# Fisica Generale B

## *Le sorgenti del magnetismo*

Scuola di Ingegneria e Architettura

UNIBO – Cesena

Anno Accademico 2014 – 2015

# Le sorgenti del magnetismo

## *Spire ed aghi magnetici*

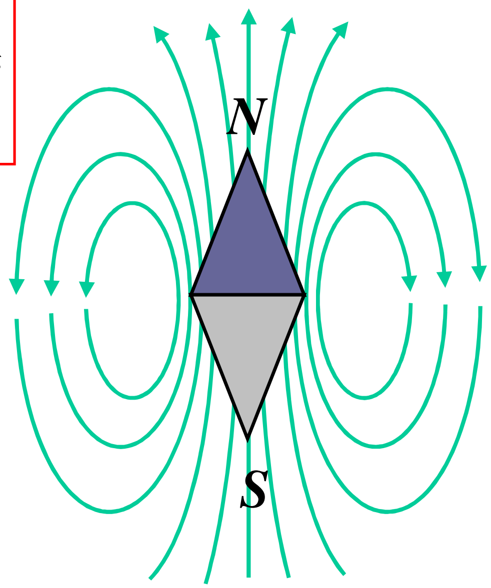
*John Michell (1750): misure su aghi magnetici lunghi e sottili, con pendolo di torsione*

$$\boxed{\vec{F}_{12} \propto \frac{q_{m1} q_{m2}}{r_{12}^2}\hat{r}_{12}}$$

*Si può scrivere*

$$\boxed{\vec{F}_{12} = \frac{\mu_0}{4\pi}\frac{q_{m1} q_{m2}}{r_{12}^2}\hat{r}_{12} = q_{m2}\vec{B}_1}$$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{r}}$$




*definendo così la "intensità" del polo magnetico:*

***$q_m$ = rapporto tra il modulo della forza che agisce su di esso in un dato campo, e l'intensità del campo stesso.***

$$\boxed{q_{m2} = \frac{\left|\vec{F}_{12}\right|}{\left|\vec{B}_1\right|}}$$

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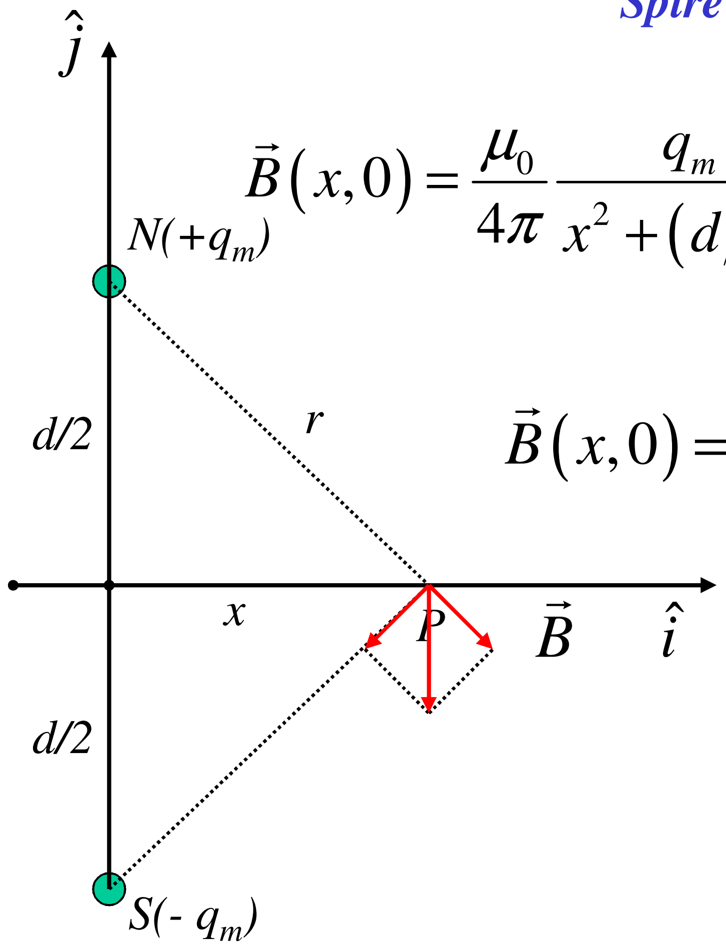
## *Spire ed aghi magnetici*

