The structure of the nucleon

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Nucleus in the 50's

Rutherford experiment: most of the nuclear mass is located in a the center of the atom → nucleus, composed by protons and neutrons

No information on the size and substructure of the nucleons

Electron is the ideal probe:

- electromagnetic interactions only
- interaction by single photon exchange

Start campaign of several e^- p scattering experiments with increasing e^- energy

- proton size (elastic scattering - E<GeV)
- proton substructure (deep inelastic scattering - E>GeV)

Elastic scattering of e$^-$ on nucleons

The scattering of relativistic e$^-$ ($E \gg m_e$) by a charge distribution can be calculated using quantum mechanics.

For spinless e$^-$ with energy E on point charge the cross section is given by the Rutherford formula

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2}$$

Taking into account the e$^-$ spin , the backscattering is supprssed to conserve the helicity and we get the Mott cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2 \theta/2}{4E^2 \sin^4 \theta/2}$$

If the e- is scattered by a static source ($M \rightarrow \infty$) its final energy E'=E. For finite mass the nucleon recoils the energy and transferred four-momentum are

$$E' = \frac{E}{1 + 2E/M \sin^2 \theta/2} \qquad q^2 = -4EE' \sin^2 \theta/2$$

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Elastic scattering of e$^-$ on nucleons

The elastic scattering of e$^-$ by a point-like massive particle is

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2}\frac{E'}{E}\left[1-\frac{q^2}{2M^2}\tan^2\theta/2\right]$$

These simple rules should be modified if the target has a spatial charge distribution ρ(r), the amplitude is modified by a form factor

$$F(q^2)=\int d^3r\, e^{i\vec{q}\cdot\vec{r}}\rho(r)$$

$F(q^2)\rightarrow 1$ for small momentum transfer $q\rightarrow 0$
$q^2>0$ to observe deviations from point-like behavior

The cross section for relativistic e$^-$ p scattering is given by the Rosenbluth formula

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2}\frac{E'}{E}\left[\left(F_1^2+\frac{\kappa^2Q^2}{4M^2}F_2^2\right)+\frac{Q^2}{2M^2}(F_1+\kappa F_2)^2\tan^2\theta/2\right]$$

$$Q^2=-q^2$$

Two form factors $F_{1,2}(q^2)$ for electric and magnetic coupling terms (κ=1.79 is the anomalous magnetic coupling of the proton in units of the nuclear magneton $e\hbar/2Mc$)

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The “size” of the proton

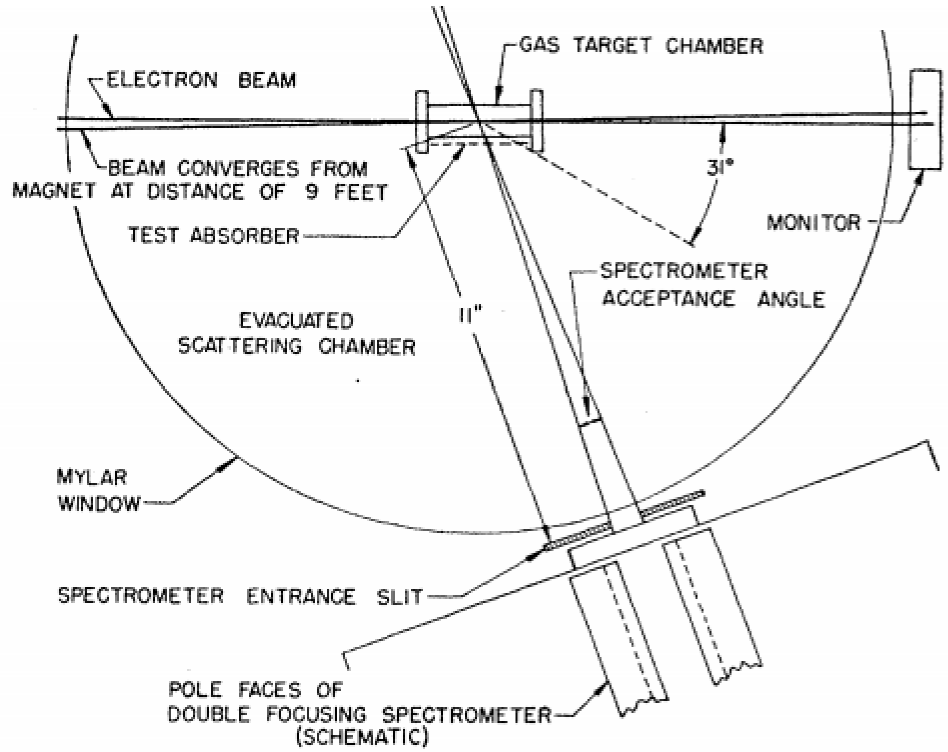
McAllister and Hofstadter (1956)

Elastic scattering of 188 MeV e- at Stanford LINAC on protons
Spectrometer can be rotated around the interaction region

Robert Hofstadter (1915-1990) Nobel prize 1961 for his studies

Sensitivity to the root mean square radius of the proton at low momentum transfers:

$$F(q^2) = \int d^3r\, e^{i\vec{q}\cdot\vec{r}} \rho(r)$$
$$= \int d^3r\, \rho(r)\left[1 + i\vec{q}\cdot\vec{r} - 1/2(\vec{q}\cdot\vec{r})^2 + \ldots\right]$$
$$= 1 - \frac{q^2}{6} \langle r^2 \rangle + \ldots$$

