Fisica Generale A

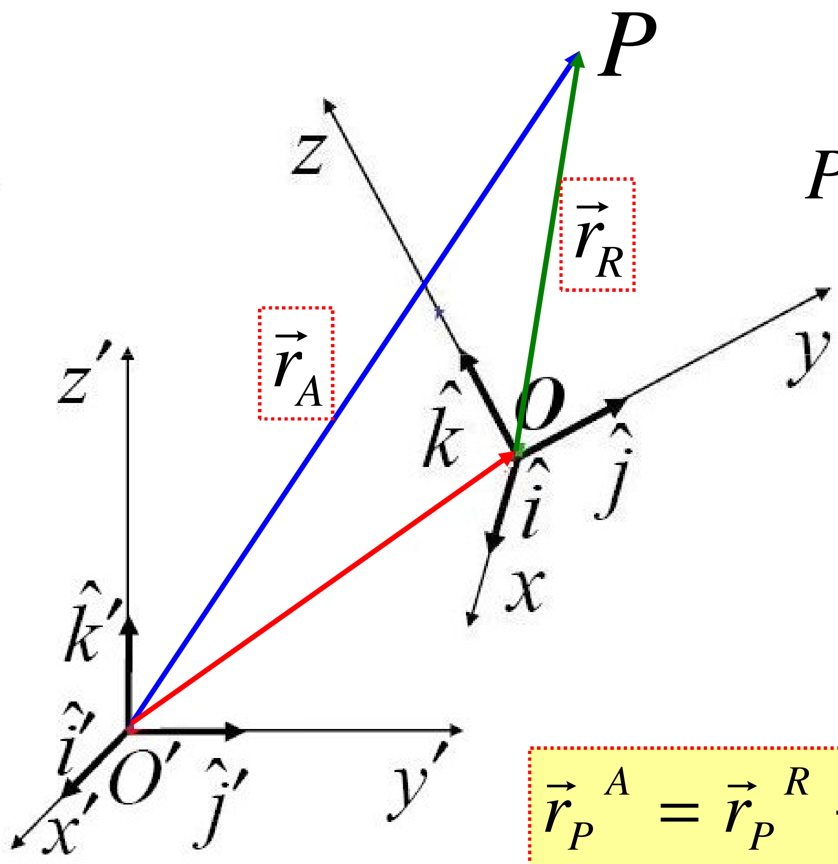
Moti relativi

Scuola di Ingegneria e Architettura

UNIBO – Cesena

Anno Accademico 2015 – 2016

Moti relativi



Posizione

$$P - O' = (P - O) + (O - O')$$

$$\vec{r}_A = \vec{r}_R + (O - O')$$

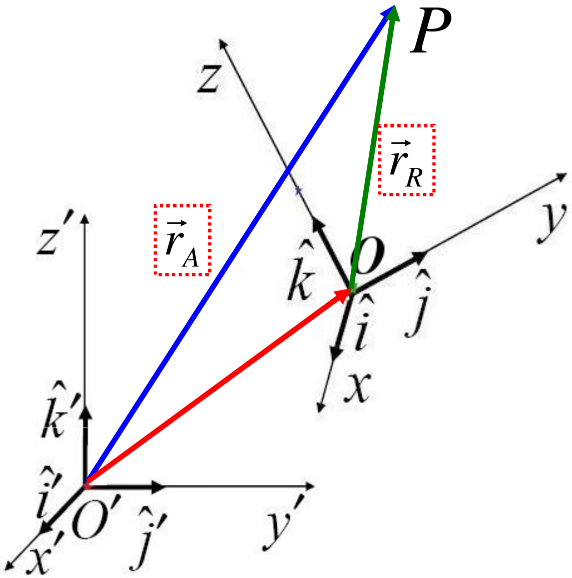
$$\vec{r}_R = \vec{r}_A + (O' - O)$$

$$\vec{r}_P^{\ A} = \vec{r}_P^{\ R} + \vec{r}_O^{\ A}$$

$$\vec{r}_P^{\ R} = \vec{r}_P^{\ A} + \vec{r}_{O'}^{\ R}$$

Moti relativi

Velocità



$$\vec{r}_A = \vec{r}_R + (O - O')$$

$$\vec{r}_R = P - O = x\hat{i} + y\hat{j} + z\hat{k} =$$
$$= (P - O') + (O' - O) = x'\hat{i}' + y'\hat{j}' + z'\hat{k}' + (O' - O)$$
$$\vec{r}_A = (P - O') = x'\hat{i}' + y'\hat{j}' + z'\hat{k}' =$$
$$= (P - O) + (O - O') = x\hat{i} + y\hat{j} + z\hat{k} + (O - O')$$

$$\vec{r}_R = x\hat{i} + y\hat{j} + z\hat{k}$$
$$\vec{r}_A = x\hat{i} + y\hat{j} + z\hat{k} + (O - O')$$

$$\vec{v}_R = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$
$$\vec{v}_A = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} + x\dot{\hat{i}} + y\dot{\hat{j}} + z\dot{\hat{k}} + \vec{v}_O$$

