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R_b, R_c and the $q\bar{q}\gamma$ rate at LEP

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Abstract

The rate of hadronic Z decays in which a photon radiates off a final state quark can be used, together with the total hadronic width, to measure the electroweak couplings of quarks to the Z. Then, with some model dependence, also the ratios of the single quark widths over the total hadronic width can be derived. Despite the limited precision, the ratios for b and c quarks obtained in this way:

$$R_b = 0.237 \pm 0.013 \quad ; \quad R_c = 0.137 \pm 0.020$$

show the same trend as the direct measurement of R_b and R_c, with respect to the SM theoretical expectations. This, far from being a further probe of the reported discrepancy in the two quark widths, is however an interesting coincidence.

1 Introduction

The Standard Model (SM) has been found to be in excellent agreement with most of the very precise electroweak measurements done at LEP. Yet, two observables deviate from this general trend, that is the ratios of the number of Z particles decaying into a b or into a c quark over the total number of hadronic Z decays. The (preliminary) experimental values obtained by averaging the results af all four collaborations are [1]:

$$R_b^{(exp)} = 0.2211 \pm 0.0016 \tag{1}$$
$$R_c^{(exp)} = 0.1598 \pm 0.0069 \tag{2}$$

are respectively 3.4 standard deviations above and 1.8 standard deviations below the SM theoretical expectations [2], when $m_t = 180 \pm 12\, GeV/c^2$ [3] is used:

$$R_b^{(SM)} = 0.2156 \pm 0.0003 \tag{3}$$
$$R_c^{(SM)} = 0.172 \tag{4}$$

The partial width of the decay of Z boson into a fermion pair is given in the SM by:

$$\Gamma_{f\bar{f}} = \frac{G_\mu M_Z^3}{2\pi\sqrt{2}} \frac{N_C}{3} \left(\beta_f \frac{3-\beta_f^2}{2} v_f^2 + \beta_f^3 a_f^2\right) \tag{5}$$

where the vector and axial couplings can be written in terms of the fermion charge, the third component of the weak isospin and the electroweak mixing angle as

$$v_f = 2I_3^f - 4Q_f \sin^2\theta_W \quad ; \quad a_f = 2I_3^f \tag{6}$$

The combinations

$$c_f = \left(\beta_f \frac{3-\beta_f^2}{2} v_f^2 + \beta_f^3 a_f^2\right) \simeq (v_f^2 + a_f^2) \tag{7}$$

are the electroweak couplings of the fermion f to the Z, and the last equality in (7) holds at LEP energies with good approximation for all quark flavours but the b. In fact, the threshold effect parametrized by β_f (the fermion velocity in units of c) leads to a partial width $\Gamma_{b\bar{b}}$ which is appreciably lower than the corresponding widths of all other charge 1/3 quarks.

There are several corrections (QCD, QED, higgs mass) that modify (5), with more or less the same strength for all quark flavours [4] and therefore they do not affect the above defined ratios. The biggest correction specific to the b sector (apart the mass threshold in (7)) is a genuine electroweak correction to the Z-$b\bar{b}$ vertex involving the heavy *top* quark.

The effect of m_t on $\Gamma_{b\bar{b}}$ can be parametrized with [4]:

$$c_b = \left(\beta_b \frac{3-\beta_b^2}{2} (v_b + \frac{2}{3}\delta r_t)^2 + \beta_b^3 (a_b + \frac{2}{3}\delta r_t)^2\right) \tag{8}$$

where

$$\delta r_t = \frac{3\sqrt{2}\, G_\mu m_t^2}{(4\pi)^2} \quad . \tag{9}$$

In the Minimal Supersymmetric extension of the SM it is possible to obtain slightly larger values of R_b without altering other EW observables by assuming that light supersimmetric particles can loop in the Z-$b\bar{b}$ vertex. In this model, it is indeed practically impossible to explain any deviation in any of the other partial decay width. However, the existence of light sparticles has almost been excluded by the 130-136 GeV LEP data (and could be more or less totally excluded by the forthcoming higher energy data).