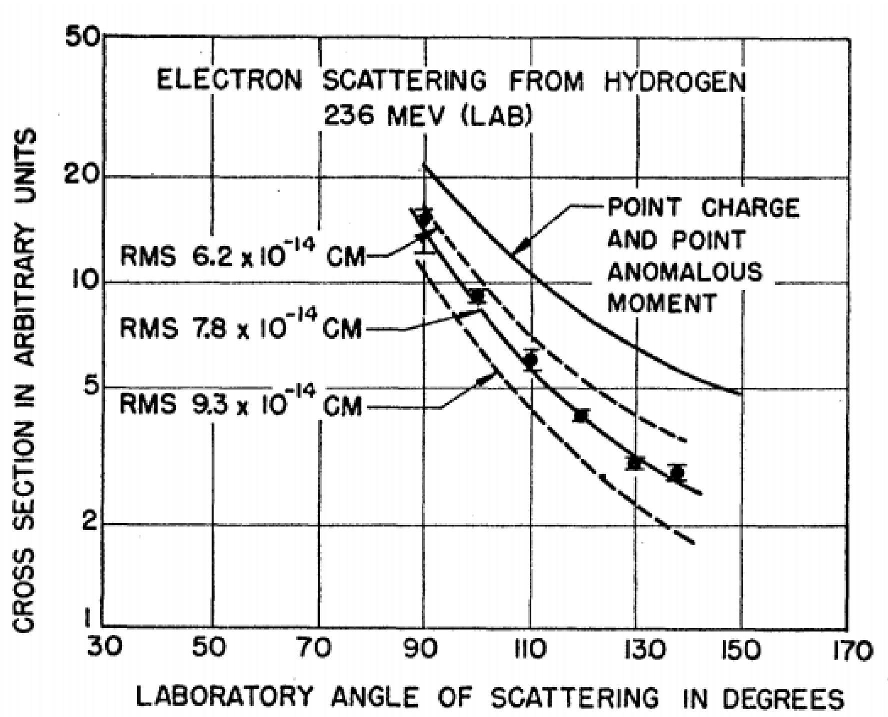


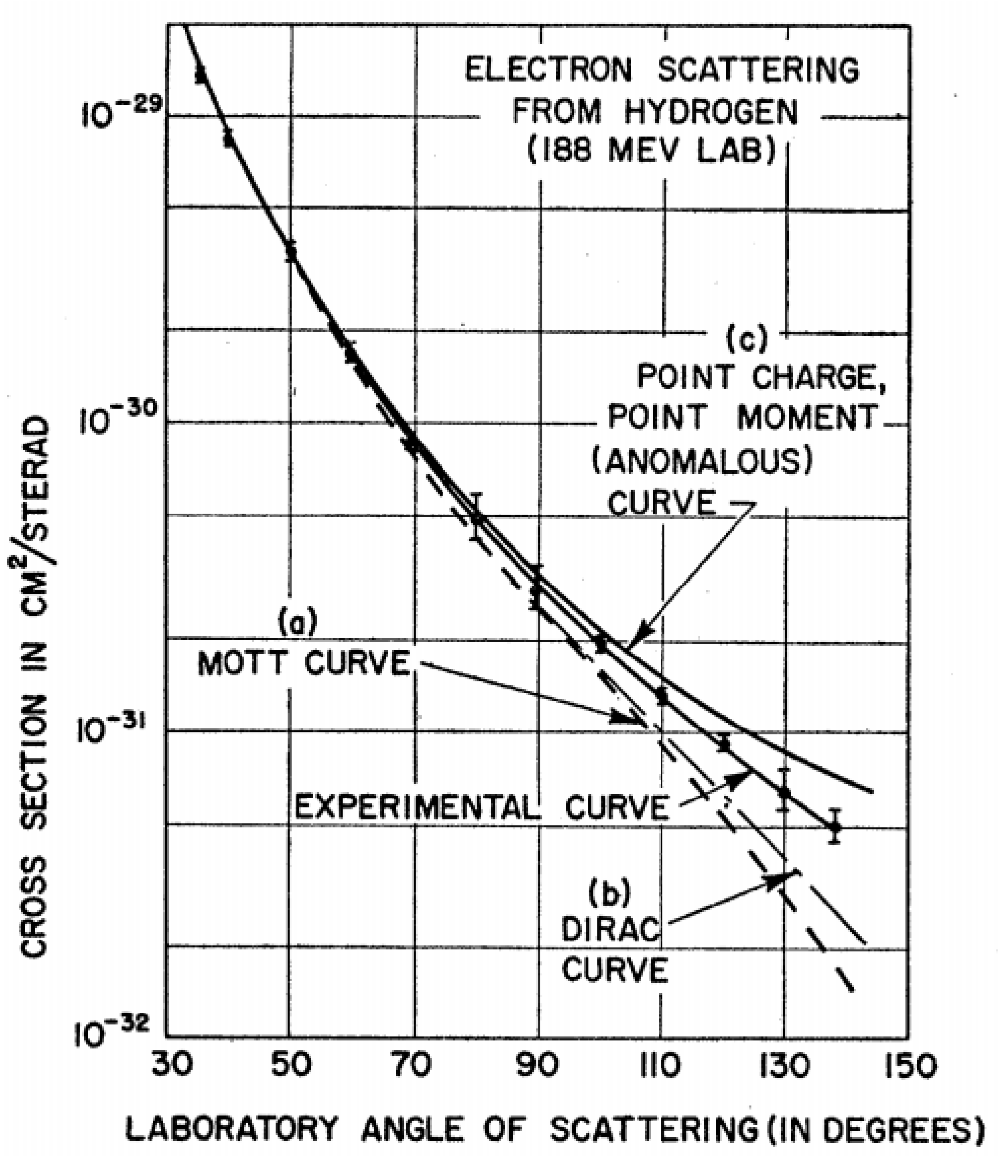
The “size” of the proton

Measurements incompatible with point-like proton

Best description of data obtained for:

$\sqrt{\langle r^2 \rangle} = 0.74 \pm 0.24$ fm

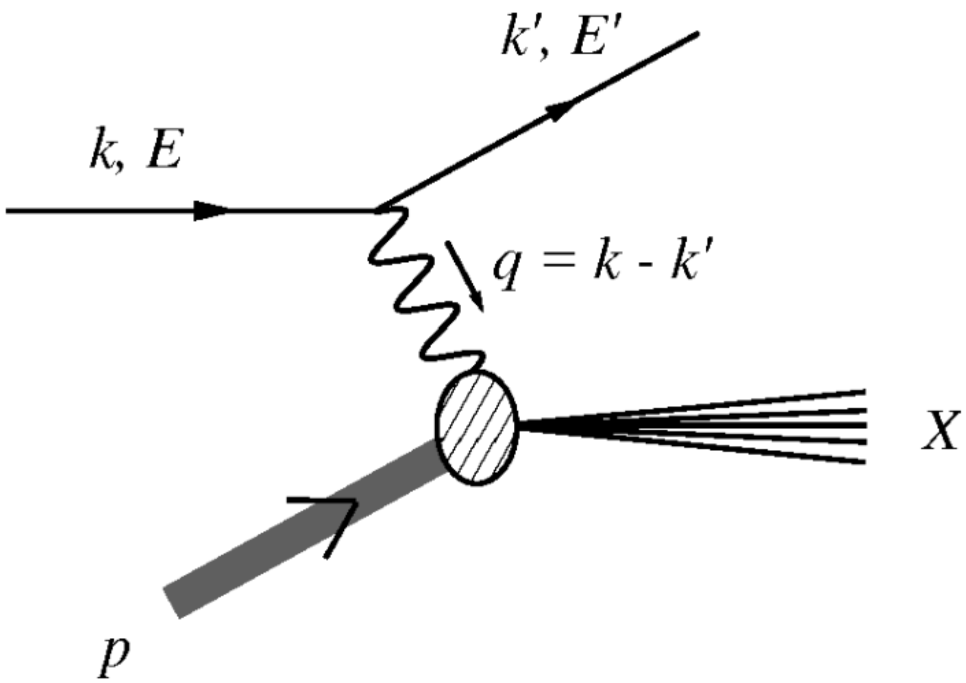




Larger e⁻ energies open a new era

In the late 60's SLAC can accelerate e⁻ up to 18 GeV
At these energies a large fraction of the scattering is inelastic