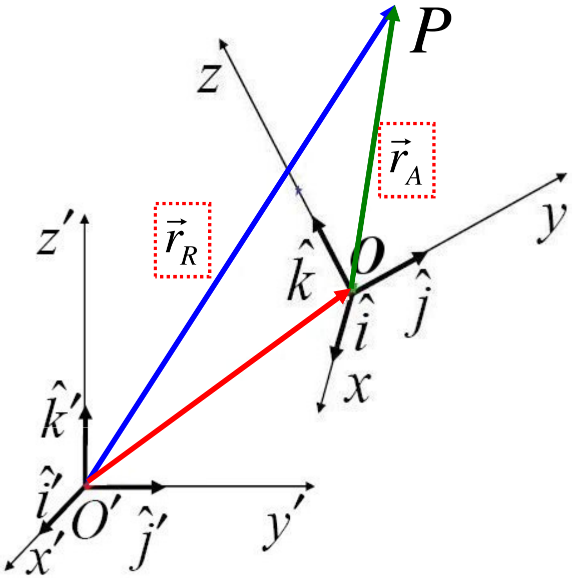
$$\vec{v}_A = \vec{v}_R + \vec{v}_O + \vec{\omega} \wedge (P - O)$$

$$x\dot{\hat{i}} + y\dot{\hat{j}} + z\dot{\hat{k}} = x\vec{\omega} \wedge \hat{i} + y\vec{\omega} \wedge \hat{j} + z\vec{\omega} \wedge \hat{k}$$
$$= \vec{\omega} \wedge \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$\vec{v}_A = \vec{v}_R + \vec{v}_T$$
$$\vec{v}_T = \vec{v}_O + \vec{\omega} \wedge (P - O)$$

$$\vec{v}_p{}^A = \vec{v}_P{}^R + \vec{v}_O{}^A + \vec{\omega} \wedge (P - O)$$

Moti relativi



Velocità

$$\vec{r}_A = \vec{r}_R + (O' - O)$$

$$\vec{r}_R = P - O' = x'\hat{i}' + y'\hat{j}' + z'\hat{k}' =$$
$$= (P - O) + (O - O') = x\hat{i} + y\hat{j} + z\hat{k} + (O - O')$$
$$\vec{r}_A = (P - O) = x\hat{i} + y\hat{j} + z\hat{k} =$$
$$= (P - O') + (O' - O) = x'\hat{i}' + y'\hat{j}' + z'\hat{k}' + (O' - O)$$
$$\vec{r}_R = x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$$
$$\vec{r}_A = x'\hat{i}' + y'\hat{j}' + z'\hat{k}' + (O' - O)$$