Recently, other scenarios have emerged to try to explain the discrepancy, the most popular involving a "leptophobic" Z', which would affect all the $q\bar{q}$ partial widths, not only $\Gamma_{b\bar{b}}$ [5].

Concerning the experimental results, one should keep in mind that the measurements of the two partial widths (1) and (2) are correlated: if one fixes R_c to its SM value, then also R_b moves slightly towards the theoretical expectation [1],

$$R_b(R_c = 0.172) = 0.2202 \pm 0.0016 \quad , \tag{10}$$

still remaining 2.9 standard deviations away from $R_b^{(SM)}$.

One should also notice that the b and, to lesser extent, the c quark are the only quark flavours that can be tagged with sufficient efficiency and purity at LEP/SLD. All other partial Z decay widths into quarks have not been yet directly measured.

In this work we be follow a suggestion originally presented in [6], where a preliminary LEP average of the final state radiation photon rate were derived from [7]. Indeed, this last paper included some result still preliminary and, more important, it did not take correctly into account the differences between R_b and $R_{d,s}$ and the phase space reduction factors due to photon emission.

2 The final state photon radiation and its rate at LEP

The electroweak couplings of quarks can be measured by comparing the rate of hadronic Z^0 decays in which a photon radiates off one of the final state quarks, with the total decay rate of Z^0 into hadrons [8].

In first approximation, according to equation (7), in the SM only two different couplings of quarks to the Z exist, depending on the charge of the quark ($-1/3$ or $+2/3$ of the proton charge). The total width of the decay $Z \to hadrons$ is then proportional to the sum of the coupling constants:

$$\Gamma_{had} \propto (2c_{2/3} + 3c_{1/3}) \tag{11}$$

On the other hand, since the photon couples to the square of the charge of the quark, the width of the decay $Z \to q\bar{q}\gamma$ is proportional to a different linear combination of the couplings:

$$\Gamma_{q\bar{q}\gamma} \propto (8c_{2/3} + 3c_{1/3}) \tag{12}$$

Thus, by comparing the total hadronic rate at LEP with the number of detected $q\bar{q}\gamma$ events one should be able to derive the single quark couplings. In fact several factors modify equations (11) and (12). As we have already seen in equation (8), c_b is not equal to c_d and c_s because of the mass threshold and vertex effects. Moreover, the mass of the radiating quark reduces the phase space available for the emission of a photon from the final state. Its influence on the measured $q\bar{q}\gamma$ cross section depends on the experimental cuts, being more relevant for higher photon energies. It will be shown in section 4 that this phase space reduction is relevant in case the emitting quark is a b or a c, leading to corrections to equation (12) that depend on the experimental cuts.

To unfold the linear combination of the couplings in equation (12) one must know the proportionality factor therein. To do that, a theoretical model for the emission of photons from the final state quark must be chosen. Several popular MonteCarlo programs include final state radiation (FSR), either with the Parton Shower (PS) approach [9], or based on a calculation of the Matrix Elements (ME) of the process at the order (α, α_s) [10].

All four experiments at LEP have published comparisons of their data with some or all the MC above; for the most recent published data, which will be used in the present work see [11, 12, 13, 14].

These measurements face several practical problems:

- in the theory, because of lack of knowledge of the photon fragmentation function, the photon is required to be isolated, according to some criteria, from the emitting quark in order to reduce the difficulty of dealing with non perturbative effects;
- on the experimental side, isolation is also mandatory, together with a cut on the photon energy, in order to cope with the hadronic background, like π^0, which is quite abundant inside jets.

