}(\mathsf{z},\bar{\mathsf{z}}|\beta)$, the solution of the associated Lax system,

$$\partial_{\mathsf{z}}\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix} = \frac{1}{4}\begin{pmatrix}-2\,\partial_{\mathsf{z}}\varphi & -e^{\theta}\,e^{-\varphi}\\ e^{\theta}e^{\varphi} & 2\,\partial_{\mathsf{z}}\varphi\end{pmatrix}\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix}$$

and

$$\partial_{\bar{\mathsf{z}}}\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix} = \frac{1}{4}\begin{pmatrix}2\,\partial_{\bar{\mathsf{z}}}\varphi & -e^{-\theta}\,e^{-\varphi}\\ e^{-\theta}e^{\varphi} & -2\,\partial_{\bar{\mathsf{z}}}\varphi\end{pmatrix}\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix}$$

with e^{θ} being the "spectral parameter [P.Finseca, AZ (2003)]

By the Lorentz invariance, $\Psi_\pm$ depend on the combination

$$Z = \mathsf{z}\, e^{\theta}\,, \qquad \bar{Z} = \bar{\mathsf{z}}\, e^{-\theta}$$

(which are the light-cone coordinates in the rest frame of a particle of the rapidity θ), and I will write

$$\Psi_\pm(Z, \bar{Z})\,.$$

$$\langle\theta_1,\theta_2|\sigma(x/2)\sigma(-x/2)|\theta_1,\theta_2\rangle_{\rm conn} =$$

$$G^{-1}\Big[\mathcal{G}(\theta_2|\theta_1)\mathcal{G}(\theta_1|\theta_2) - \mathcal{G}(\theta_1|\theta_1)\mathcal{G}(\theta_2|\theta_2) + \mathcal{G}(\theta_1,\theta_2)\mathcal{G}(\theta_1,\theta_2)\Big].$$

where

$$\begin{aligned}
\mathcal{G}(\theta_1,\theta_2) &= -\frac{i}{2}\left[G\,\frac{e^{\theta_1}-e^{\theta_2}}{e^{\theta_1}+e^{\theta_2}}\,\Psi_s(\theta_1,\theta_2) - \tilde{G}\,\Psi_a(\theta_1,\theta_2)\right],\\
\mathcal{G}(\theta_1|\theta_2) &= -\frac{1}{2}\left[G\,\frac{e^{\theta_1}+e^{\theta_2}}{e^{\theta_1}-e^{\theta_2}}\,\Psi_a(\theta_1,\theta_2) - \tilde{G}\,\Psi_s(\theta_1,\theta_2)\right],
\end{aligned}$$

$G = \langle 0 \mid T\sigma(x/2)\sigma(-x/2) \mid 0\rangle$, $\tilde{G} = \langle 0 \mid T\mu(x/2)\mu(-x/2) \mid 0\rangle$, and

$$\begin{aligned}
\Psi_s(\theta_1,\theta_2) &= \Psi_+(Z_1,\bar{Z}_1)\Psi_-(Z_2,\bar{Z}_2) + \Psi_-(Z_1,\bar{Z}_1)\Psi_+(Z_2,\bar{Z}_2)\,,\\
\Psi_a(\theta_1,\theta_2) &= \Psi_+(Z_1,\bar{Z}_1)\Psi_-(Z_2,\bar{Z}_2) - \Psi_-(Z_1,\bar{Z}_1)\Psi_+(Z_2,\bar{Z}_2)\,.
\end{aligned}$$

Here

$$(Z_1,\bar{Z}_1) = \left(e^{\theta_1}\mathsf{z},\, e^{-\theta_1}\bar{\mathsf{z}}\right), \qquad (Z_2,\bar{Z}_2) = \left(e^{\theta_2}\mathsf{z},\, e^{-\theta_2}\bar{\mathsf{z}}\right),$$

In particular

$$\mathcal{G}(\theta|\theta) = \tilde{G}\ K(Z,\bar{Z}) - G\ L(Z,\bar{Z})$$

where

$$K = \Psi_{+}(Z,\bar{Z})\Psi_{-}(Z,\bar{Z})\,,$$
$$L = \Psi_{-}(Z,\bar{Z})\partial_\theta\Psi_{+}(Z,\bar{Z}) - \Psi_{+}(Z,\bar{Z})\partial_\theta\Psi_{-}(Z,\bar{Z})$$

Still too complicated to be evaluated in a closed form ...

But simplifies in the high-energy limit $\theta_{12} = \theta_1 - \theta_2 \to \infty$:

- The rational factors $\frac{e^{\theta_1}+e^{\theta_2}}{e^{\theta_1}-e^{\theta_2}}$ can be dropped, so that

$\langle\theta_1,\theta_2|\sigma(x/2)\sigma(-x/2)|\theta_1,\theta_2\rangle_{\text{conn}} \to$

$$G\left[L(Z_1,\bar{Z}_1)\,L(Z_2,\bar{Z}_2) + K(Z_1,\bar{Z}_1)\,K(Z_2,\bar{Z}_2)\right] -$$
$$\tilde{G}\left[L(Z_1,\bar{Z}_1)\,K(Z_2,\bar{Z}_2) + K(Z_1,\bar{Z}_1)\,L(Z_2,\bar{Z}_2)\right]$$

• At large $|Z+\bar{Z}| >> m^{-1}$ $\Psi_\pm(Z,\bar{Z})$ become essentially "dressed plane waves", e.g. at $Z \to +\infty$ and $Z >> \bar{Z}$

$$\Psi_+ \to 2\, e^{\varphi/2} \ \ \cos\left(\frac{Z-\bar{Z}}{4} - \frac{\pi}{4}\right),$$
$$\Psi_- \to 2\, e^{-\varphi/2} \cos\left(\frac{Z-\bar{Z}}{4} + \frac{\pi}{4}\right).$$

It follows

$$K(Z,\bar{Z}) \to 2\cos\left(\frac{Z-\bar{Z}}{2}\right), \qquad L(Z,\bar{Z}) \to |Z+\bar{Z}|\,.$$

The leading high-energy behavior is dominated by the term

$$G\, L(Z_1,\bar{Z}_1)L(Z_1,\bar{Z}_1)$$

in the matrix element (the rest of the terms are suppressed by at least one factor $e^{-\theta_{12}}$).

