Weak interactions and the decay of $\pi^{\pm}$ and K^0 mesons

Matteo Negrini
INFN-Bologna

INFN

Parity (P)

Parity operator P → inversion of the spatial coordinates: $P\,\psi(x) \rightarrow \psi(-x)$

$P\,\psi(x) = +\,\psi(x)$ Even parity (P=+1)

$P\,\psi(x) = -\,\psi(x)$ Odd parity (P=-1)

$P\,\psi(x) \neq \pm\,\psi(x)$ No definite parity eigenvalue

For the spherical harmonic functions (angular momentum): $P=(-1)^l$

$$P\,Y_l^m(\theta,\phi) = Y_l^m(\pi-\theta,\pi+\phi) = (-1)^l\,Y_l^m(\theta,\phi)$$

Particles have an intrinsic parity +1 or -1

- Fermions: particles and antiparticles have opposite parity
- Bosons: particles and antiparticles have the same parity

Parity is a multiplicative quantum number (angular and intrinsic parts)
It is conserved by the strong and electromagnetic interactions and this allows to determine the relative parity of particles

Charge conjugation (C)

The charge conjugation reverse the charge and the magnetic moment of a particle (all other coordinates remain unchanged).
In relativistic quantum mechanics it corresponds to the replacement of a particle with its anti-particle.

$$C\,|\,\text{particle}\,\rangle \rightarrow |\,\text{anti-particle}\,\rangle$$

it therefore implies a change of sign in the lepton/baryon number

Only neutral mesons may be C eigenstates:

$$C\,|\,\pi^+\rangle \rightarrow |\,\pi^-\rangle \neq \pm|\,\pi^+\rangle$$
$$C\,|\,\pi^0\rangle = \eta|\,\pi^0\rangle$$

$\eta^2=1$; the decay $\pi^0 \rightarrow \gamma\gamma$ implies $\eta=+1$ ($\pi^0 \rightarrow 3\gamma$ not possible since $C_\gamma=-1$)

INFN

Time reversal (T)

Physics laws may be invariant or not under time reversal

- filming a free falling object in gravitational field looks realistic when observed forward and backward
- time direction cannot be reversed for heat diffusion

Elementary interactions obey time reversal invariance (almost… we will see later when this is not true)

Experimentally this can be verified by checking inverse interactions

$$a + b \leftrightarrow c + d$$

P violation in weak interactions

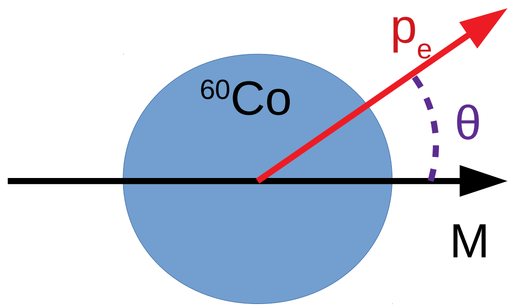
τ-θ puzzle: strange mesons with the same mass decay to $\pi^+ \pi^+ \pi^-$ and $\pi^+ \pi^0$, which have different parity

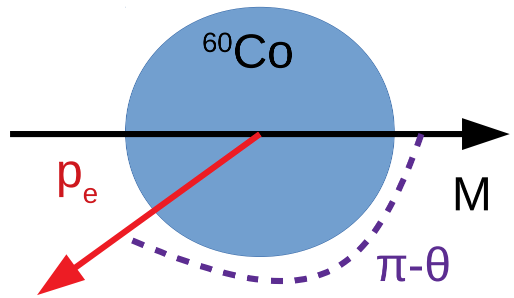
In 1956 Lee and Yang proposed that weak interactions are not invariant under P, so a single particle can decay in both ways