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{r}$$




$$\vec{B}(x,0) = \frac{\mu_0}{4\pi}\frac{q_m}{x^2+(d/2)^2}\left(-\frac{d/2}{\sqrt{x^2+(d/2)^2}}-\frac{d/2}{\sqrt{x^2+(d/2)^2}}\right)\hat{j}$$

$$\vec{B}(x,0) = -\frac{\mu_0}{4\pi}\frac{q_m d}{r^3}\hat{j} \qquad B(x,0)\underset{\lim\limits_{x\to\infty}}{\longrightarrow}\propto\frac{1}{\left|x^3\right|}$$

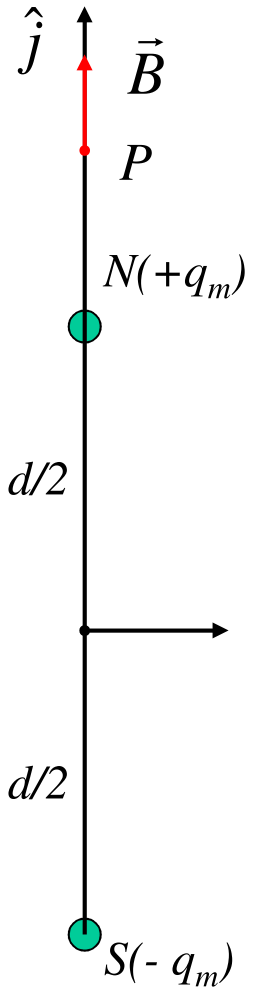
*Momento di dipolo magnetico:* $\boxed{\vec{m} = q_m d\,\hat{j}}$

$$\boxed{\vec{B}(x,0) = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3}}$$

# Le sorgenti del magnetismo

## *Spire ed aghi magnetici*

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{r}}$$




$$\vec{B}(0,y) = \frac{\mu_0 q_m}{4\pi}\left(\frac{1}{[y-d/2]^2} - \frac{1}{[y+d/2]^2}\right)\hat{j}$$

$$\vec{B}(0,y) = \frac{\mu_0 q_m}{4\pi}\left(\frac{[y+d/2]^2 - [y-d/2]^2}{[y-d/2]^2[y+d/2]^2}\right)\hat{j}$$

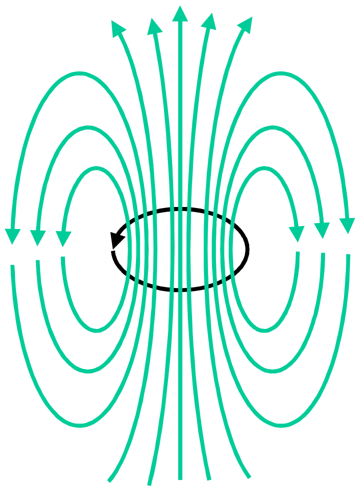
$$\vec{B}(0,y) \underset{\substack{\lim \\ y \gg d}}{\approx} \frac{\mu_0 q_m}{4\pi}\left(\frac{2yd}{y^4}\right)\hat{j}$$

$$\boxed{\vec{B}(0,y) \underset{\substack{\lim \\ y \gg d}}{\approx} \frac{\mu_0}{2\pi}\frac{\vec{m}}{y^3}}$$

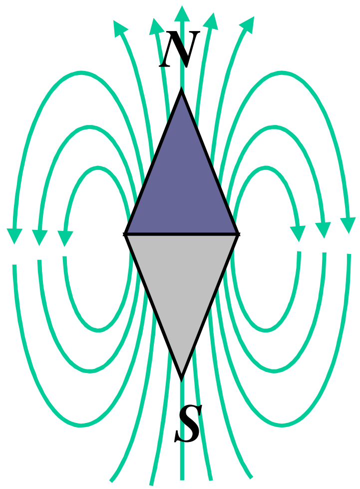
# Le sorgenti del magnetismo

### *Spire ed aghi magnetici*

*Campo magnetico sull'asse di simmetria, distanti dalla sorgente*



$\hat{k}$ $\uparrow$




$$\boxed{\vec{B} \underset{r \gg R}{\overset{\lim}{\approx}} \frac{\mu_0}{2\pi}\frac{\vec{m}}{r^3}}$$

$$\vec{m} = iS\,\hat{k}$$

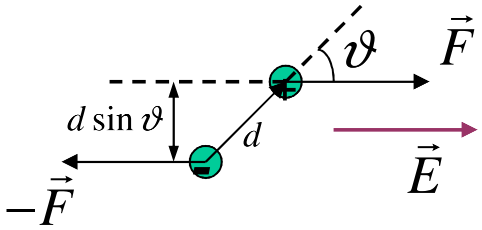
$$\boxed{\vec{B} \underset{r \gg d}{\overset{\lim}{\approx}} \frac{\mu_0}{2\pi}\frac{\vec{m}}{r^3}}$$