Therefore, one can write:

$$iA(\theta_{12}) = -\frac{1}{2}\int d^2x\,|UV|\,G(\rho + i0) + 0(1)\,,$$

$$\begin{aligned} d^2x &= d\mathsf{x}d\mathsf{t} = \frac{1}{2}\,d\mathsf{z}d\bar{\mathsf{z}}\,, \qquad \rho = \mathsf{z}\bar{\mathsf{z}}\,, \\ U &= Z_1 + \bar{Z}_1 = e^{\theta_1}\,\mathsf{z} + e^{-\theta_1}\,\bar{\mathsf{z}}\,, \\ V &= Z_2 + \bar{Z}_2 = e^{\theta_2}\,\mathsf{z} + e^{-\theta_2}\,\bar{\mathsf{z}}\,. \end{aligned}$$

The integral is handled as follows: write

$$|UV| = UV - 2UV\,\Theta(-UV)$$

with Θ being the usual step function. The first term is analytic in the coordinates; it can be evaluated by the Wick rotation $\mathsf{t} \to -i\mathsf{y}$, yielding

$$-\frac{1}{2}\int UV\,G(\rho + i0)\,d^2x = 2\pi i\,G_3\,\cosh\theta_{12} \to i\pi\,G_3\,e^{\theta_{12}}\,,$$

where

$$G_3 = \frac{1}{2}\int_0^\infty \rho\,G(\rho)\,d\rho = \int_0^\infty r^3\,\langle\sigma(r)\sigma(0)\rangle\,dr\,.$$

In

$$\int \Theta(-UV)\, UV\, G(\rho)\, d^2x$$

the integration domain is limited to the domain $UV < 0$, laying between the lines $U = 0$ and $V = 0$. For the part $U < 0$, $V > 0$ one can use the "Lorentz polar coordinates"

$$\mathsf{z} = -\sqrt{-\rho}\, e^{-\phi}, \qquad \bar{\mathsf{z}} = \sqrt{-\rho}\, e^{\phi}$$

with the integration over ϕ limited to the domain $\theta_2 < \phi < \theta_1$. As the result

$$\int_{U<0<V} UV\, G(\rho + i0)\, d^2x =$$
$$-\theta_{12}\, e^{\theta_{12}} \int_{-\infty}^{0} \rho\, G(\rho + i0)\, d\rho = \theta_{12}\, e^{\theta_{12}} \int_{0}^{\infty} \rho\, G(\rho)\, d\rho$$

Finally

$$\frac{iA(\theta_{12})}{\sinh \theta_{12}} = -4\, G_3 \left[\theta_{12} - i\pi/2 + \theta_0\right] + O\Big(e^{-\theta_{12}}\Big),$$

(COM energy $E = 2\cosh(\theta_{12}/2)$); θ_0 is a constant whose evaluation requires the knowledge about $\Psi_\pm$ at $|Z + \bar{Z}| \leq m^{-1}$).

By optical theorem

$$\frac{2\,\Im m\, A}{\sinh\theta} = \sigma_{\rm tot}^{(2)}$$

$$\sigma_{\rm tot} \;\equiv\; \begin{array}{l}\text{probability of all inelastic processes in the}\\ \text{2-particle scattering } = h^2\,\sigma_{\rm tot}^{(2)} + h^4\,\sigma_{\rm tot}^{(2)} + \ldots\end{array}$$

$$\sigma_{\rm tot}(E) \sim 8\,G_3\,h^2\,\log(E^2) + O(h^4) \qquad\qquad E \to \infty$$

Suggests that

$$\sigma_{\rm tot}^{(n)} \sim \log^n(E^2)$$

The $E \to \infty$ asymptotic of A corresponds to "quasi-classical" approximation:

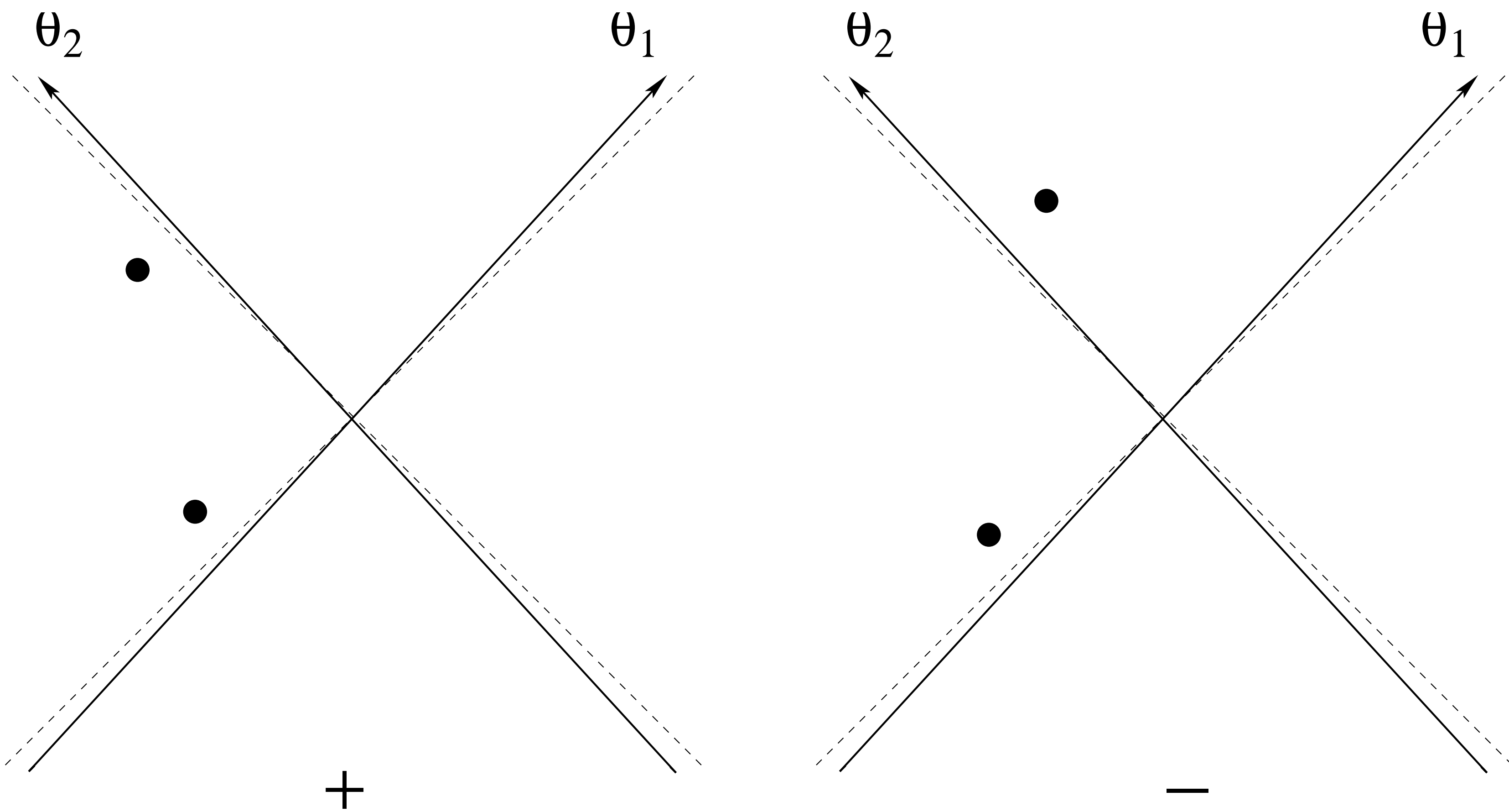
- The particles 1 and 2 are represented by classical trajectories

$$u \equiv e^{\theta_1}\,\mathsf{z} + e^{-\theta_1}\,\bar{\mathsf{z}} = 0\,,$$
$$v \equiv e^{\theta_2}\,\mathsf{z} + e^{-\theta_2}\,\bar{\mathsf{z}} = 0\,.$$