P conservation in weak decays was experimentally tested by C.S. Wu et al in 1957

Observation of P violation in ^{60}Co decay

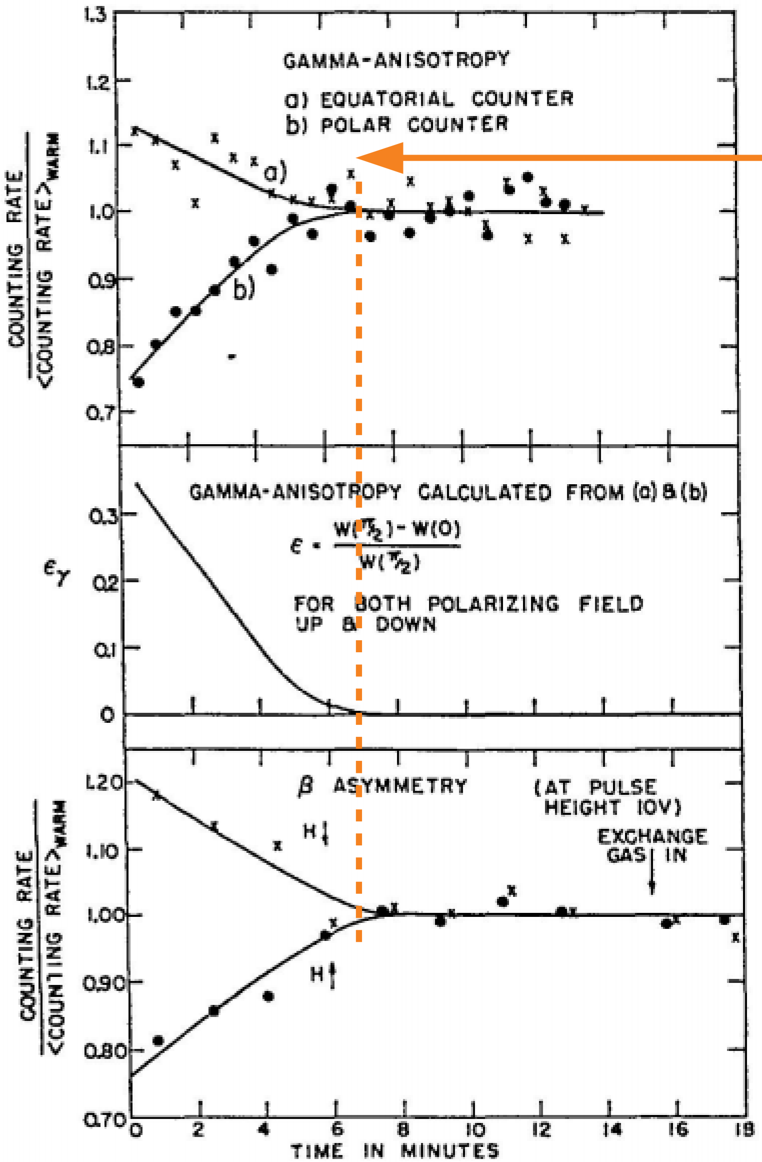
Wu et al. (1957)
Sample of ^{60}Co (J=5) at T=0.01 K inside a solenoidal magnetic field
In these conditions the spins of Co nuclei are highly aligned
^{60}Co undergo β^- decay to ^{60}Ni* (J=4)
The degree of alignment can be determined from the angular distribution of γ rays from ^{60}Ni* decay





θ: angle of emission of the e^- with respect to the polarization direction
P inverts the e^- momentum but leaves the spin (polarization of the nuclei) unchanged
An asymmetry in the distribution between θ and π-θ implies P violation

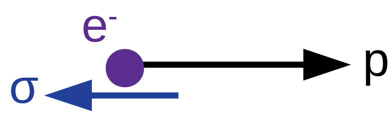
C.S. Wu et al results



In the warm-up time of ~6 min the gamma anisotropy disappears

A beta asymmetry is observed, disappearing with the gamma anisotropy

1) P violation in weak decay
2) e^- polarized in the direction opposite to their momentum



Theory indicates a degree of polarization proportional to *v/c*


Chien-Shiung Wu in 1958 at Columbia University

Experiment done by C.S. Wu with a team at the National Bureau of Standards (experts able to reach the necessary low temperatures and obtain polarized nuclei)

The V-A theory of weak interactions

In analogy with the electromagnetic interaction, Fermi proposed that weak interactions proceed through currents.
The original theory considers point-like interactions.

For neutron decay the matrix element can be written as:

$$M = G\, J_{baryon} \cdot J_{lepton}$$

where *J* are the leptonic an baryonic currents

$$J_{baryon} = \bar{p}\, O\, n$$
$$J_{lepton} = \bar{e}\, O\, \nu$$

In general the Lorentz invariant operator *O*, that involve spin transitions, can have 5 possible forms in terms of Dirac matrices:

$$J_{lepton} = \begin{cases} \bar{e}\,\nu & \text{Scalar} \\ \bar{e}\,\gamma_5\,\nu & \text{Pseudoscalar} \\ \bar{e}\,\gamma^\mu\,\nu & \text{Vector} \\ \bar{e}\,\gamma^\mu\,\gamma_5\,\nu & \text{Axial} \\ i/2\,\bar{e}\,[\gamma^\mu, \gamma^\nu]\,\nu & \text{Tensor} \end{cases}$$

Names associated to the transformation properties of the weak currents under space inversion

The V-A theory of weak interactions

It was noticed that:

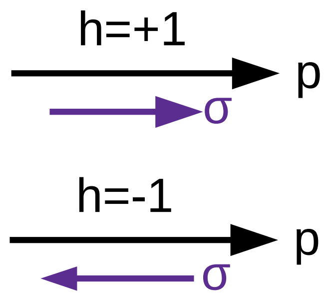
- for S and V spin transitions of the nucleus are not allowed
- for A and T produce spin transition of the nucleus of one unit

but both type of beta decays are observed, so the weak interactions should contain at least one term of the two types.

Different terms also have consequences on the helicity of the leptons

- V and A interactions result in leptons and antileptons of opposite helicities
- S, T and P interactions produce leptons and antileptons with the same helicity

$$h = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{\sigma}|\cdot|\vec{p}|}$$



The theory must not have well defined parity + determination of lepton helicities yield to the V-A theory

$$J_{lepton} = \bar{e}\,\gamma^{\mu}(1 \pm \gamma_5)\,\nu$$

- $1+\gamma_5 \rightarrow$ right handed neutrino (left handed antineutrino)
- $1-\gamma_5 \rightarrow$ left handed neutrino (right handed antineutrino)