The mass of X (W) recoiling against the electron can be inferred from the energy and the deflection of the outgoing electron



The process can be described in terms of fundamental quantities:

$$Q^2 = -q^2 = 4EE'\sin^2\theta/2$$

$$\nu = E - E'$$

The invariant mass of the final hadronic state is:

$$W^2 = (p+q)^2$$

$$W^2 = M^2 + 2M\nu - Q^2$$

For the case of elastic scattering:

$$Q^2 = 2M\nu$$

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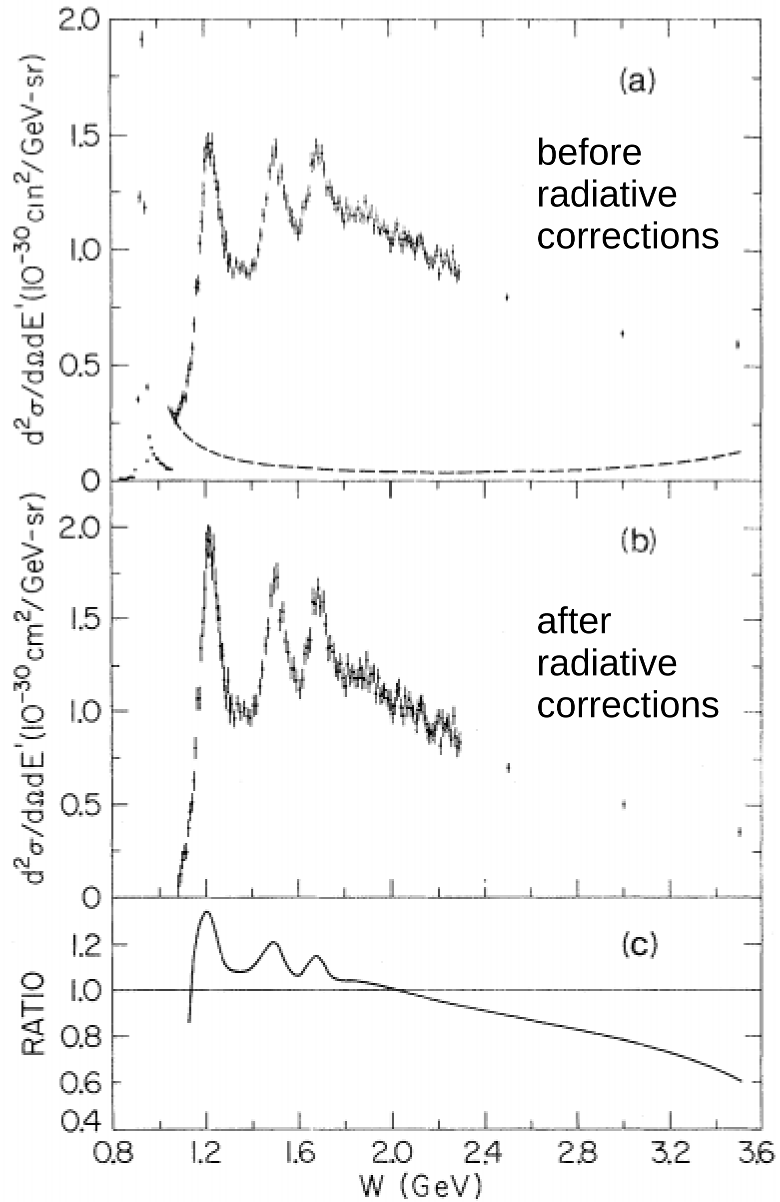
Nuclear resonances in e-p scattering

SLAC-MIT team (1969)

e^- with incident energies 7-17 GeV
Observed scattering angles 6°-10°

Covered range Q^2 up to 7.4 GeV^2

Data showed peaks in correspondence with the masses of the N (I=1/2, S=0) and Δ (I=3/2, S=0) baryons



Deep inelastic e-p scattering

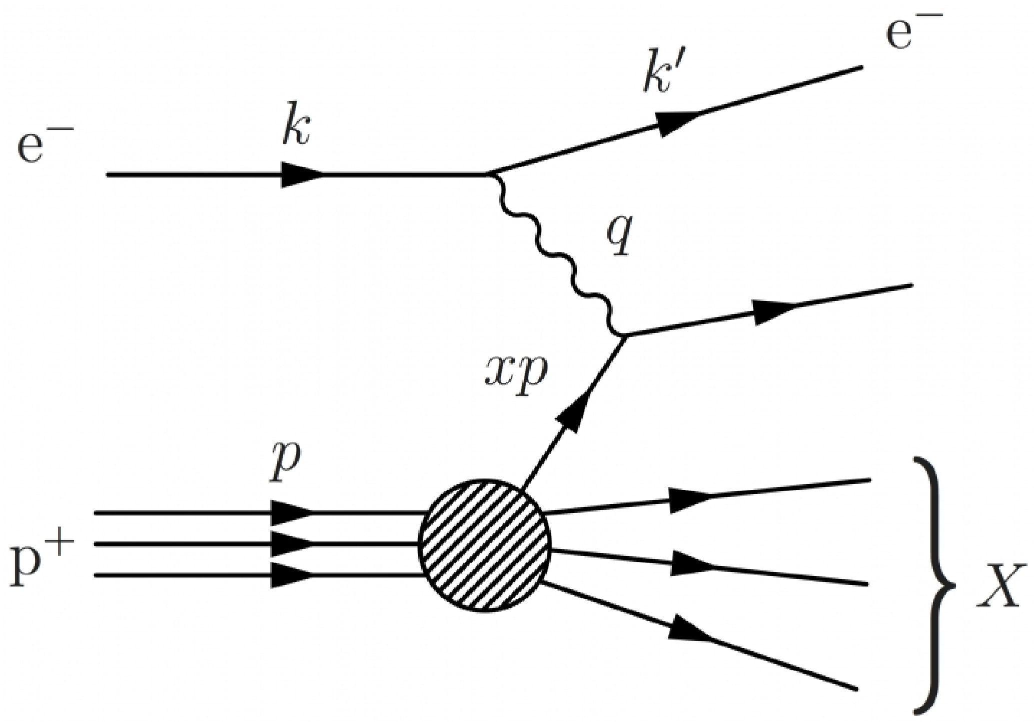
Full differential cross section extends Mott formula incorporating new effects in the structure functions $W_1(Q^2,\nu)$ and $W_2(Q^2,\nu)$