$$\vec{v}_R = \dot{x}'\hat{i}' + \dot{y}'\hat{j}' + \dot{z}'\hat{k}'$$
$$\vec{v}_A = \dot{x}'\hat{i}' + \dot{y}'\hat{j}' + \dot{z}'\hat{k}' + x'\dot{\hat{i}}' + y'\dot{\hat{j}}' + z'\dot{\hat{k}}' + \vec{v}_{O'}$$

$$\vec{v}_A = \vec{v}_R + \vec{v}_O + \vec{\omega}' \wedge (P - O')$$

$$x'\dot{\hat{i}}' + y'\dot{\hat{j}}' + z'\dot{\hat{k}}' = x'\vec{\omega}' \wedge \hat{i}' + y'\vec{\omega}' \wedge \hat{j}' + z'\vec{\omega}' \wedge \hat{k}'$$
$$= \vec{\omega}' \wedge \left(x'\hat{i}' + y'\hat{j}' + z'\hat{k}'\right)$$

$$\vec{v}_A = \vec{v}_R + \vec{v}_T$$
$$\vec{v}_T = \vec{v}_{O'} + \vec{\omega}' \wedge (P - O')$$

$$\vec{v}_p^{\,A} = \vec{v}_P^{\,R} + \vec{v}_{O'}^{\,A} + \vec{\omega}' \wedge (P - O')$$

Moti relativi

Accelerazione

$$\vec{v}_R = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$
$$\vec{v}_A = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} + x\dot{\hat{i}} + y\dot{\hat{j}} + z\dot{\hat{k}} + \vec{v}_O$$

$$\vec{a}_R = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$
$$\vec{a}_A = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k} + 2\left(\dot{x}\dot{\hat{i}} + \dot{y}\dot{\hat{j}} + \dot{z}\dot{\hat{k}}\right) + x\ddot{\hat{i}} + y\ddot{\hat{j}} + z\ddot{\hat{k}} + \dot{\vec{v}}_O$$

$$\dot{x}\dot{\hat{i}} = \dot{x}\vec{\omega}\wedge\hat{i} = \vec{\omega}\wedge\dot{x}\hat{i}$$

$$x\ddot{\hat{i}} = x\frac{d}{dt}\dot{\hat{i}} = x\frac{d}{dt}\left(\vec{\omega}\wedge\hat{i}\right) =$$
$$= x\dot{\vec{\omega}}\wedge\hat{i} + x\vec{\omega}\wedge\dot{\hat{i}} =$$
$$= x\dot{\vec{\omega}}\wedge\hat{i} + x\vec{\omega}\wedge\left(\vec{\omega}\wedge\hat{i}\right) =$$
$$= \dot{\vec{\omega}}\wedge x\hat{i} + \vec{\omega}\wedge\left(\vec{\omega}\wedge x\hat{i}\right)$$

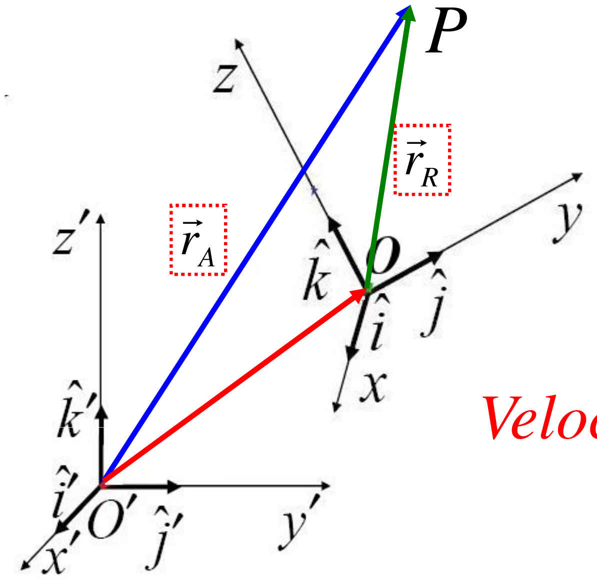
$$\vec{a}_A = \vec{a}_R + 2\vec{\omega}\wedge\left(\dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}\right) + \dot{\vec{\omega}}\wedge\left(x\hat{i} + y\hat{j} + z\hat{k}\right) + \vec{\omega}\wedge\left[\vec{\omega}\wedge\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\right] + \dot{\vec{v}}_O$$