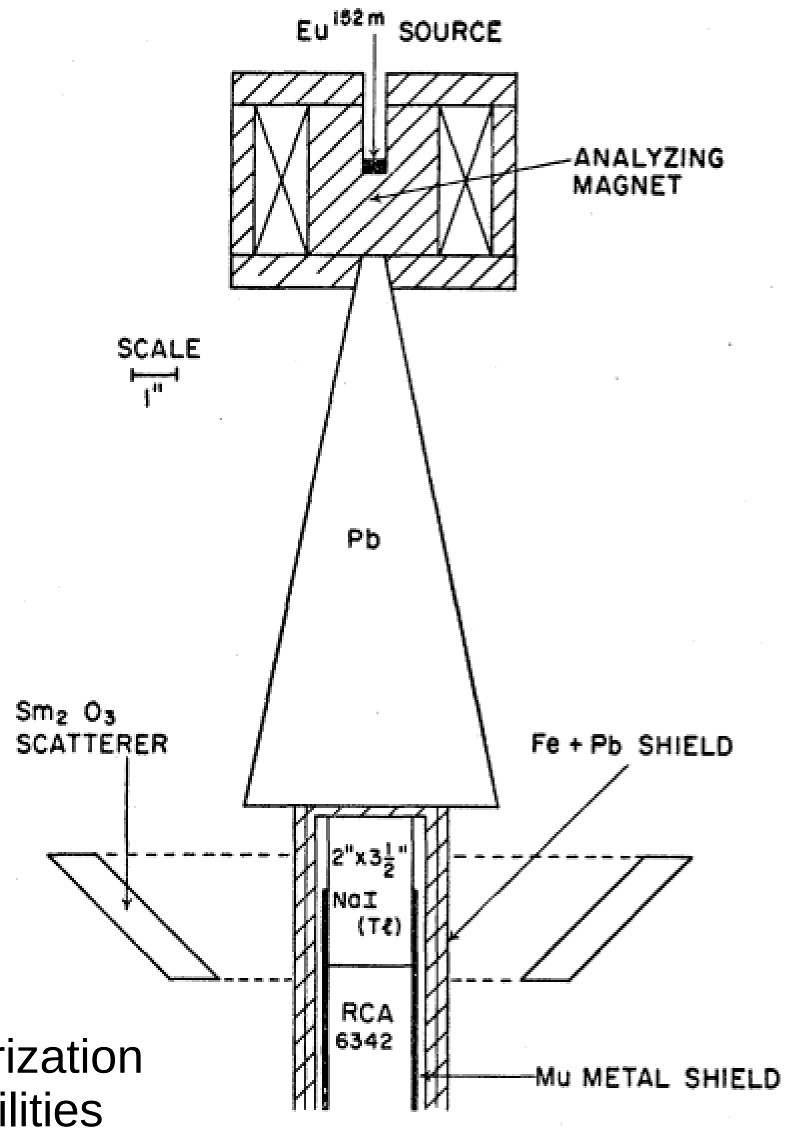
Helicity of neutrinos - 1

Goldhaber, Grodzins, Sunyar (1957)

^{152m}Eu (J=0) decays by electron capture

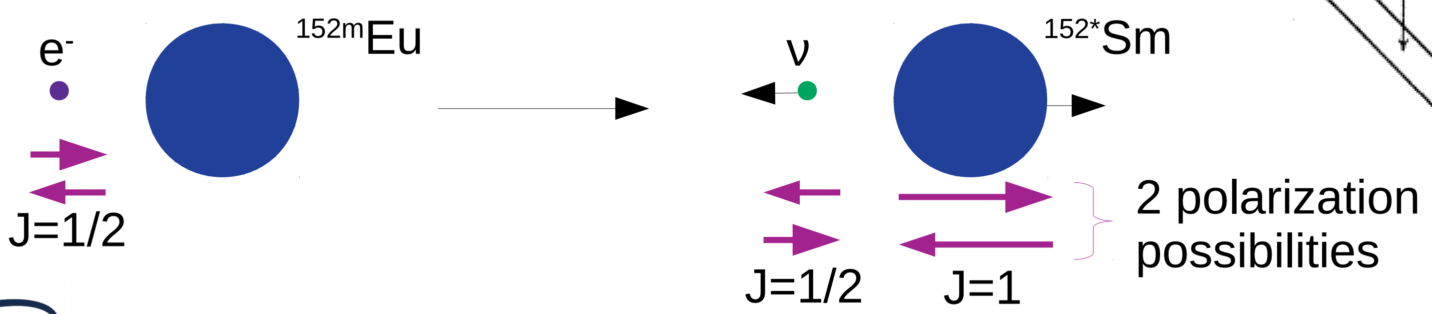
$$e^- \quad {}^{152m}Eu \rightarrow {}^{152*}Sm \quad \nu$$