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2 \cos^2\theta/2}{4E^2 \sin^4\theta/2\, E}\left[W_2(Q^2,\nu) - 2W_1(Q^2,\nu)\tan^2\theta/2\right]$$

Hypothesis: scattering on pointlike constituents of the proton, carrying a fraction x of the proton momentum

$$(xp+q)^2 = m^2$$

$$x^2 p^2 + q^2 + 2xp\cdot q = m^2$$



At q^2 larger than the mass scale M^2:

$$|x^2 p^2| = x^2 M^2 \ll q^2 \qquad m^2 \ll q^2$$

$$x = \frac{Q^2}{2M\nu} \qquad 0 < x < 1$$

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Scale invariance

Bjorken scale invariance (1967): in principle W_1 and W_2 depend on the values of Q^2 and ν. Bjorken showed that, under the hypothesis of elastic scattering on pointlike quarks, W_1 and νW_2 should depend only on the quantity $x=q^2/2M\nu$

Independently developed Feyman model: partons can be not only valence quarks and includes the possibility of quark-antiquark pairs. The distribution functions for the varoius quarks are: u(x), d(x), $\bar{u}$(x), …

The momenta should add to the total proton momentum:

$$\int dx\, x[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)+\ldots]=1$$

and the correct quantum number are obtained for:

$$\int dx[u(x)+\bar{u}(x)]=2$$
$$\int dx[d(x)+\bar{d}(x)]=1$$
$$\int dx[s(x)+\bar{s}(x)]=0$$

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SLAC-MIT experiment

SLAC-MIT group (1969) - Nobel prize 1990

Electron energy range 7-17 GeV
Spectrometers with heavy shielding

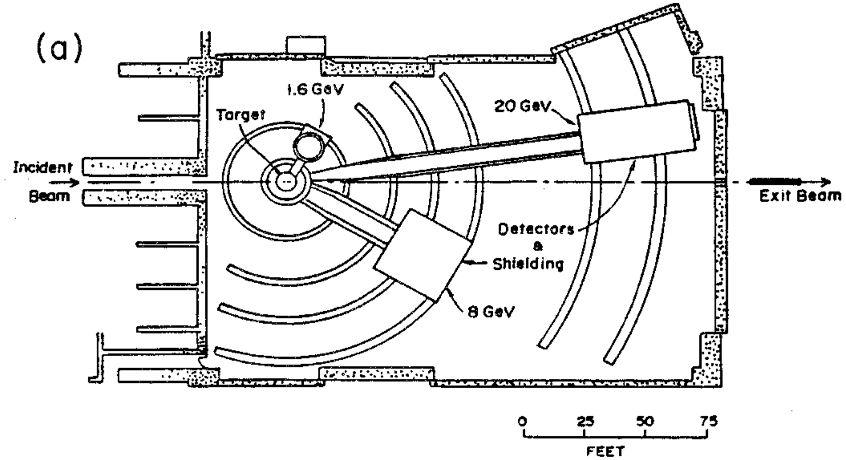
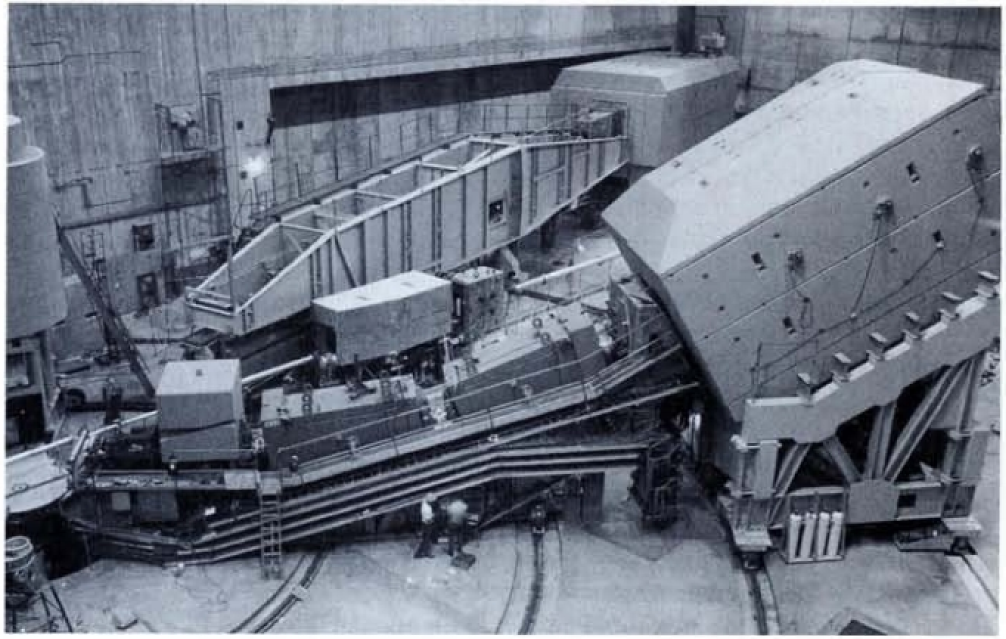
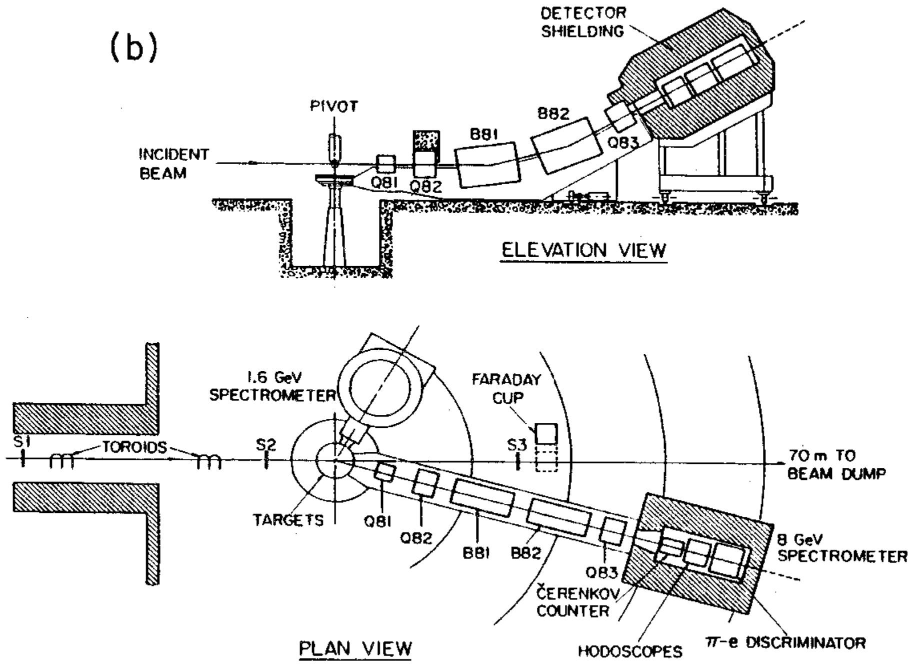