$$\vec{a}_A = \vec{a}_R + \vec{a}_O + \dot{\vec{\omega}}\wedge(P-O) + \vec{\omega}\wedge\left[\vec{\omega}\wedge(P-O)\right] + 2\vec{\omega}\wedge\vec{v}_R$$

$$\vec{a}_A = \vec{a}_R + \vec{a}_T + \vec{a}_C$$
$$\vec{a}_T = \vec{a}_O + \dot{\vec{\omega}}\wedge(P-O) + \vec{\omega}\wedge\left[\vec{\omega}\wedge(P-O)\right]$$
$$\vec{a}_C = 2\vec{\omega}\wedge\vec{v}_R$$

$$\vec{a}_P{}^A = \vec{a}_P{}^R + \vec{a}_O{}^A + \dot{\vec{\omega}}\wedge(P-O) + \vec{\omega}\wedge\left[\vec{\omega}\wedge(P-O)\right] + 2\vec{\omega}\wedge\vec{v}_P{}^R$$

Moti relativi



Posizione

$$\vec{r}_A = \vec{r}_R + (O - O')$$

$$\vec{r}_P{}^A = \vec{r}_P{}^R + \vec{r}_O{}^A$$

Velocità

$$\vec{v}_A = \vec{v}_R + \vec{v}_T$$

$$\vec{v}_T = \vec{v}_O + \vec{\omega} \wedge (P - O)$$

Vedi Corpo Rigido

$$\vec{v}_p{}^A = \vec{v}_P{}^R + \vec{v}_O{}^A + \vec{\omega} \wedge (P - O)$$

Accelerazione

$$\vec{a}_A = \vec{a}_R + \vec{a}_T + \vec{a}_C$$

$$\vec{a}_T = \vec{a}_O + \dot{\vec{\omega}} \wedge (P - O) + \vec{\omega} \wedge [\vec{\omega} \wedge (P - O)]$$

$$\vec{a}_C = 2\vec{\omega} \wedge \vec{v}_R$$

$$\vec{a}_P{}^A = \vec{a}_P{}^R + \vec{a}_O{}^A + \dot{\vec{\omega}} \wedge (P - O) + \vec{\omega} \wedge [\vec{\omega} \wedge (P - O)] + 2\vec{\omega} \wedge \vec{v}_P{}^R$$

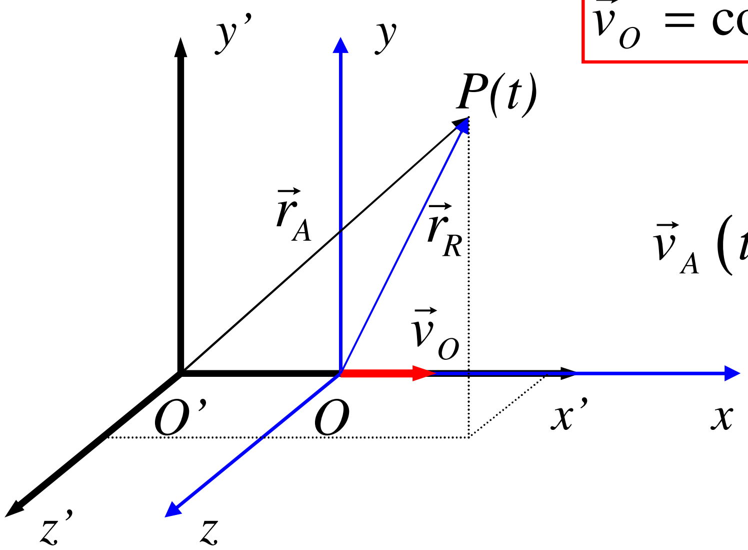
Moti relativi

Trasformazioni di Galileo

$$\vec{r}_A = \vec{r}_R + (O - O')$$

$$\vec{v}_A = \vec{v}_R + \vec{v}_T$$
$$\vec{v}_T = \vec{v}_O + \vec{\omega} \wedge (P - O)$$