^{152*}Sm (J=1) decays rapidly to the ground state
^{152}Sm (J=0) by γ emission



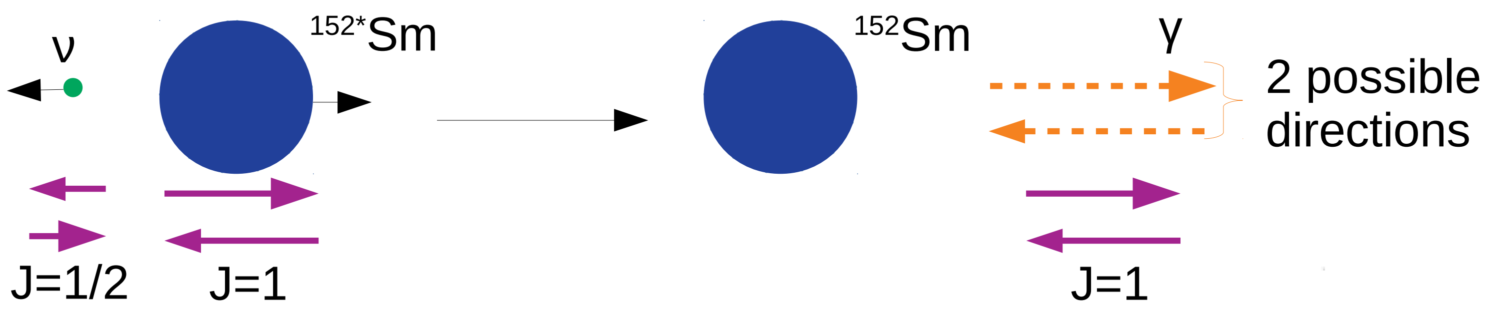
e$^-$ capture prominently in s-wave
The spin of ^{152*}Sm is aligned with the one of the original e$^-$ and opposite to that of the ν

^{152*}Sm recoils on ν → 2 possible polarizations

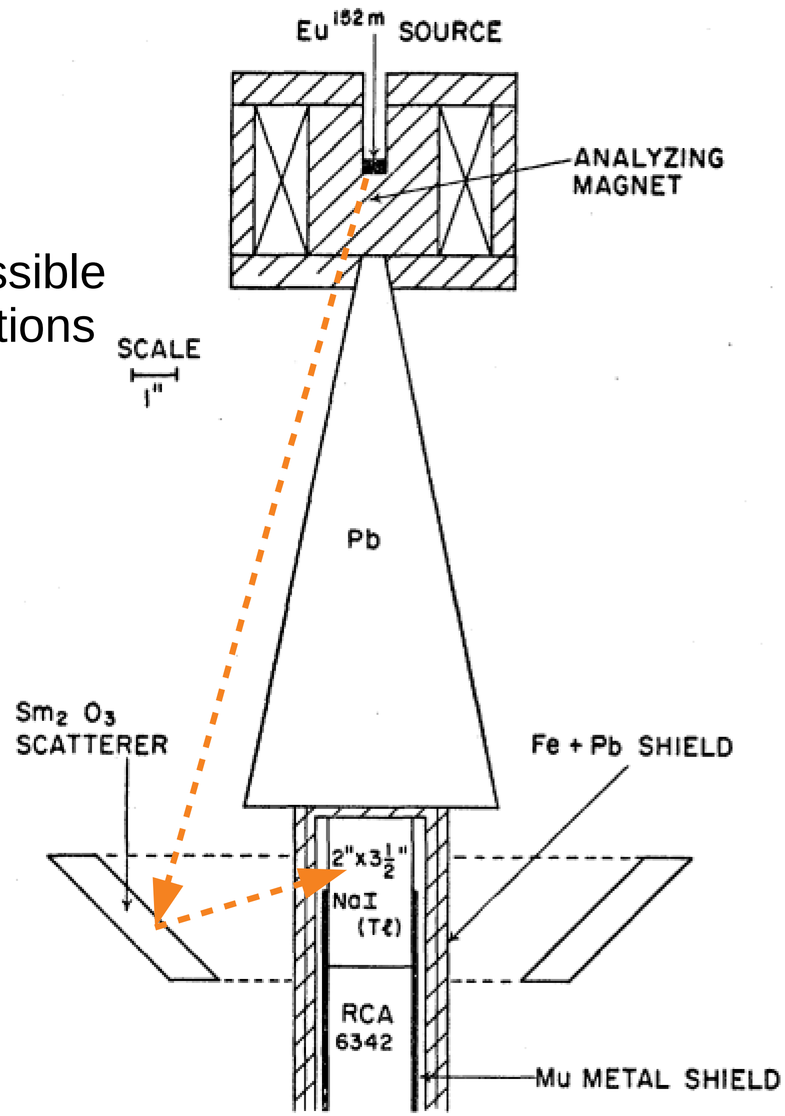


Helicity of neutrinos - 2

"Immediate" transition $^{152*}Sm \rightarrow {}^{152}Sm\ \gamma$ ($\tau \sim 10^{-13}$s), before Sm cnahges its velocity
J is taken by the gamma



- γ emitted in the direction of flight of ^{152*}Sm will be polarized as the ν
- γ emitted in the opposite direction will have polarization opposite to the ν

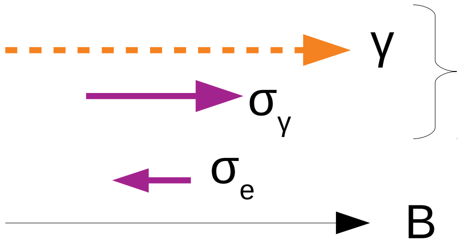


Observe resonant scattering of γ in ^{152}Sm target (NaI counter)
Because of doppler effect, only γ in the "forward" direction can produce resonant scattering