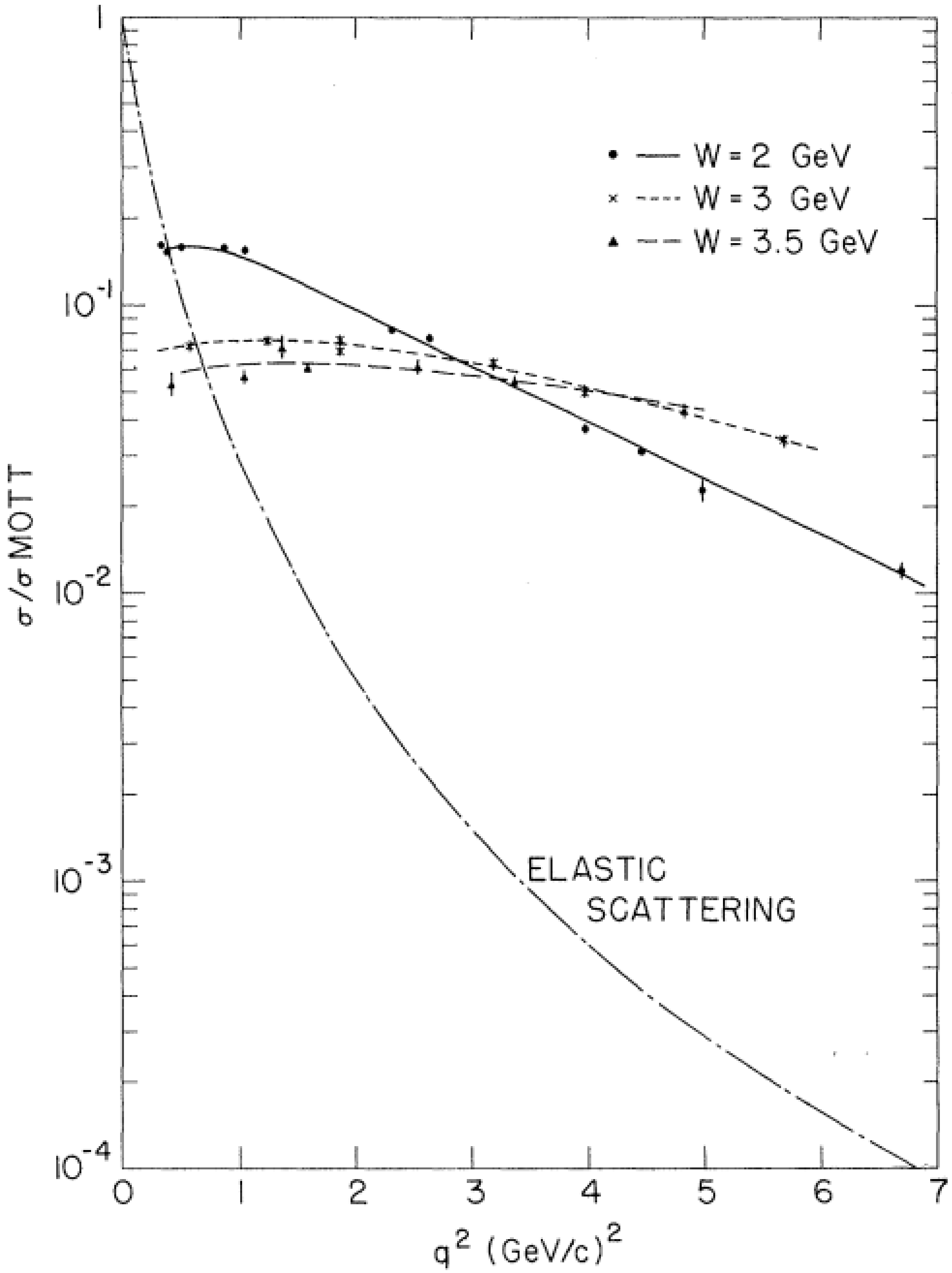
Fig. 1 — The two magnetic spectrometers used for the SLAC-MIT experiment. The 8 GeV spectrometer is in the foreground and the 20 GeV unit is to the rear. The bulk of the detectors comprise shielding (weighing 450 tons for the 8 GeV device).





SLAC-MIT experiment

Comparison of the observed cross sections for different values of the recoil mass W compared with elastic e-p scattering (computed for 10°)