The reported results are based on a total number of $447,706$, $1,483,906$, $323,674$ and $353,160$ hadronic decays, respectively for ALEPH, DELPHI, L3 and OPAL. The main cuts used in the different analyses are summarized in table 1. Whenever the jet resolution parameter $y_{\gamma j}$ appears in the table under the column of the photon isolation with respect to the jets, it means that jets are formed according to the chosen recombination algorithm, JADE-E0 [15] or Durham [16], and then the distance of the photon to the jets is checked as a function of the same resolution parameter y. DELPHI and OPAL also show results for different y_{cut} values; only the one used to derive the quark couplings is reported in table 1. For all other details the reader should refer to the original papers [11, 12, 13, 14].

It is widely accepted that for simulating the rate of final state radiation photon emitted at large angle from the parent quark a ME approach is more appropriate than a PS approach, the latter containing a lot of phenomenological parameters tuned by using global shape variables in different phase space regions with respect to the ones relevant for FSR studies (see for example [8], [17] or the discussions raised at the Annecy workshop on photon radiation from quarks and reported in [18]). Thus, it is intriguing that when experiments compare their total $q\bar{q}\gamma$ rate with some PS model, they find generally a better agreement than when they compare their rate with a ME model [11, 12]. In fact, as we will see later, ME calculations seem to overestimate the FSR rate. On the other hand, the agreement with the PS models is not perfect, since the predicted photon energy or the photon isolation parameter spectra do not match with the data. On the contrary, when data are compared with ME MC, apart from the lower rate one finds better agreement for all photon related spectra. The rates of $e^+e^- \to q\bar{q}\gamma$ predicted by the different ME's tested at LEP do not significantly differ among themselves.

In the following we assume that the ME approach is the correct one for dealing with FSR and that the uncertainties arising from neglecting higher order in α and α_s can be

	E_γ^{min}	θ_γ^{min}	γ isolation from other particles	γ isolation from jets
ALEPH	5 GeV	18.2^o	ECAL < 500 MeV HCAL < 2 GeV in 20^o	$\theta_{\gamma j} > 40^o$ $(\theta_{\gamma j} > 20^o$ if $E_\gamma > 20 GeV)$
DELPHI	5.5 GeV	45^o	$E <$ 500 MeV in 20^o	$y_{\gamma j} > 0.02$ (Durham)
L3	5 GeV	45^o	$E <$ 500 MeV in 15^o	$\theta_{\gamma j} > 20^o$ $y_{\gamma j} > 0.05$ (JADE-E0)
OPAL	7.5 GeV	43.9^o	$p_T <$ 250 MeV $E <$ 250 MeV in 15^o	$y_{\gamma j} > 0.06$ (JADE-E0)

Table 1: *Main cuts applied by the four collaborations on photon energy, polar and isolation angle.*

kept under control by varying the internal generator parameters and $\alpha_s^{(1)}$, as explained in [17]. As a consequence, the discrepancy with the rate in the data can be solved only by modifying the electroweak quark coupling constants used in the programs. It will be shown that the resulting coupling constants move from the SM predictions towards the same direction as suggested by the direct measurements of R_b and R_c.

3 Compilation of the published results and other inputs used

All four LEP experiments have published some comparison with the available ME MC predictions. However, ALEPH does not make any quantitative comparison of the experimental rate with the ones predicted by any ME: it is thus impossible to derive the electroweak couplings from their published data. This is the reason why in the following only the FSR rate measured by DELPHI, L3 and OPAL will be used in order to extract the LEP mean value.

If we define $S_{q\bar{q}\gamma}$ using equation (12) as $(8c_{2/3} + 3c_{1/3})$ [12, 13, 14]:

$$S_{q\bar{q}\gamma}^{DELPHI} = 11.71 \pm 0.43 \pm 0.70 \pm 0.56 \tag{13}$$
$$S_{q\bar{q}\gamma}^{L3} = 11.88 \pm 0.70 \pm 0.94 \pm 0.63 \tag{14}$$
$$S_{q\bar{q}\gamma}^{OPAL} = 12.36 \pm 0.70 \pm 0.78 \pm 0.29 \tag{15}$$

The three errors are respectively the uncorrelated, partially correlated, totally correlated ones and have been derived from the detailed breakdowns published by the collaborations. In the uncorrelated class enter all statistical errors and selection efficiencies. In the partially correlated error class enter all errors coming from background rejection and conversion factors from parton to hadron level in the theory. All those errors directly coming from the theoretical models are considered totally correlated: they are usually obtained by varying in a reasonable range the internal parameters of the model.