Helicity of neutrinos - 3

Study the dependence on the gamma polarization

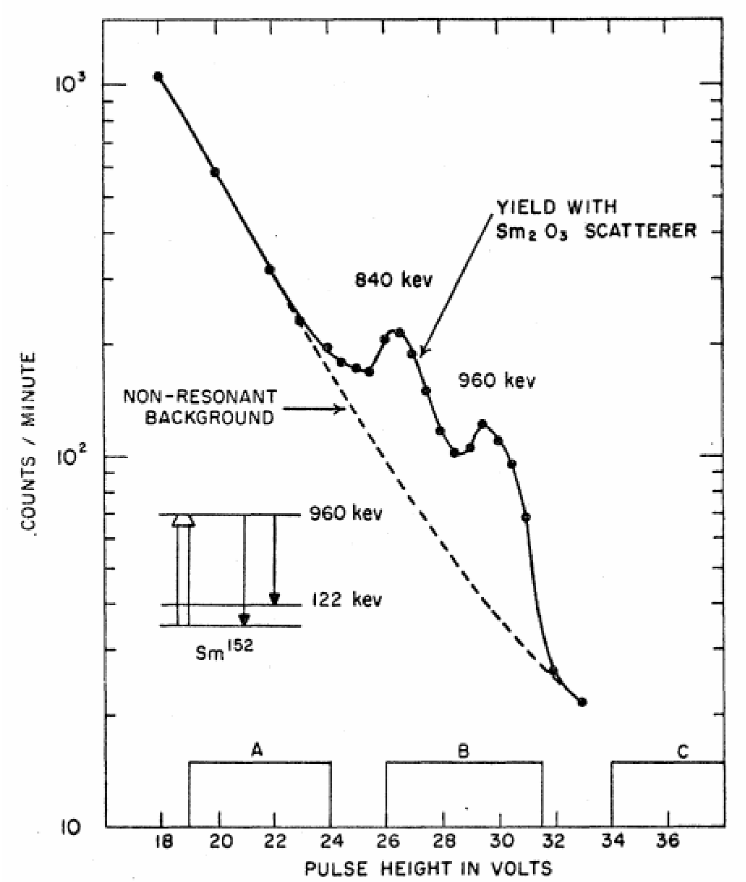
γ must pass through magnetized iron before interacting with the Sm absorber
If the spin of the electron in the iron is opposite to that of the photon they can interact producing an e- spin flip, otherwise they don't



Right-handed gamma can produce electron spin flip in iron

The transmission in the iron is larger for left-handed gammas than for right-handed

Comparison of the counting rates reverting the magnetic field → results compatible with 100% left-handed helicity neutrinos



Counting rate of resonant scattered gammas, observed in the NaI counter in absence of magnetic field by Goldhaber et al.

Charged pion decay

Pion decays branching fractions provide an important test of V-A theory

$$\pi^- \to \mu^- \bar{\nu}_\mu \qquad \pi^- \to e^- \bar{\nu}_e$$

The two processes are similar in principle, but the mass of the μ (105 MeV) is much closer to the one of the π (140 MeV) than the electron one (0.511 MeV)

- The π has spin 0, so the lepton and the neutrino are emitted with the same helicity
- If the neutrino has well-defined helicity, the lepton must have the "wrong" helicity
- The polarization degree depends on the velocity, therefore the decay into muon is largely favored

The ratio of branching fraction computed in the V-A theory is:

$$\frac{\pi^- \to e^- \bar{\nu}_e}{\pi^- \to \mu^- \bar{\nu}_\mu} = 1.2 \times 10^{-4}$$

Charged pion decay

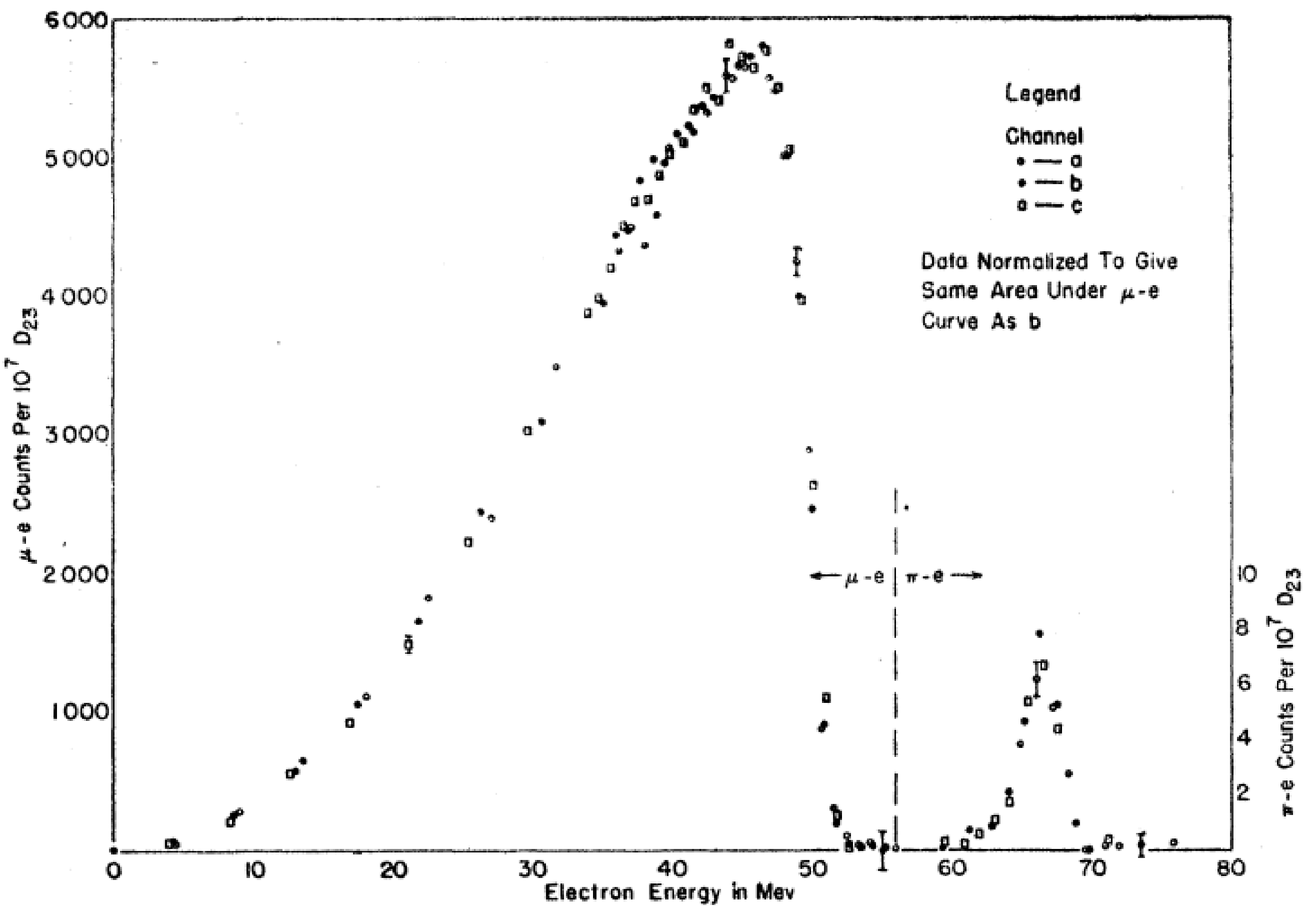
Anderson, Fujii, Miller, Tau (1960)