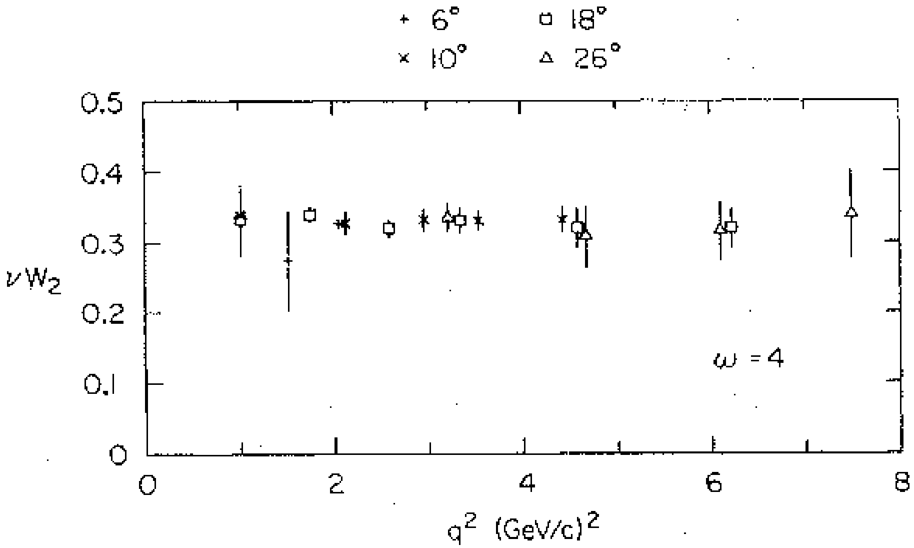


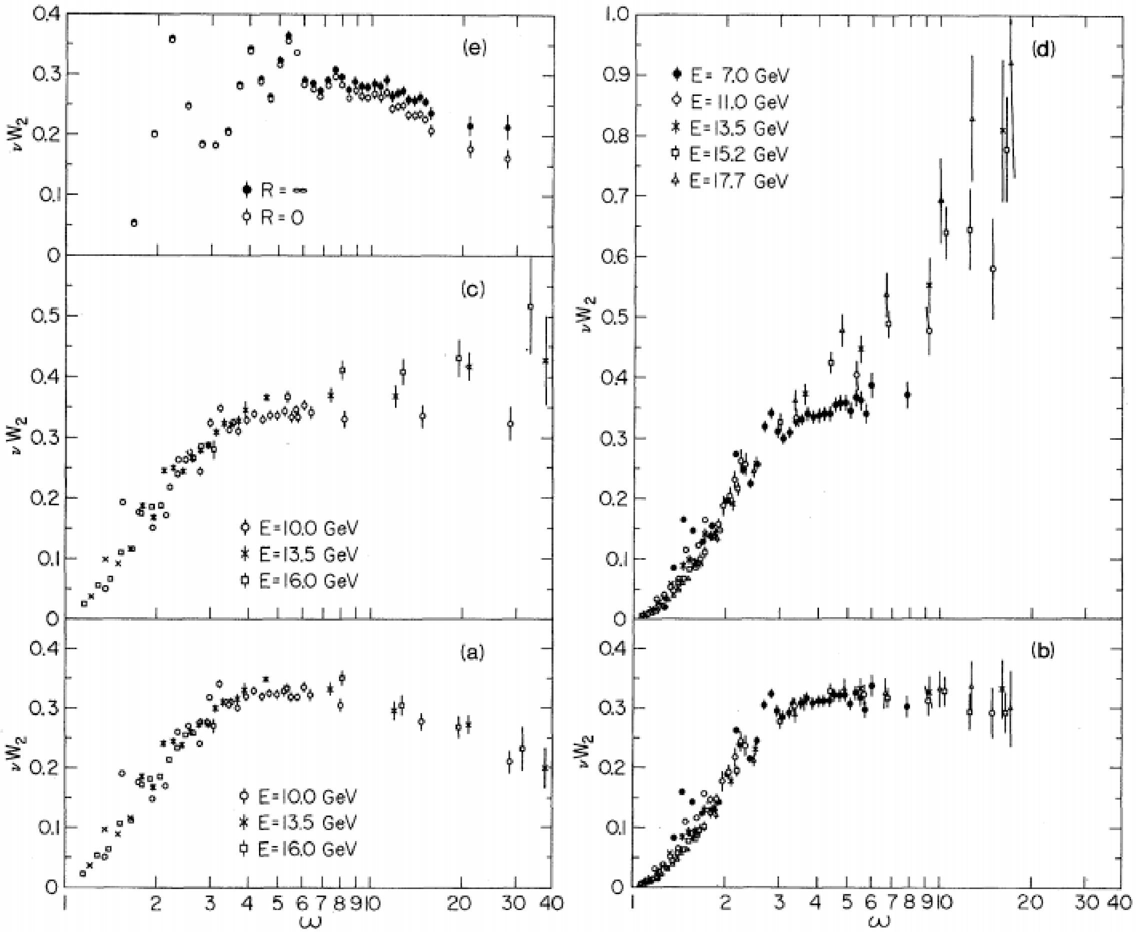
SLAC-MIT experiment

Observed behaviors consistent with Bjorken's expectations for elastic scattering on partons

νW_2 dependent on x ($\omega = x^{-1}$)

νW_2 independent on q^2





The Callan-Gross relation

The connection between the structure functions and the parton distributions can be obtained expressing the cross sections in terms of Lorentz invariant variables: $s=2ME$, $x=q^2/2M\nu$, $y=\nu/E$

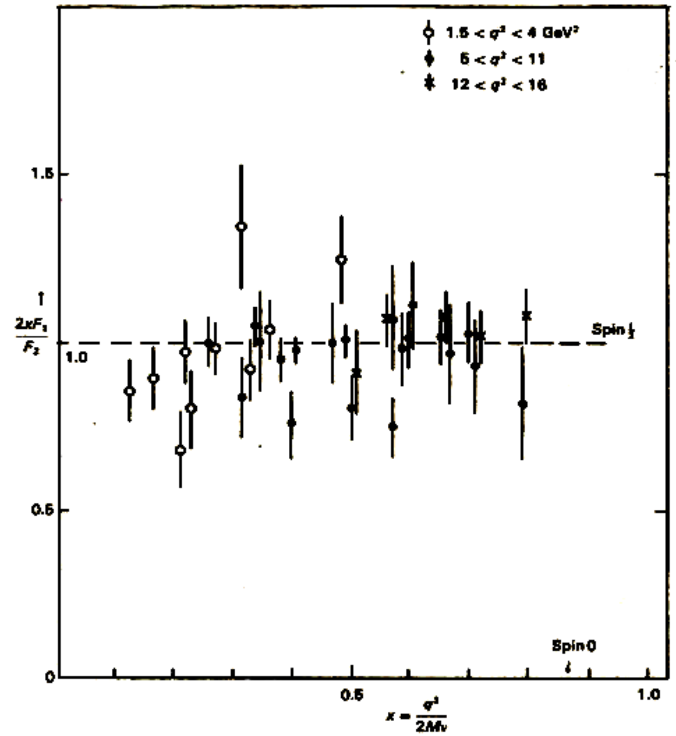
$$\frac{d\sigma}{dx\,dy}=\frac{4\pi\alpha^2 s}{Q^4}\left[\frac{1}{2}\left(1+(1-y)^2\right)2xF_1+(1-y)\left(F_2-2xF_1\right)-\frac{M}{2E}xy\,F_2\right]$$