- The $2 \to 2$ matrix elements are approximated as

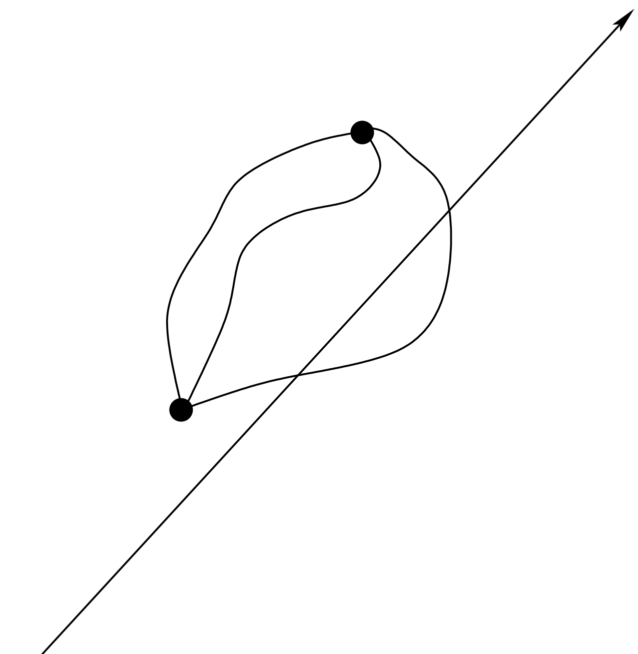
$$\langle \theta_1, \theta_2 \mid T\sigma(x_1)\sigma(x_2) \mid \theta_1, \theta_2 \rangle \simeq$$
$$\mathrm{sign}(u_1)\mathrm{sign}(u_2)\mathrm{sign}(v_1)\mathrm{sign}(v_2)\, \langle 0 \mid T\sigma(x_1)\sigma(x_2) \mid 0 \rangle\,,$$

((u_i, v_i) are the (u, v) coordinates of the insertion points x_i), as long as x_1, x_2 are not too close to the trajectories

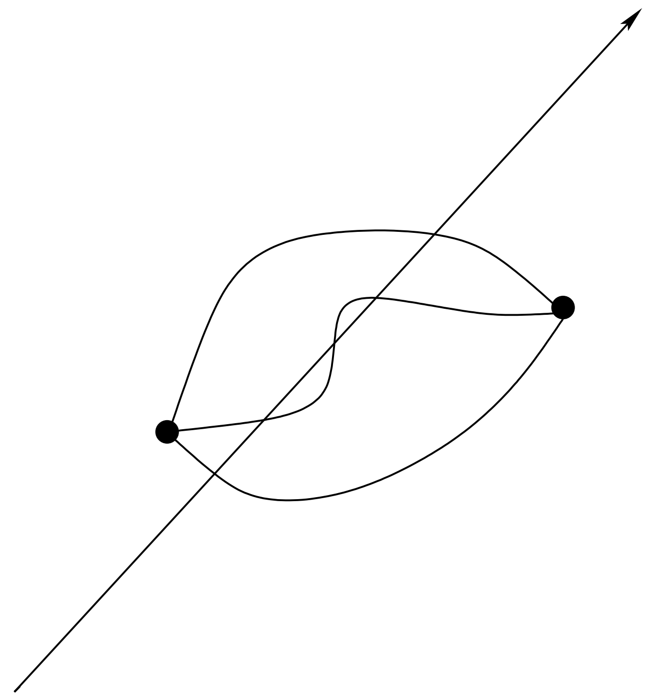

θ2
θ1
θ2
θ1
+
−

(i) Correlations between $\sigma(x_1)$ and $\sigma(x_2)$ is due to the exchanges of odd numbers of particles

(ii) Intersections generate the minus sign, e.g.



$+$

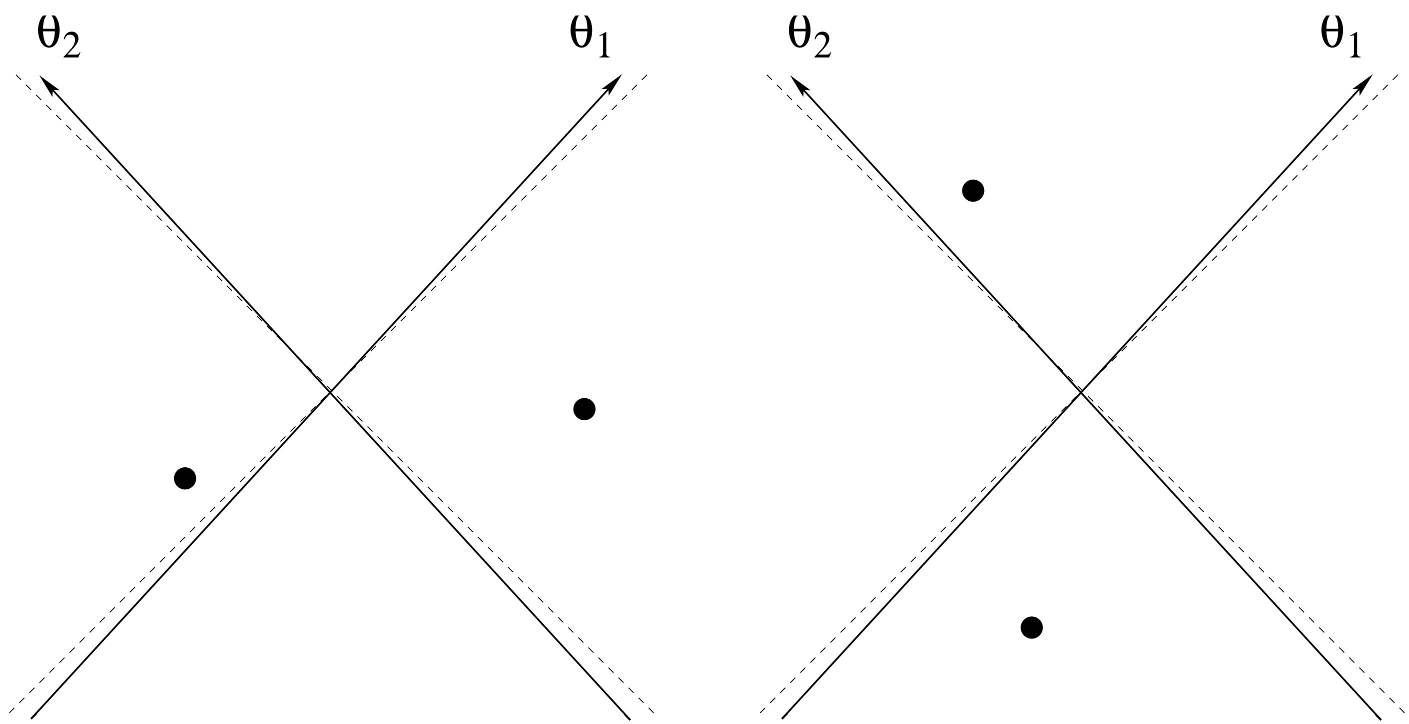


$-$

$$\langle\theta_1,\theta_2 \mid T\sigma(x_1)\sigma(x_2) \mid \theta_1,\theta_2\rangle_{\text{conn}} =$$
$$\langle\theta_1,\theta_2 \mid T\sigma(x_1)\sigma(x_2) \mid \theta_1,\theta_2\rangle - \text{disconnected parts}\,,$$