$$\vec{m} = q_m d\,\hat{k}$$

# Le sorgenti del magnetismo

### *Spire ed aghi magnetici*

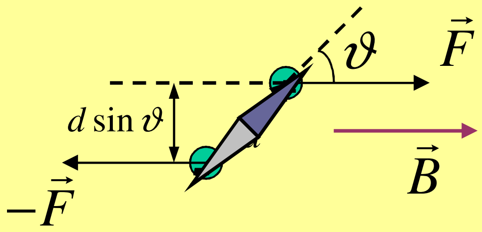
*Ago magnetico – analogia con il dipolo elettrico in un campo elettrostatico*




$$\vec{\mathcal{M}} = \vec{p} \wedge \vec{E}$$

$$U_E = q\Delta V_+ - q\Delta V_- = -\frac{d}{2}Eq\cos\vartheta - \frac{d}{2}Eq\cos\vartheta$$

$$U_E = -\vec{p} \cdot \vec{E}$$



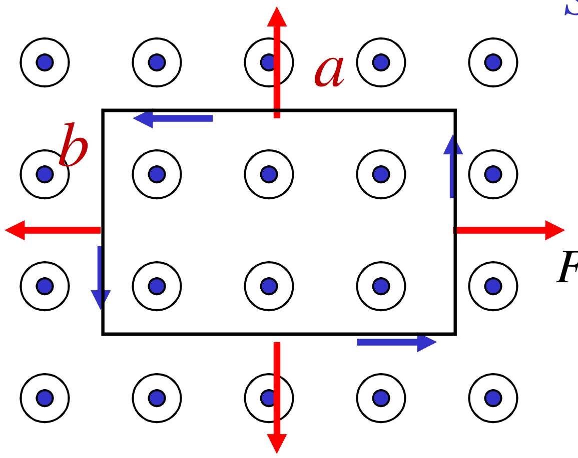

$$\vec{\mathcal{M}} = \vec{m} \wedge \vec{B}$$

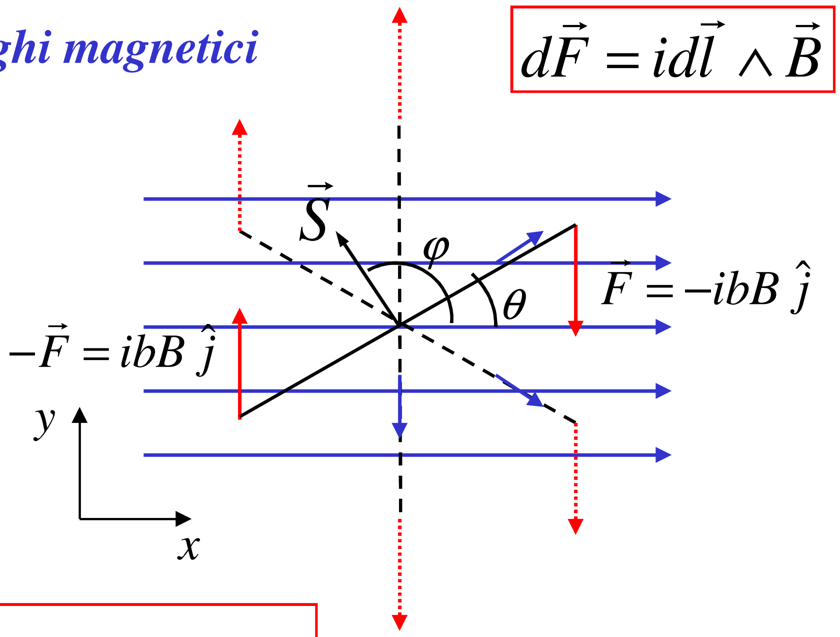
$$U_B = -\vec{m} \cdot \vec{B}$$

# Le sorgenti del magnetismo

***Spire ed aghi magnetici***

$$d\vec{F} = i d\vec{l} \wedge \vec{B}$$




$$\begin{aligned}\vec{\mathcal{M}} &= \vec{a} \wedge \vec{F} \\ &= \vec{a} \wedge \left(-ibB\,\hat{j}\right) = -iBba\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right) \wedge \hat{j} \\ &= -iSB\cos\theta\hat{k} = -iSB\sin\varphi\hat{k} = i\vec{S} \wedge \vec{B}\end{aligned}$$

$$\vec{\mathcal{M}} = \vec{m} \wedge \vec{B}$$

*Il risultato è coerente con* $U_B = -\vec{m} \cdot \vec{B}$

$\mathbf{B} = \nabla \times \mathbf{A}$