π+ beam stopped in target → energy spectrum of emerging electrons

Measured:

$$\frac{\pi^- \rightarrow e^- \ \bar{\nu}_e}{\pi^- \rightarrow \mu^- \ \bar{\nu}_\mu} = 1.21 \pm 0.07 \times 10^{-4}$$

Perfect agreement with expectations for V-A interaction



Weak decay of strange particles

Extremely successful theory of weak interaction
The strength for ΔS=0 processes is larger than for ΔS=1 ones
How to include the weak decay of strange particles to compare the 2 different processes?

Cabibbo proposed that in processes involving hadrons, the hadronic current is composed by 2 terms:

$$J_{hadron} = \cos\theta_c J^{\mu}_{\Delta S=0} + \sin\theta_c J^{\mu}_{\Delta S=1}$$

The Cabibbo angle is θ_c~13°, so ΔS=1 processes are suppressed
The interaction can be described introducing effective couplings for hadronic currents:

$$\Delta S=0: \quad G_{\pi}^2 = G_n^2 = G_{\mu}^2 \cos^2\theta_c$$

$$\Delta S=1: \quad G_K^2 = G_{\mu}^2 \sin^2\theta_c$$

The exact meaning of the Cabibbo angle become clearer in the following years

The neutral kaon system

K^0 and $\bar{K}^0$ and particle and antiparticle, with opposite strangeness.

They can be produced by the strong interaction, that conserves strangeness and isospin, in different processes, so K^0 and $\bar{K}^0$ are eigenstates of the strong interacion

They can both decay to $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$, which violates P, but this is not surprising now because K decay is due to the weak interaction

Assuming that the combination of charge conjugation and parity, CP, is a symmetry of the weak interaction, and knowing that C applied to K^0 produces a $\bar{K}^0$, we can write (fixing an arbitrary phase):

$$CP|K^0> \; = \; |\bar{K}^0>$$

If CP is conserved, the physical states, with definite mass and lifetime, are CP eigenstates:

CP=+1 : $|K_1^0> \; = \frac{1}{\sqrt{2}}[\; |K^0> \; + \; |\bar{K}^0> \;]$ CP=-1 : $|K_2^0> \; = \frac{1}{\sqrt{2}}[\; |K^0> \; - \; |\bar{K}^0> \;]$

INFN

The neutral kaon system

Being produced in s-wave, the $\pi^+\pi^-$ system from K^0 decay has CP=+1 while the $\pi^+\pi^-\pi^0$ has CP=-1. It turns out that the lifetime of K^0_2 is significantly longer than the one of K^0_1.

First observation of by Lande et al. (1956) at the Brookhaven Cosmotron, placing a cloud chamber (need to observe decays, not interactions!) at 6 m from the collision points. Observed events with 2 tracks and non-coplanar with the line of flight, requiring a 3rd neutral object to escape undetected.
The events are predominantly $\pi^\pm e^\pm \nu$, sometimes $\pi^\pm \mu^\pm \nu$, rarely $\pi^+\pi^-\pi^0$.