$$\vec{a}_A = \vec{a}_R + \vec{a}_T + \vec{a}_C$$
$$\vec{a}_T = \vec{a}_O + \dot{\vec{\omega}} \wedge (P - O) + \vec{\omega} \wedge [\vec{\omega} \wedge (P - O)]$$
$$\vec{a}_C = 2\vec{\omega} \wedge \vec{v}_R$$



$$\vec{v}_O = \text{costante} \quad , \quad \vec{\omega} = \vec{0}$$

$$x'(t) = x(t) + v_O t$$
$$y'(t) = y(t)$$
$$z'(t) = z(t)$$

$$\vec{v}_A(t) = \vec{v}_R(t) + \vec{v}_O \quad \Rightarrow \quad \begin{cases} v_{x'}(t) = v_x(t) + v_O \\ v_{y'}(t) = v_y(t) \\ v_{z'}(t) = v_z(t) \end{cases}$$

$$\vec{a}_A = \vec{a}_R$$

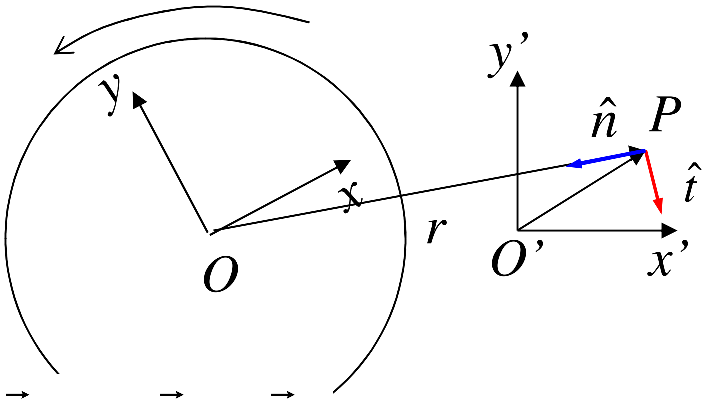
Moti relativi

$$\vec{r}_A = \vec{r}_R + (O - O')$$

$$\vec{v}_A = \vec{v}_R + \vec{v}_T$$
$$\vec{v}_T = \vec{v}_O + \vec{\omega} \wedge (P - O)$$

$$\vec{a}_A = \vec{a}_R + \vec{a}_T + \vec{a}_C$$
$$\vec{a}_T = \vec{a}_O + \dot{\vec{\omega}} \wedge (P - O) + \vec{\omega} \wedge [\vec{\omega} \wedge (P - O)]$$
$$\vec{a}_C = 2\vec{\omega} \wedge \vec{v}_R$$

$$\vec{v}_O = \vec{0} \quad , \quad \vec{\omega} = \text{costante} \quad , \quad \vec{v}_A = \vec{0}$$



$$\vec{v}_R(t) = -\vec{v}_T(t)$$
$$\vec{v}_R(t) = -\vec{\omega} \wedge (P - O)$$
$$\vec{v}_R(t) = |\omega r| \hat{t}$$

$$\vec{a}_R = -\vec{a}_T - \vec{a}_C$$
$$\vec{a}_R = -\vec{\omega} \wedge [\vec{\omega} \wedge (P - O)] - 2\vec{\omega} \wedge \vec{v}_R$$
$$\vec{a}_R = -\vec{\omega} \wedge [\vec{\omega} \wedge (P - O)] + 2\vec{\omega} \wedge [\vec{\omega} \wedge (P - O)]$$

$$\vec{a}_R = \vec{\omega} \wedge [\vec{\omega} \wedge (P - O)]$$
$$\vec{a}_R(t) = |\omega^2 r| \hat{n}$$