DELPHI and OPAL obtain their results by comparing their total $q\bar{q}\gamma$ rate with the theoretical expectations, after having extracted the other free parameter, $\alpha_s^{(1)}$, from the relative amount of three jets plus γ and two jets plus γ events: there, also the variation of $S_{q\bar{q}\gamma}$ when $\alpha_s^{(1)}$ moves in its 1σ range enters in the class of totally correlated errors. The result of L3 is instead derived from a two parameter fit of the energy spectrum of the FSR photon, the angle from the photon to the nearest jet and the transverse photon energy with respect to the thrust axis.

In [12] a further error is reported, that is the spread of $S_{q\bar{q}\gamma}$ results when the y_{cut} is moved over a wide range. In fact, this error should be mostly reabsorbed by computing $\alpha_s^{(1)}$ separately for each y_{cut}, this being related to the higher order corrections which become more relevant for y_{cut} values far from the 'optimal' ones [17]. No other experiment take it into account, and also in the DELPHI result it is believable that it can be reabsorbed into the theoretical one: it will not be taken into account in the present compilation.

Other quantities relevant for the analysis are the total hadronic Z width and the strong coupling constant.

For the total hadronic width, which has to be compared with the FSR partial width, we use the mean LEP value published in [1]:

$$\Gamma_{had} = (1.7450 \pm .0030)\ GeV \tag{16}$$

For the strong coupling constant one can't use any estimate which is based on the measured total hadronic width, since if we want to measure the quark coupling constants we must be ready to admit that they could be different from the theoretical ones. A reasonable estimate of α_s coming only from jet shapes analyses and low energy measurements is [6]:

$$\alpha_s = 0.118 \pm .006 \tag{17}$$

The *top* mass is left to vary from 168 to 192 GeV, according to the CDF and D0 measurements [3].

4 Phase space reduction for photon radiation because of the heavy quarks masses

In the ME calculations [10] all quarks are considered massless. In the L3 work [13] it is suggested to parametrize the effect of the reduction of the phase space available for photon radiation, because of the non vanishing mass of the emitting quark, by using in equation (12) not the electroweak coupling constants themselves, but first multiply them for a factor ϵ_q, where $\epsilon_q \leq 1$ and $\epsilon_q = 1$ only if the mass of the radiating quark has no effect on the FSR rate for the chosen cuts. They make an estimate, based on the insertion of the mass in some MC distribution, that $\epsilon_b^{L3} = 0.8 \pm 0.1$. The DELPHI result given in [12], which uses similar cuts (but the method of extracting $S_{q\bar{q}\gamma}$ is different), relies on this value of ϵ_b. OPAL does not take into account any phase space reduction factor. None of the experiments try to compute a factor linked to the c quark mass, ϵ_c: in fact, the deviation from unity is expected to be lower than for ϵ_b, nevertheless the coupling of the FSR photon with the square of the quark charge leads to a factor of 4 in front of ϵ_c in the expression of $S_{q\bar{q}\gamma}$.

Given the small but not negligible effect that those factors have on the FSR rate, before any attempt of averaging all results one should try to compute the specific ϵ_q for the different experimental cuts. The method used in the present work is the following. One of the ME MC programs used in the analyses, EEPRAD, has been run in order to extract the expected FSR rates with the DELPHI and OPAL experimental cuts. The cross section is the one expected for massless quarks. Then the beam energy in the simulation has been reduced according to the final state quark mass; the angular and photon energy cuts have been kept as before, while the quark-photon mass cuts have been rescaled accordingly. By scaling the FSR cross section obtained in such a way to the previous center of mass energy value, one obtain a good approximation of the effect of the reduced phase space on the FSR rate. Using this method, the following results for the DELPHI-like and OPAL-like cuts have been derived:

$$\epsilon_c^{DELPHI} = 0.975 \pm 0.025 \quad ; \quad \epsilon_b^{DELPHI} = 0.94 \pm 0.03 \tag{18}$$
$$\epsilon_c^{OPAL} = 0.96 \pm 0.03 \quad ; \quad \epsilon_b^{OPAL} = 0.89 \pm 0.05 \tag{19}$$