Strangeness oscillations

In vacuum, propagation and decay of the neutral K are described in terms of CP eigenstates:

$$|K_1^0(t)\rangle = e^{-iE_1 t - \Gamma_1 t/2} \frac{1}{\sqrt{2}} [\ |K^0(0)\rangle + |\bar{K}^0(0)\rangle\ ]$$

$$|K_2^0(t)\rangle = e^{-iE_2 t - \Gamma_2 t/2} \frac{1}{\sqrt{2}} [\ |K^0(0)\rangle - |\bar{K}^0(0)\rangle\ ]$$

Quantum mechanics allows us to consider the K^0 and $\bar{K}^0$, produced in strong interactions, as superposition of K^0_1 and K^0_2.
K^0_1 decays more rapidly so, after some time, only the K^0_2 part remain.

The amplitudes for K^0_1 and K^0_2 after a time t, and in the center of mass frame (E=m and we can use the proper lifetime):

$$\langle K_1^0 |K_1^0(t)\rangle = e^{-im_1 t - \Gamma_1 t/2} \langle K_1^0 |K_1^0(0)\rangle$$

$$\langle K_2^0 |K_2^0(t)\rangle = e^{-im_2 t - \Gamma_2 t/2} \langle K_2^0 |K_2^0(0)\rangle$$

Strangeness oscillations

Starting from a pure K^0 sample (initial K^0_1 and K^0_2 amplitudes are $1/\sqrt{2}$), after a time t:

$$<K^0|K^0(t)> = \frac{1}{2}\left(e^{-im_1 t - \Gamma_1 t/2} + e^{-im_2 t - \Gamma_2 t/2}\right)$$

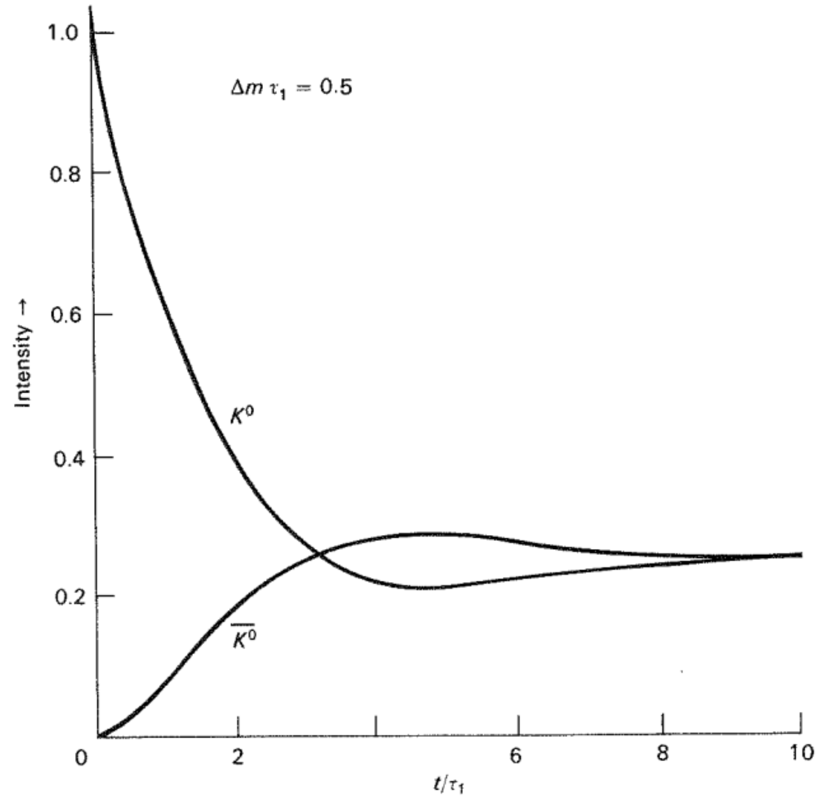
$$<\bar{K}^0|K^0(t)> = \frac{1}{2}\left(e^{-im_1 t - \Gamma_1 t/2} - e^{-im_2 t - \Gamma_2 t/2}\right)$$

so the probability to have K^0 and $\bar{K}^0$ after a time t is:

$$|<K^0|K^0(t)>|^2 = \frac{1}{4}\left(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-(\Gamma_1+\Gamma_2)t/2}\cos\Delta m t\right)$$

$$|<\bar{K}^0|K^0(t)>|^2 = \frac{1}{4}\left(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_1+\Gamma_2)t/2}\cos\Delta m t\right)$$

so K^0 and $\bar{K}^0$ intensities oscillate with frequency Δm.



K^0 regeneration

Pais and Piccioni (1955) suggested that a K^0_2 passing through a layer of material, should regenerate a K^0_1 component.

K^0 and $\bar{K}^0$ interact differently with the matter (strong interaction so the two states with definte strangeness come into play)
for example $\bar{K}^0 p \rightarrow \Lambda \pi^+$ is allowed while $K^0 p \rightarrow \Lambda \pi^+$ is not, so they are absorbed differently in matter.

It can be observed that the K^0 beam after emerging from the material has an increased K^0_1 component (small effect, order 10^{-3}).