Where $F_1(x,Q^2)=MW_1$ and $F_2(x,Q^2)=\nu W_2$ (should not be confused with the previous form factors of the elastic scattering)

The point-like expression for spin ½ particles is obtained for

$$\frac{2xF_1(x)}{F_2(x)}=1$$

Callan-Gross relation



Structure functions and parton distributions

In the parton model the nucleon in the e-N interaction can be considered as a the result of partonic currents, such as:

$$J_{u,\mu} = -\frac{2i}{3}\bar{u}(x)\gamma_\mu u(x)$$

$$J_{d,\mu} = \frac{i}{3}\bar{d}(x)\gamma_\mu d(x)$$

Assuming only u,d,s quarks in the proton this corresponds to:

$$F_1 = \frac{1}{2}\left[\left(\frac{2}{3}\right)^2 (u(x)+\bar{u}(x)) + 2\left(\frac{1}{3}\right)^2 (d(x)+\bar{d}(x)) + \left(\frac{1}{3}\right)^2 (s(x)+\bar{s}(x))\right]$$

$$F_2 = x\left[\left(\frac{2}{3}\right)^2 (u(x)+\bar{u}(x)) + 2\left(\frac{1}{3}\right)^2 (d(x)+\bar{d}(x)) + \left(\frac{1}{3}\right)^2 (s(x)+\bar{s}(x))\right]$$

The same formulas apply for the neutron, but with different parton distributions, allowing to derive relations between the e-p and e-n F_1 and F_2 structure functions

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Inelastic ν-N scattering

The parton model also allows to write the ν-N cross sections
The structure functions should be rewritten ($F \rightarrow F^{\nu}$) following V-A theory, resulting in an additional structure function F^{ν}_3 and unique coupling G_F (neutrinos not sensitive to electric charge)

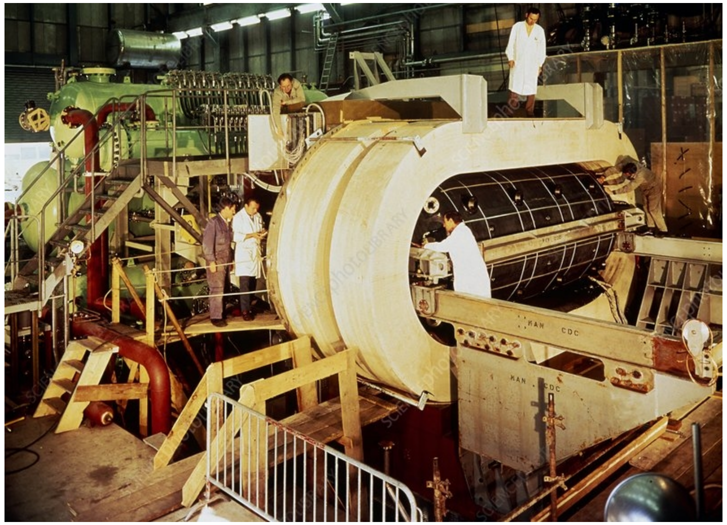
$$\frac{d\sigma^{\nu}}{dx\,dy} = \frac{G_F^2 M E}{\pi}\left[(1-y)F_2^{\nu} + y^2 x F_1^{\nu} + (y - y^2/2)\,x F_3^{\nu}\right]$$
$$\frac{d\sigma^{\bar{\nu}}}{dx\,dy} = \frac{G_F^2 M E}{\pi}\left[(1-y)F_2^{\nu} + y^2 x F_1^{\bar{\nu}} - (y - y^2/2)\,x F_3^{\bar{\nu}}\right]$$

For the interactions $\nu_\mu\, p \rightarrow \mu^- X$ and $\bar{\nu}_\mu\, p \rightarrow \mu^+ \bar{X}$, ν can interact with d or $\bar{u}$, while $\bar{\nu}$ can interact with $\bar{d}$ or u, so the cross sections can be written as:

$$\frac{d\sigma^{\nu}}{dx\,dy} = \frac{2 G_F^2 M E}{\pi}\, x\left[d(x) + (1-y)^2 \bar{u}(x)\right]$$
$$\frac{d\sigma^{\bar{\nu}}}{dx\,dy} = \frac{2 G_F^2 M E}{\pi}\, x\left[\bar{d}(x) + (1-y)^2 u(x)\right]$$

Gargamelle at CERN

ν and $\bar{\nu}$ cross sections measured using the Gargamelle liquid bubble chamber, filled with heavy freon (CF_3Br) exposed to ν and $\bar{\nu}$ beams produced at the CERN PS.