$$\text{disc. parts} =$$
$$\left[1 + \Big(\text{sign}(u_1)\text{sign}(u_2) - 1\Big) + \Big(\text{sign}(v_1)\text{sign}(v_2) - 1\Big)\right] \times$$
$$\langle 0 \mid T\sigma(x_1)\sigma(x_2) \mid 0\rangle$$

$$\langle\theta_1,\theta_2 \mid T\sigma(x_1)\sigma(x_2) \mid \theta_1,\theta_2\rangle_{\text{conn}} =$$
$$4\,\Theta(-u_1v_1)\Theta(-u_2v_2)\,\langle 0 \mid T\sigma(x_1)\sigma(x_2) \mid 0\rangle\,,$$


θ_2
θ_1
θ_2
θ_1

Since $\langle 0 \mid T\sigma(x_1)\sigma(x_2) \mid 0\rangle$ depends on $x_1 - x_2$

$$-\frac{1}{2}\int d^2x_1\, d^2x_2\, \langle \theta_1, \theta_2 \mid T\sigma(x_1)\sigma(x_2) \mid \theta_1, \theta_2\rangle_{\text{conn}} =$$

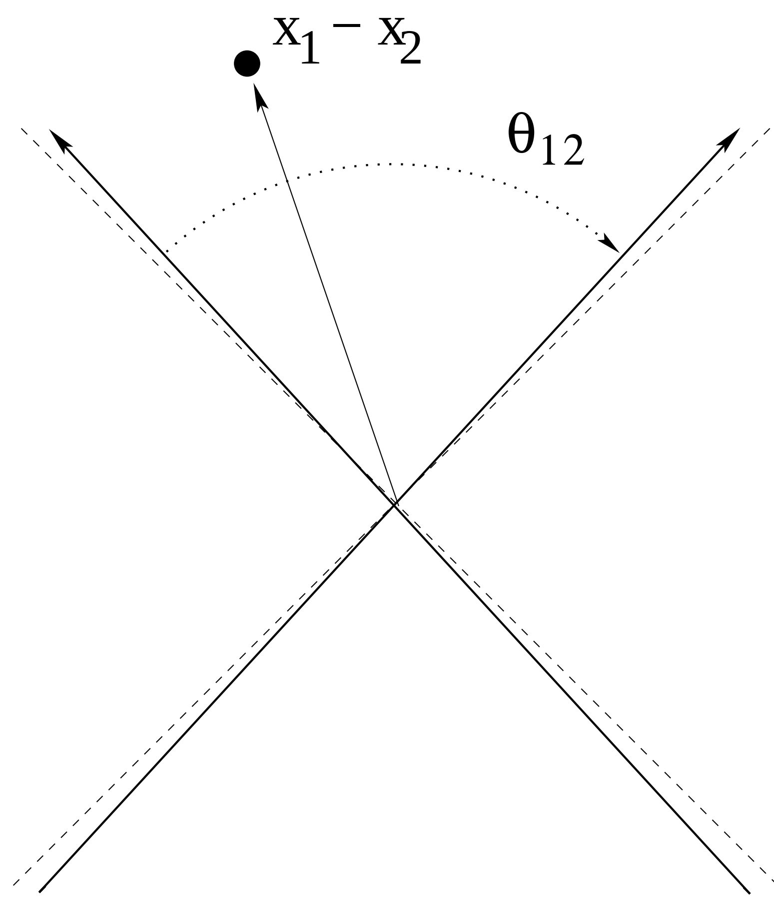
$$-\frac{1}{2\,\sinh\theta_{12}}\int d^2x\, |UV|\, \langle 0 \mid T\sigma(x/2)\sigma(-x/2) \mid 0\rangle \;\sim \log(E^2)$$

($U = u_1 - u_2$, $V = v_1 - v_2$ are the (u, v) coordinates associated with the separation $x = x_1 - x_2$).

Origin of the log(E) behavior: If $\theta_1 \to +\infty$, $\theta_2 \to -\infty$

$$U = e^{\theta_1}\,\mathsf{z} + e^{-\theta_1}\,\bar{\mathsf{z}} \simeq e^{\theta_1}\,\mathsf{z}\,,$$
$$V = e^{\theta_2}\,\mathsf{z} + e^{-\theta_2}\,\bar{\mathsf{z}} \simeq e^{-\theta_2}\,\bar{\mathsf{z}}\,,$$

unless z (or $\bar{\mathsf{z}}$) is too small. If one replaces $|UV| \to e^{\theta_{12}}\,|\mathsf{z}\bar{\mathsf{z}}|$ the integrand would be Lorentz invariant $\Rightarrow$ infinite volume of the lorentz group. At finite θ_{12} it is $\sim \theta_{12} \sim \log(E^2)$.



x1 − x2
θ12

Higher orders in h^2:

$$S(\theta_{12}) = -\langle \theta_1, \theta_2 \mid T \exp\Big\{ -ih \int \sigma(x)\, d^2x \Big\} \mid \theta_1, \theta_2 \rangle_{\text{conn}}$$

At $\theta_{12} \to \infty$

$$\langle \theta_1, \theta_2 \mid T\sigma(x_1)\sigma(x_2)...\sigma(x_{2n}) \mid \theta_1, \theta_2 \rangle \approx \left[\prod_{i=1}^{2n} \text{sign}(u_i)\text{sign}(v_i) \right] \langle 0 \mid T\sigma(x_1)\sigma(x_2)...\sigma(x_{2n}) \mid 0 \rangle \,.$$

The corr. function is Lorentz invariant $\Rightarrow$ integration over $\prod_{i=1}^{2n} d^2x_i$ produces the factor θ_{12} from the Lorentz boost.

The corr. function contains disconnected parts

$$\langle 0 \mid T\sigma(x_1)\sigma(x_2) \mid 0 \rangle \langle 0 \mid T\sigma(x_3)\sigma(x_4) \mid 0 \rangle ... \langle 0 \mid T\sigma(x_{2n-1})\sigma(x_{2n}) \mid 0 \rangle + \text{permutations}\,,$$

which are invariant w.r.t. n copies of the Lorentz group $\Rightarrow$ θ_{12}^n.

Exponential series $\Rightarrow$

$$S(\theta) \sim -\exp\left\{ -4G_3\, h^2 \Big(\theta - i\pi/2\Big) \right\} \qquad \Re e\, \theta \to +\infty$$

Summary and Remarx:

- At $h^2/m^{15/4} << 1$ the $2 \to 2$ elastic scattering amplitude has the power-like asymptotic at $E \to \infty$,

$$S_{2\to2}(E) \to -\left(\frac{e^{-\frac{i\pi}{4}}\,E}{E_0}\right)^{-8G_3\,h^2}$$

with G_3 - third moment of the spin-spin corr. function.