ϵ_u, ϵ_d and ϵ_s are not expected to differ significantly from 1. One can observe that the tighter energy cut which is used in the OPAL analysis results in an increased sensitivity to the phase space reduction. Moreover, also the effect due to the mass of the c quark is not negligible. Thus, a factor ϵ_c has been added also to the L3 result, scaled with their measured ϵ_b of a factor similar to the ones verified in DELPHI and OPAL and wearing a 100% uncertainity on its difference from unity:

$$\epsilon_c^{L3} = 0.94 \pm 0.06 \quad ; \quad \epsilon_b^{L3} = 0.80 \pm 0.10 \tag{20}$$

Apparently the effect is more relevant for the L3 analysis than for the other two. Given the different approach used, no attempt is done here to verify the factors in (20).

5 Results

The procedure is the following: first the quark couplings are derived for each experiment; then a mean LEP value is extracted, taking correlations into account; finally, with very few model dependent assumptions, these couplings are used to derive the values of R_b and R_c.

5.1 Extraction of the electroweak couplings

For any single experiment a two equations system must be solved:

$$2\,c_{2/3} + (2 + \eta_b)\,c_{1/3} = \frac{\Gamma_{had}}{k_1} \tag{21}$$
$$4\,(1 + \epsilon_c^{exp})\,c_{2/3} + (2 + \eta_b\,\epsilon_b^{exp})\,c_{1/3} = S_{q\bar{q}\gamma}^{exp} \tag{22}$$

where with the suffix exp are indicated the quantities which depend on each experiment, $\eta_b = c_b^{SM}/c_{1/3}^{SM} = 0.974 \pm .002$ in the SM and

$$k_1 = \frac{3M_Z^3 G_\mu}{24\pi\sqrt{2}} \left[1 + \frac{\alpha_s}{\pi} + 1.4\left(\frac{\alpha_s}{\pi}\right)^2\right] \quad . \tag{23}$$

In table 2 are reported the values of $c_{1/3}$ and $c_{2/3}$ obtained using (21) and (22) for the three experiments taken into account, together with their total error. Because the error comes predominantly from the uncertainties associated to $S_{q\bar{q}\gamma}$, the two couplings are almost 100% (negatively) correlated. The tabulated values can differ from the ones reported in the original papers, because of the different value of Γ_{had} used here and, mainly, because in those papers the factors η_b and ϵ_q were either absent or different.

	$c_{1/3}$	$c_{2/3}$
DELPHI	$1.69 \pm .12$	$0.86 \pm .18$
L3	$1.63 \pm .15$	$0.95 \pm .22$
OPAL	$1.60 \pm .12$	$0.99 \pm .18$

Table 2: *Values of the quark coupling constants obtained starting from the $q\bar{q}\gamma$ rates measured by each single experiment.*

5.2 Averaging procedure and results

The couplings reported in table 2 have been averaged by using a procedure derived from the BLUE (Best Linear Unbiassed Estimate) method [19]. The covariance matrix has been constructed by putting on its diagonal the total variance ($\mathcal{C}_{ii} = \sigma_i^2$, i stands for the experiment number), and on the non diagonal terms:

$$\mathcal{C}_{ij} = \sum_p \sigma_i(p)\sigma_j(p) \quad ,$$

where p runs over all sources of correlated errors.