CERN PS neutrino beamline

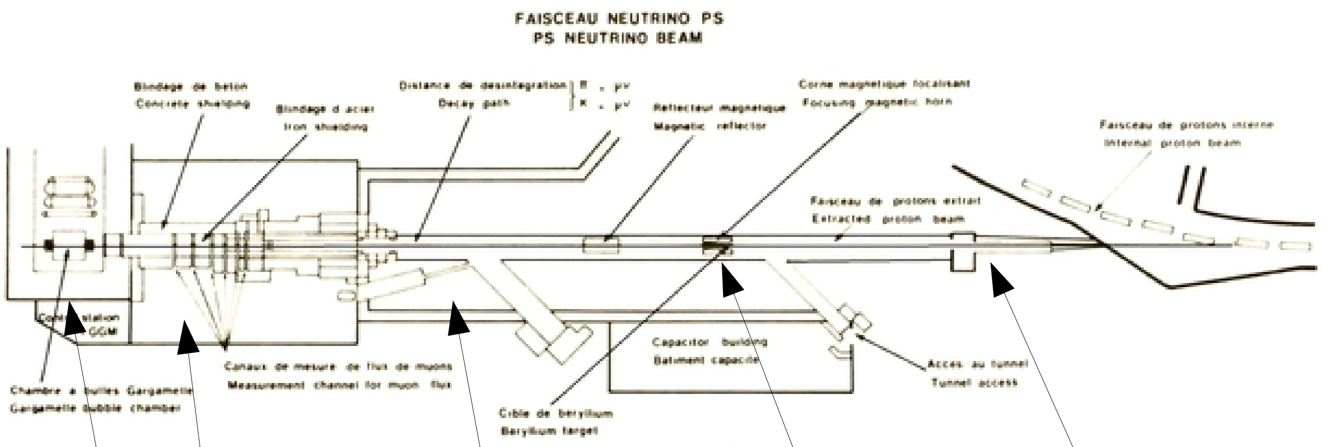


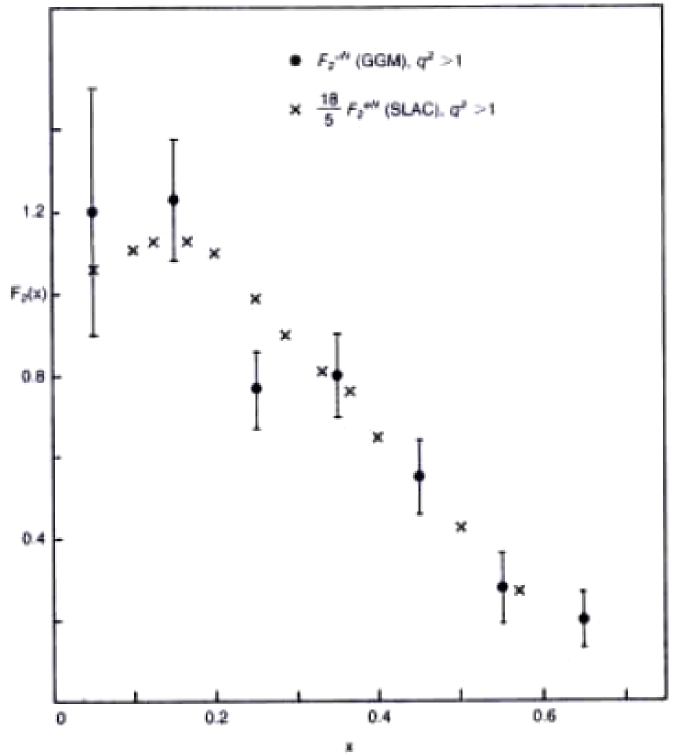
Fig. 1-2. The CERN neutrino beam lay-out.

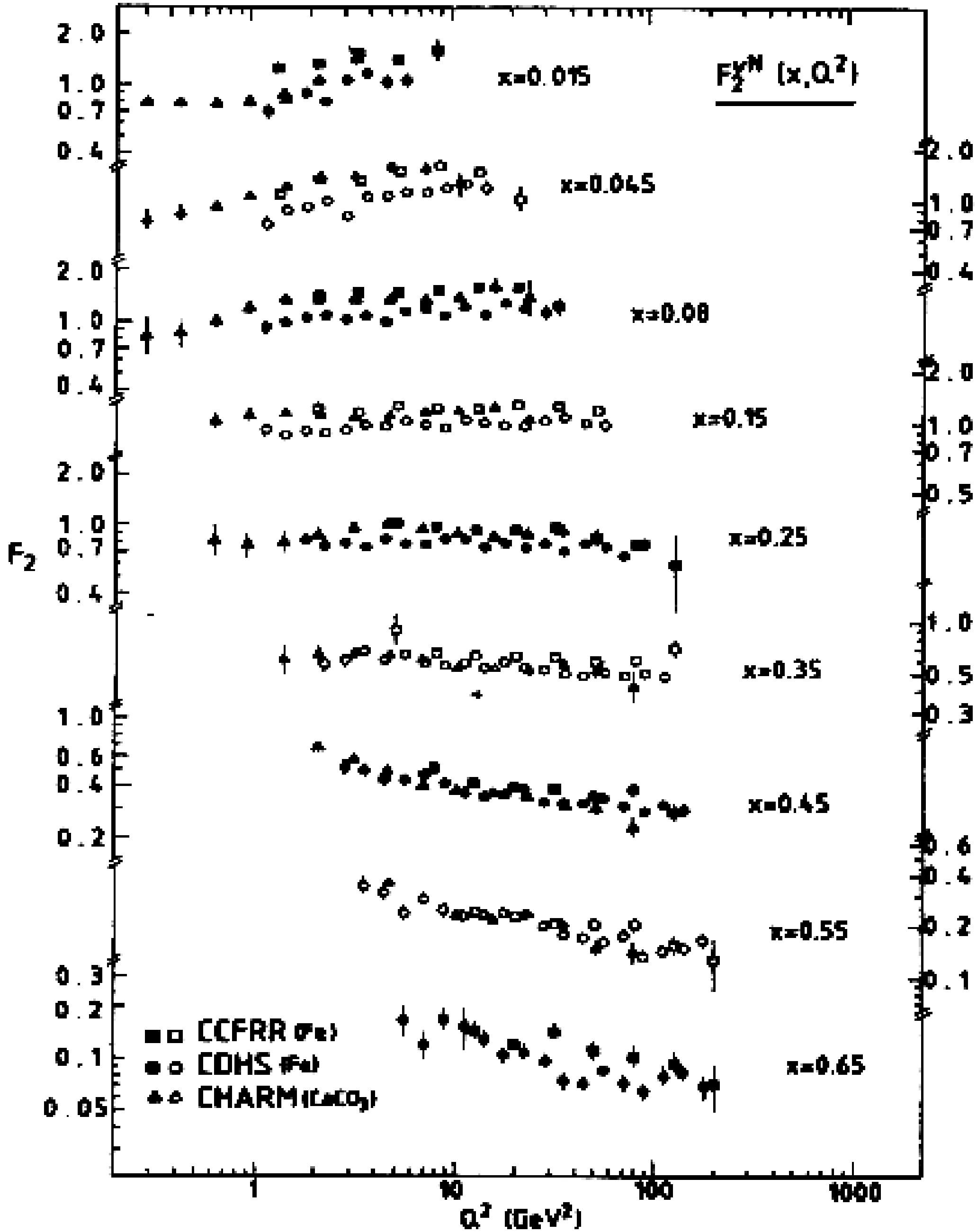
Inelastic ν-N scattering

Fν are also functions of x

Data confirm that e and ν see the same internal structure of nucleons

$$F_2^{\nu N}(x) \leq \frac{18}{5} F_2^{eN}(x)$$





Proton PDF

For each parton in the proton there is one Parton Distribution Function (PDF): $u(x,Q^2)$, $d(x,Q^2)$, $g(x,Q^2)$, …

Interpretation: probability to finding a parton of a given flavor that carries a fraction x of the total proton's momentum

The momentum sum rule (pag. 10) should be modified to include the gluon:

$$\int dx\, x[q(x)+\bar{q}(x)+g(x)]=1$$