- In principle, using this approach, one can collect the "subleading logarithms" like $h^{2n+2}\log^n(E^2)$, yielding

$$S_{2\to2}(E) \to -\left(\frac{e^{-\frac{i\pi}{4}}\,E}{E_0}\right)^{-\alpha(h^2)},$$

with $\alpha(h^2) = 8G_3\,h^2 + G_5\,h^4 + \ldots$, where e.g. G_5 is expressed in terms of certain 5-th moment of *connected* 4-spin correlation function, etc.

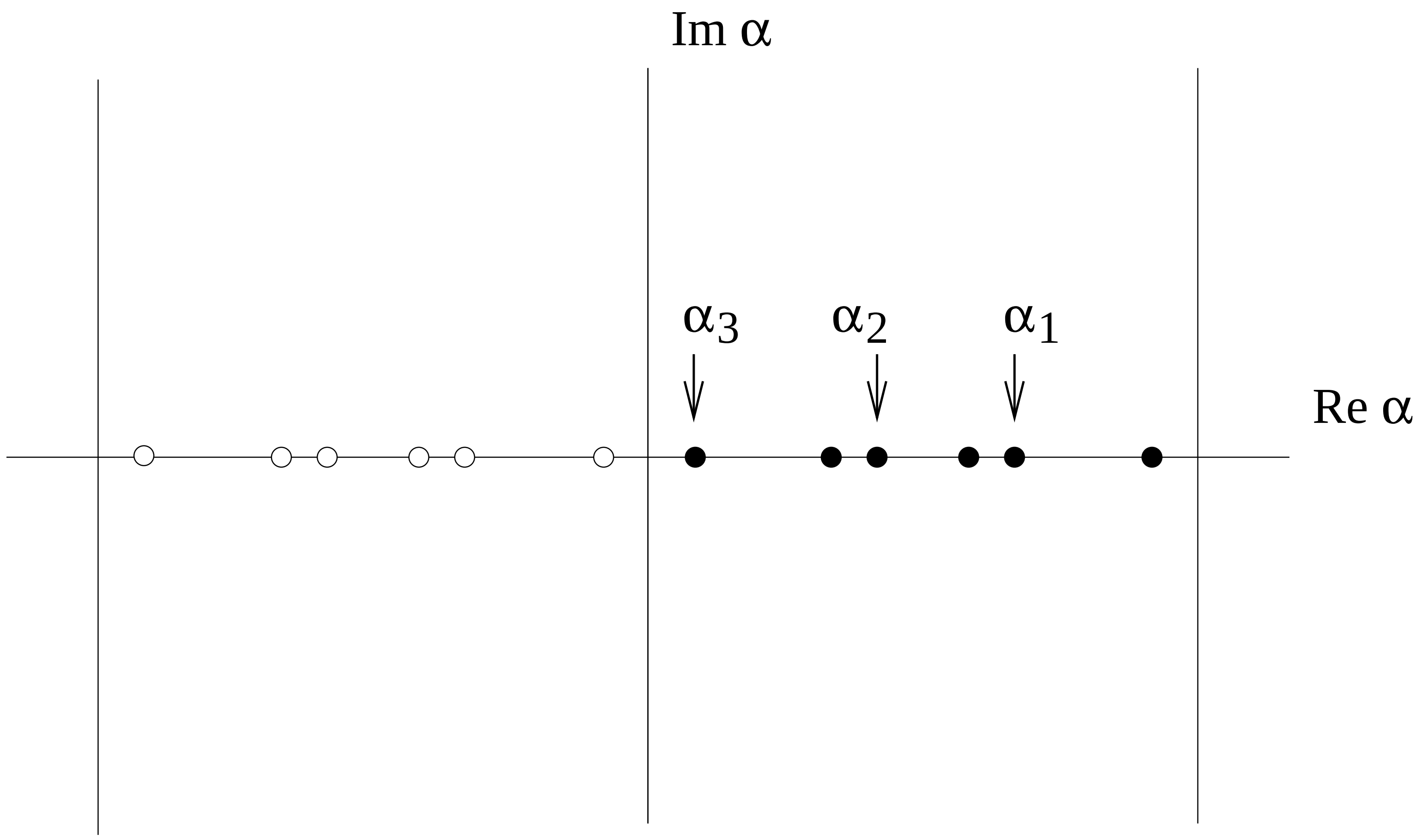
The power-like asymptotic suggests that there is no resonance whose energy grows to infinity as $h^2 \to 0$. This is puzzling: seems to violate "resonance number parity" in the IFT.

IFT is integrable in two special cases:

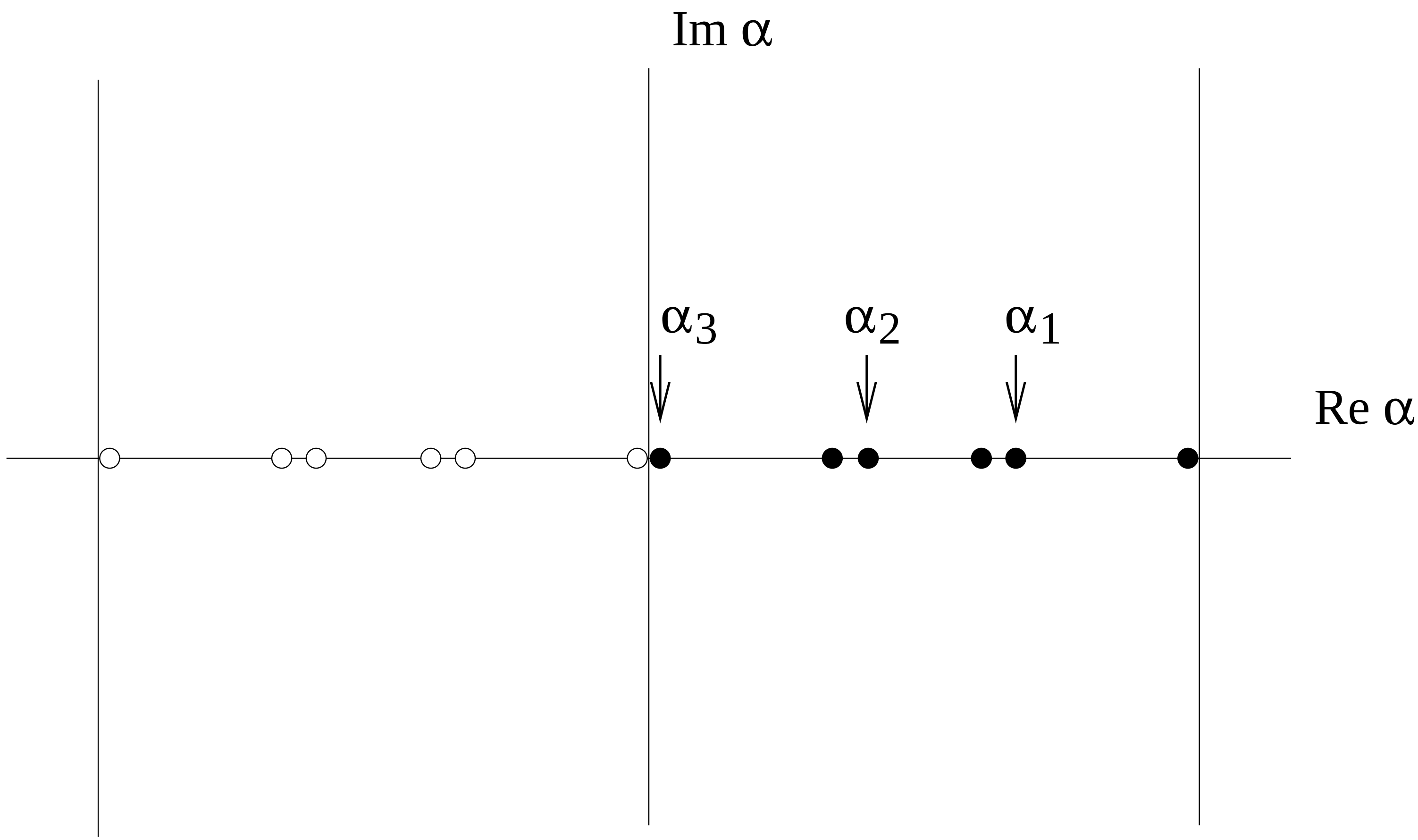
• $m = 0,\ h \neq 0 \Rightarrow$ 8 stable particles (and no resonances), non-trivial factorizable scattering theory with "E8 structure".