One problem arises when dealing with the sources of errors which are known to be partially correlated, and where it is difficult to determine the exact degree of correlation. However, any linear combination of the single results is an unbiassed estimate of the true value, provided the weights add up to 1. By including the exact degree of correlation, one could obtain the estimate of the true value which has the smaller variance. If one supposes that those errors defined as being partially correlated are instead fully correlated, then one get an unbiassed estimate of the true value, with an error which is not smaller than the one associated to the best possible estimate.

By using this procedure for $c_{1/3}$ and $c_{2/3}$, one finally gets:

$$c_{1/3}^{LEP} = 1.64 \pm 0.09 \tag{24}$$
$$c_{2/3}^{LEP} = 0.93 \pm 0.14 \tag{25}$$

The Standard model expectations for the above quantities are:

$$c_{1/3}^{SM} = 1.477 \quad ; \quad c_{2/3}^{SM} = 1.145 \tag{26}$$

The combined down-type coupling is above and the combined up-type coupling is below the SM, exactly as direct measurements of R_b and R_c do.

5.3 Effect on R_b and R_c

From the mean values obtained for the electroweak quark couplings it is straightforward to derive the single partial widths of the Z or the ratios R_b and R_c.

$$\frac{\Gamma_u + \Gamma_c}{2} = (240 \pm 36)\ MeV \tag{27}$$
$$\frac{\Gamma_d + \Gamma_s + \Gamma_b}{3} = (422 \pm 24)\ MeV \tag{28}$$

$$R_c^{from\ q\bar{q}\gamma} = 0.137 \pm 0.020 \tag{29}$$
$$R_b^{from\ q\bar{q}\gamma} = 0.237 \pm 0.013 \tag{30}$$

A 99.8 % negative correlation between R_b and R_c is found.

The 1σ and 2σ ranges in the R_b-R_c plane allowed by the direct measurements, the ones allowed by the present work, and the SM point are shown in figure 1.

At a first glance, although the error is large, it seems that the data extracted from the measured rates of $e^+e^- \to q\bar{q}\gamma$ show the same trend and are in better agreement with the directly measured R_b and R_c rather than with the SM predictions.

5.4 Model independent $c_{1/3}$ and $c_{2/3}$ couplings

One can avoid any model dependence because of possible physics effects hidden in the $Z - b\bar{b}$ vertex by using in the formulas the experimental result (1) for R_b. In that way, only the couplings of lighter quarks (u, d, s and c) remain as unknown parameter in the two equation system and one can much more safely assume that quarks of the same charge have the same electroweak coupling. When fixing $R_b = R_b^{(exp)}$, the equations (21) and (22) modify to:

$$2\,c_{2/3} + 2\,c_{1/3} = \tfrac{\Gamma_{had}}{k_1} \cdot (1 - R_b) \tag{31}$$
$$4\,(1 + \epsilon_c^{exp})\,c_{2/3} + 2\,c_{1/3} = S_{q\bar{q}\gamma}^{exp} - \epsilon_b^{exp}\,\tfrac{\Gamma_{had}}{k_1}\,R_b \tag{32}$$

By repeating the whole treatment, the electroweak quark coupling constants can be derived once more:

$$(c_{1/3}^{LEP})' = 1.71 \pm 0.14 \tag{33}$$
$$(c_{2/3}^{LEP})' = 0.92 \pm 0.14 \tag{34}$$