This again can be interpreted as another effect of the superimposition of two quantum mechanical states

INFN

CP violation in the K^0 system

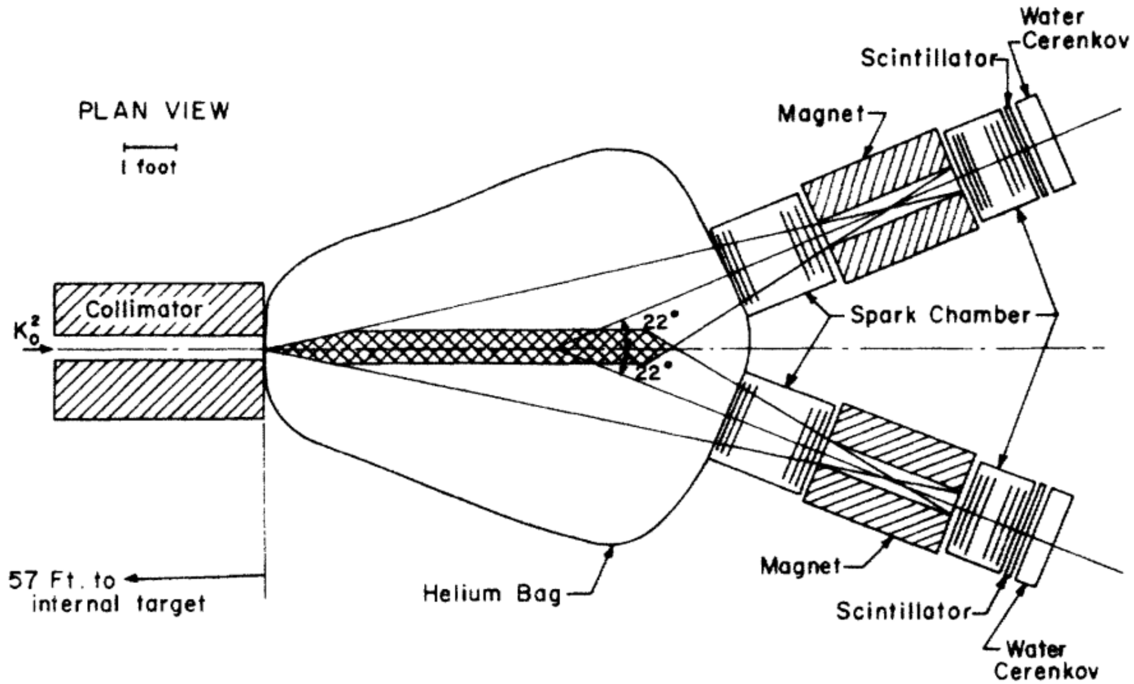
Christenson, Cronin, Fitch, Turlay (1964)

Observation of the decay $K^0_2 \rightarrow \pi^+\pi^-$ with branching fraction ~10^{-3}
If CP is a good symmetry this should be forbidden

30 GeV protons on Be target
at Brookhaven

Two arms spectrometer
at 17 m from the target

PID to separate $K^0_2 \rightarrow \pi^+\pi^-$
from other decays channels
2 charged tracks



CP violation in the K^0 system

The equation of motion can be generalized by introducing a new phase convention, so that the time evolution is described by a matrix:

$$i\frac{d}{dt}\begin{pmatrix} a(t)\ |K^0(t)> \\ b(t)\ |\bar{K}^0(t)> \end{pmatrix} = \begin{pmatrix} m_{11} - i\Gamma_{11}/2 & m_{12} - i\Gamma_{12}/2 \\ m_{12}^* - i\Gamma_{12}^*/2 & m_{22} - i\Gamma_{22}/2 \end{pmatrix} \begin{pmatrix} a(t)\ |K^0(t)> \\ b(t)\ |\bar{K}^0(t)> \end{pmatrix}$$

in which m_{12} and Γ_{12} are complex, and if their ratio is complex CP is violated.

Since CP violation is small, we can write the new terms as small deviations from the original CP conserving ones

$$M_{12} = R\,\Gamma_{12}(1 + i\,\kappa)$$

$$\Gamma_{12} = e^{-2i\phi}\Delta\Gamma/2$$

with $\kappa \ll 1$ and $R = \Delta m/\Delta\Gamma$ and introducing the phase ϕ.

CP violation in the K^0 system

The physical states, named K^0_S and K^0_L, are different from the CP eigenstates and given by

$$|K_S^0> = \frac{1}{\sqrt{2}}[e^{-i\phi}(1+\epsilon)\ |K^0(0)> + e^{i\phi}(1-\epsilon)\ |\bar{K}^0(0)>\ ]$$

$$|K_L^0> = \frac{1}{\sqrt{2}}[e^{-i\phi}(1+\epsilon)\ |K^0(0)> - e^{i\phi}(1-\epsilon)\ |\bar{K}^0(0)>\ ]$$

where $\epsilon = \frac{i\kappa R}{2R - i}$

The amplitude ratios $\eta_{+-} = \epsilon + \epsilon'$ and $\eta_{00} = \epsilon - 2\epsilon'$ are complex quantities that can be extracted from the branching ratios

$$\eta_{+-} = \frac{\Gamma(K_L^0 \to \pi^+\ \pi^-)}{\Gamma(K_S^0 \to \pi^+\ \pi^-)} \simeq 2.3 \times 10^{-3}$$

$$\eta_{00} = \frac{\Gamma(K_L^0 \to \pi^0\ \pi^0)}{\Gamma(K_S^0 \to \pi^0\ \pi^0)} \simeq 2.3 \times 10^{-3}$$

ϵ is determined by the mass mixing matrix, while ϵ' is due to direct CP violation in decay. Experimentally accessed measuring the ratio

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \approx 1 - 6\,\Re\,\frac{\epsilon'}{\epsilon}$$