• $m \neq 0,\ h = 0 \Rightarrow$ one stable particle (and no resonances), with trivial scattering: $S(\theta) = -1$.

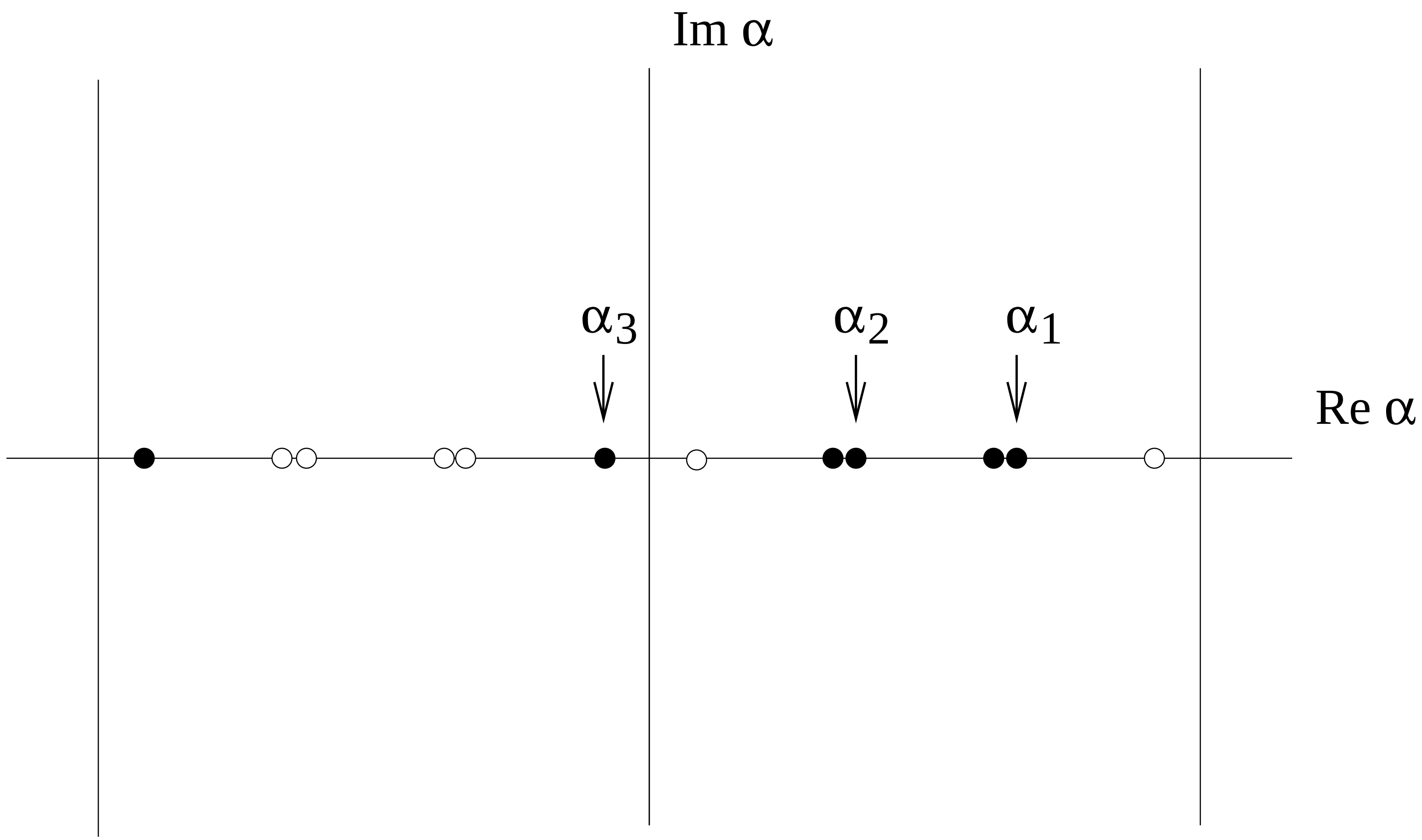
$\eta = 0$:



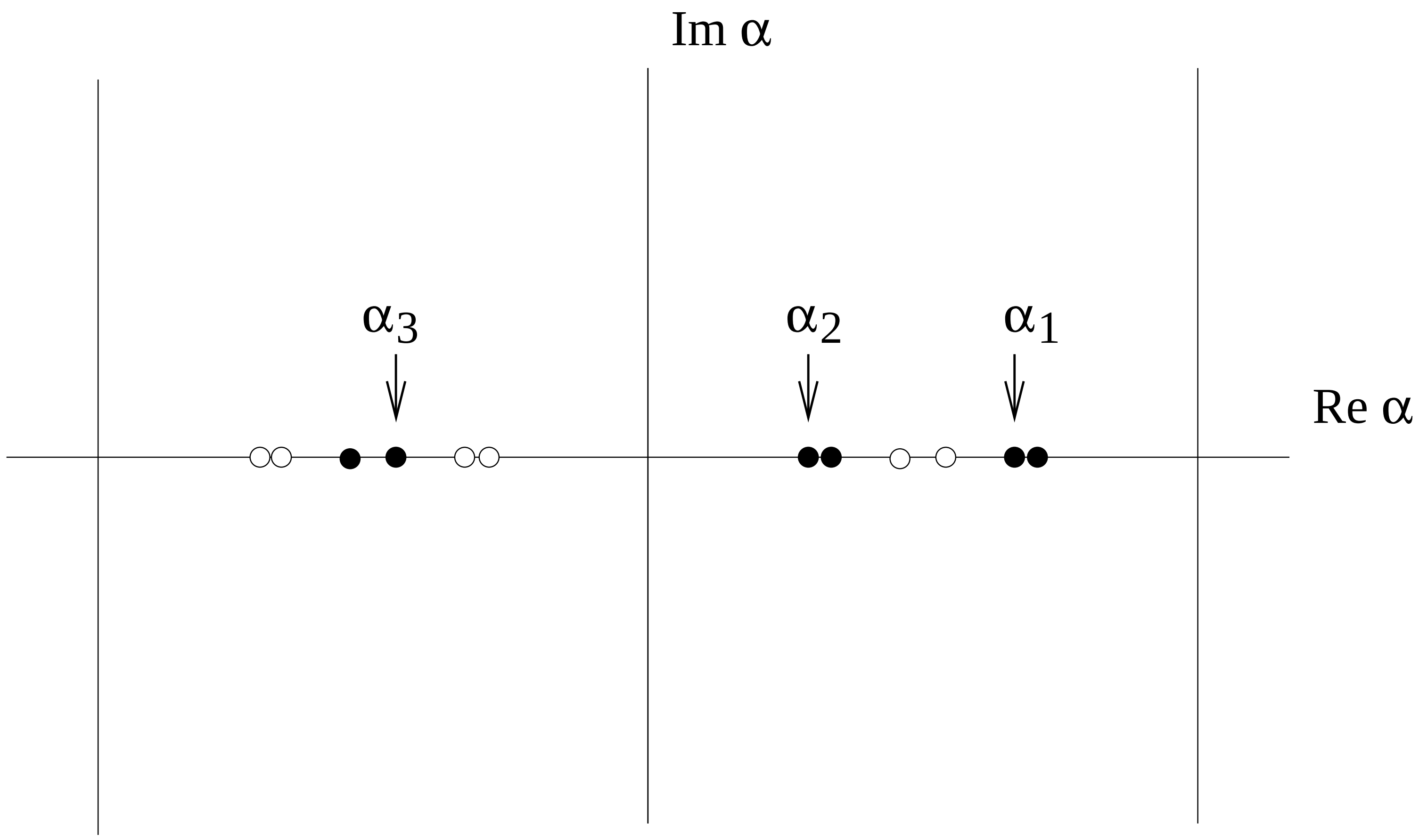
$\eta = -0.08$:



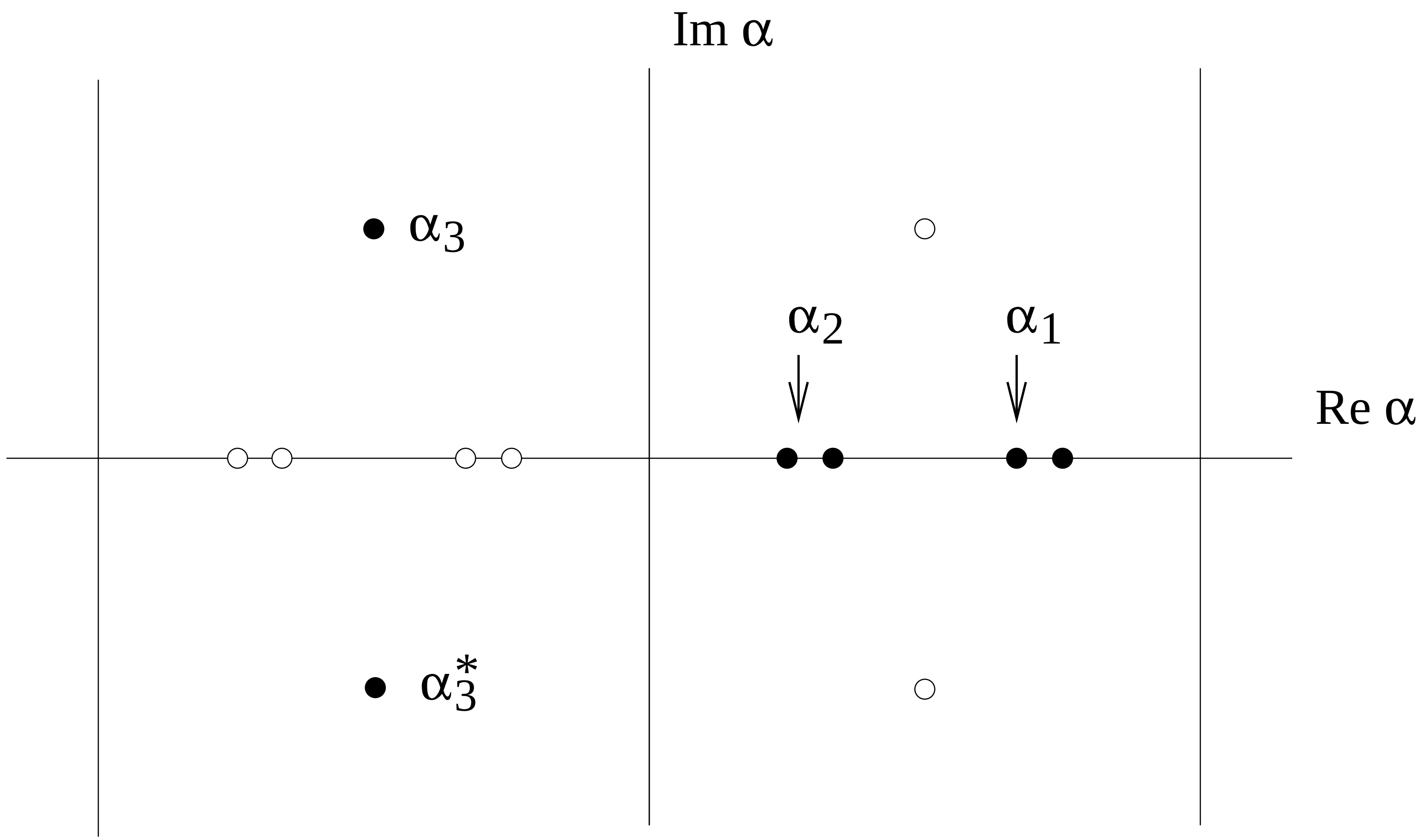
$\eta = -0.27$:



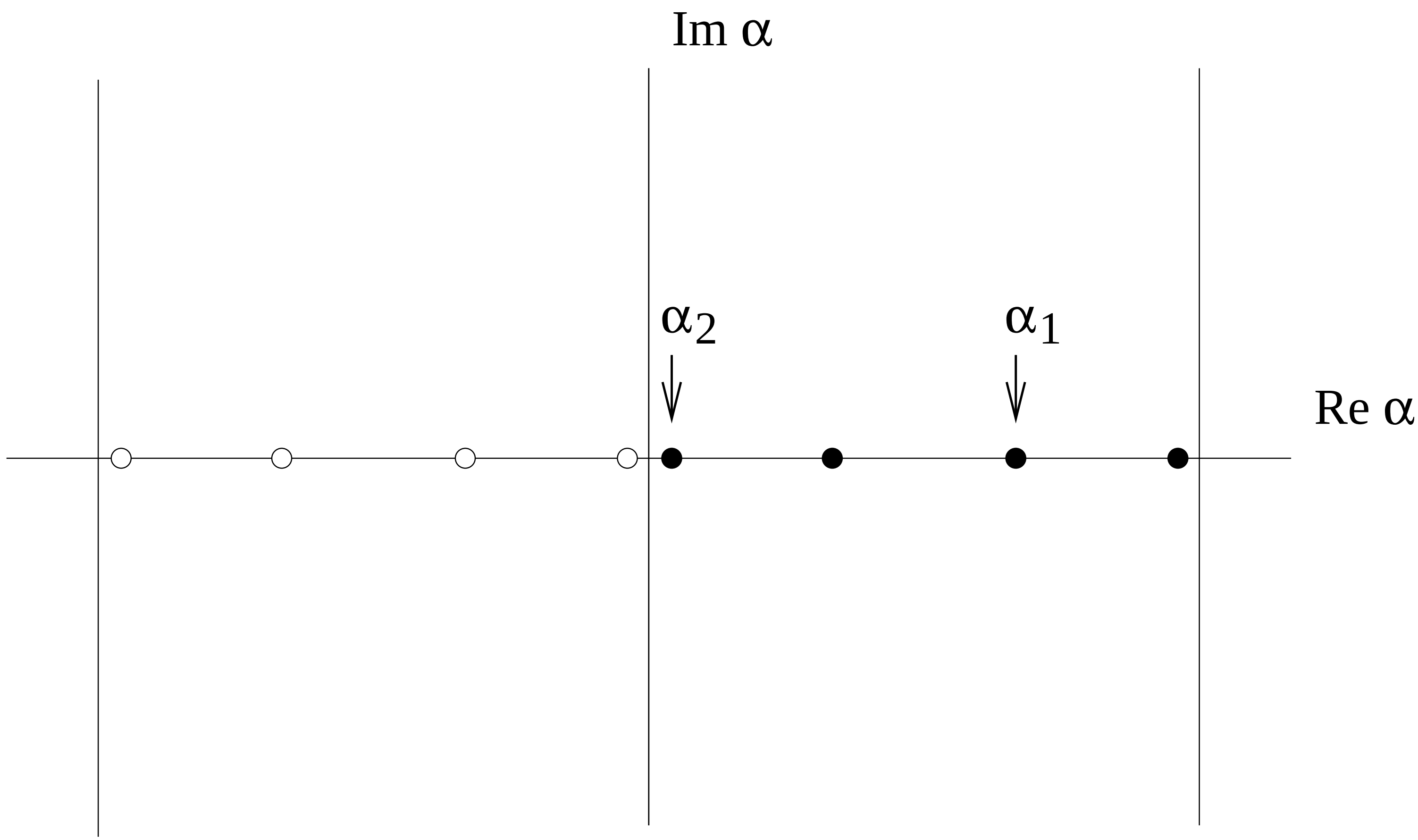
$\eta = -0.49$:



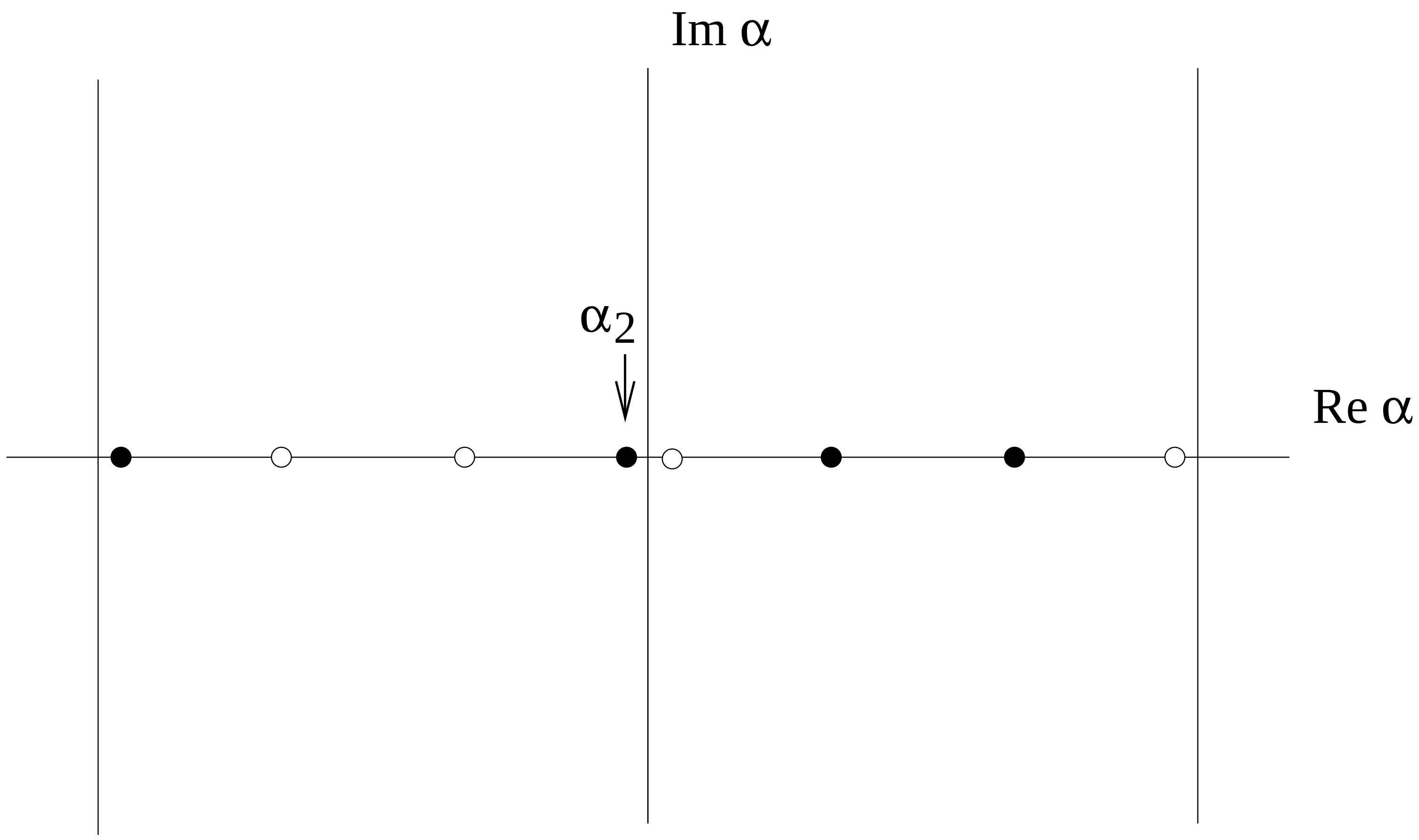
$\eta = -0.94$:



$\eta = -1.87$



$\eta = -2.29$



$\eta = -4.35$