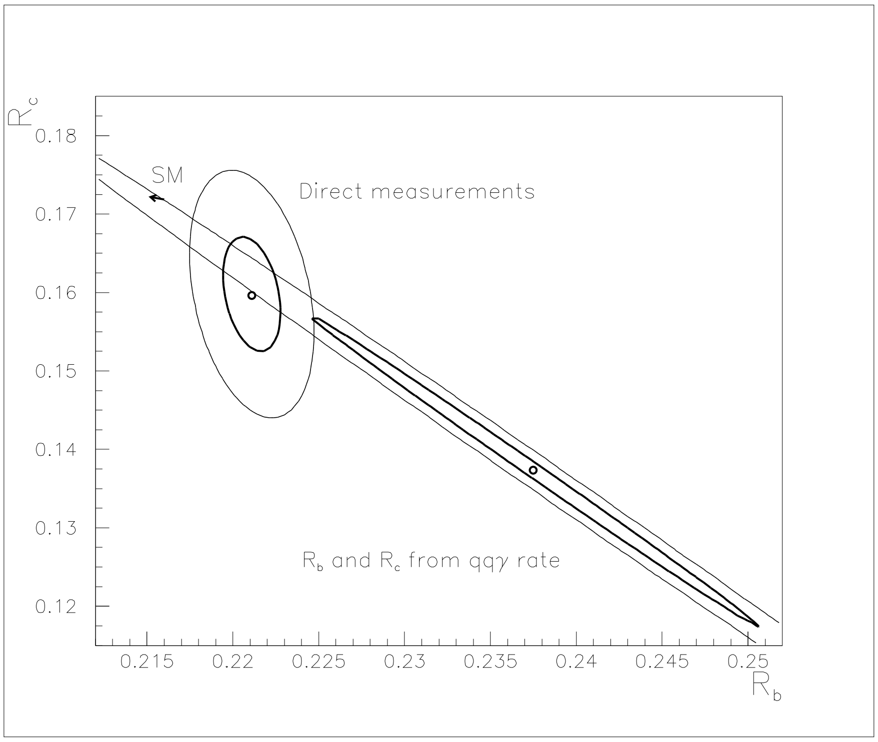

Figure 1: 1σ *(thick) and* 2σ *(thin) contour plots in the* R_b *and* R_c *plane from the direct measurements and from the present comparison between* Γ_{had} *and* $S_{q\bar{q}\gamma}$.

with a negative correlation of 99.9%. The error includes also the contribution from R_b.

In that way, no dependence remains from the assumptions made on the quark couplings, only $c_d = c_s$ and $c_u = c_c$ is required. Off course, these results still depend on the hypothesys that the $\mathcal{O}(\alpha, \alpha_s)$ ME description of photon radiation from quarks in the Z decays is correct, within the theoretical error associated.

6 Comments and conclusions

By combining the results extracted from the $q\bar{q}\gamma$ rates in the DELPHI, L3 and OPAL experiments and comparing them with the well measured Γ_{had} one gets:

$$R_b^{from\ q\bar{q}\gamma} = 0.237 \pm 0.013$$

$$R_c^{from\ q\bar{q}\gamma} = 0.137 \pm 0.020$$

These results rely on several assumptions. In particular, it is assumed that the $\mathcal{O}(\alpha, \alpha_s)$ ME approach is the correct one for dealing with final state photon radiation from quarks and that the theoretical error can be reliably calculated with the method suggested by the authors of the programs. Moreover, the ratio $R_b/R_{d,s}$ is assumed to be the same one expects from the Standard Model; possible deviations would enter in the factor η_b in (21) and (22). No significant enhancements in the single errors are foreseen whenever releasing this constraint on $R_b/R_{d,s}$; however, the correlation coefficient would lower, finally resulting in a fatter $q\bar{q}\gamma$ contour in figure 1.

When fixing $R_b = R - b^{(exp)}$, then one can intersect again Γ_{had} with $\Gamma_{q\bar{q}\gamma}$, as before. This gives:

$$c_{1/3}^{LEP} = 1.71 \pm 0.14$$

$$c_{2/3}^{LEP} = 0.92 \pm 0.14$$

independently from the model assumed for explaining the deviation of R_b from $R_{d,s}$.

Despite the large error, these measurements suggest that $c_{1/3}$ and $c_{2/3}$ could both deviate from the SM expectations and their shifts are found compatible with the ones observed in the direct measurements of R_b and R_c at LEP and SLD. The analysis of all the available data and an improved precision of the theoretical models for final state photon radiation from quarks are needed in order to confirm this trend.

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