Resonances

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πp scattering

Anderson, Fermi, Long, Nagle (1952)

Striking difference between π^+p and π^-p observed at the Chicago Cyclotron:

- π^-p cross section: rising from few mb to ~60 mb at E_π=180 MeV
- π^+p cross section: about 3 times the π^-p cross section

In a series of paper the 3 scattering process were investigated:

A) $\pi^+p \rightarrow \pi^+p$ (elastic scattering)

B) $\pi^-p \rightarrow \pi^0n$ (charge exchange)

C) $\pi^-p \rightarrow \pi^-p$ (elastic scattering)

A has the largest cross section,

C the smallest.

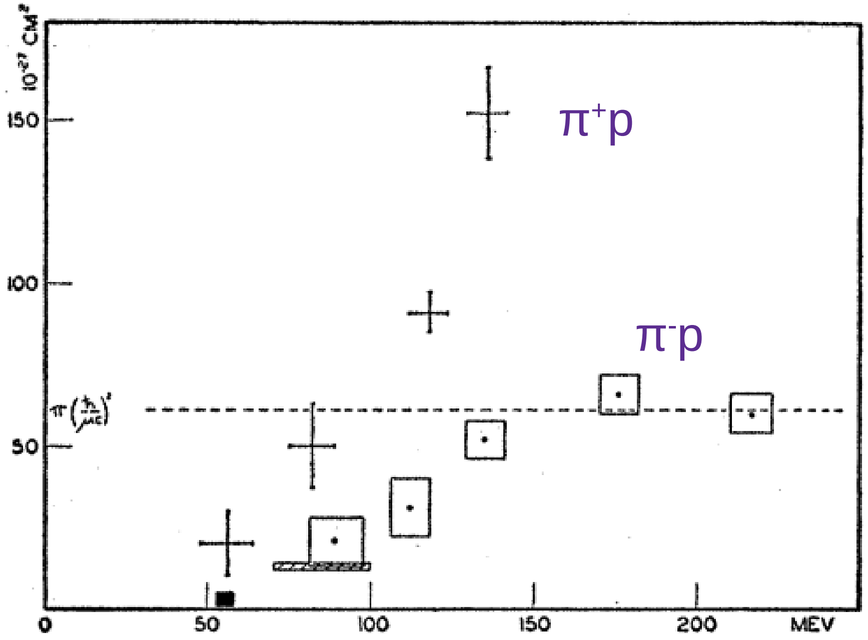


FIG. 1. Total cross sections of negative pions in hydrogen (sides of the rectangle represent the error) and positive pions in hydrogen (arms of the cross represent the error). The cross-hatched rectangle is the Columbia result. The black square is the Brookhaven result and does not include the charge exchange contribution.

πp scattering: isospin

From the point of view of the strong interaction, the previous processes can be considered as the same interaction between particles in different isospin states.

$|I,I_3\rangle = |1,+1\rangle = |\pi^+\rangle$ $|1,0\rangle = |\pi^0\rangle$ $|1,-1\rangle = |\pi^-\rangle$

$|I,I_3\rangle = |1/2,+1/2\rangle = |p\rangle$ $|1/2,-1/2\rangle = |n\rangle$

The πN system can have I=3/2, 1/2

Each state expressed as superposition of isospin states, obtained using the Clebsch-Gordan coeff.

For example, for the 3 states of interest:

$|p\pi^+\rangle = |3/2,+3/2\rangle$

$|p\pi^-\rangle = \sqrt{(1/3)}\,|3/2,-1/2\rangle - \sqrt{(2/3)}\,|1/2,-1/2\rangle$

$|p\pi^0\rangle = \sqrt{(2/3)}\,|3/2,+1/2\rangle - \sqrt{(1/3)}\,|1/2,+1/2\rangle$

πp scattering: isospin

The cross section is proportional to the square of the amplitude between the initial and final state:

$$\sigma \propto |\langle f|H|i\rangle|^2$$

Where H is an isospin conserving operator, that can proceed to bot isospin 3/2 and ½ channels, so it can be expressed as:

$$H = H_{1/2} + H_{3/2}$$

So for the 3 processes we obtain:

$<p\pi^+|H|p\pi^+> = <3/2,+3/2|H_{3/2}|3/2,+3/2> = M_{3/2}$

$<n\pi^0|H|p\pi^-> = \sqrt{2}/3 <3/2,-1/2|H_{3/2}|3/2,-1/2> + \sqrt{2}/3 <1/2,-1/2|H_{1/2}|1/2,-1/2>$

$= \sqrt{2}/3\ M_{3/2} + \sqrt{2}/3\ M_{1/2}$

$<p\pi^-|H|p\pi^-> = 1/3 <3/2,-1/2|H_{3/2}|3/2,-1/2> + 2/3 <1/2,-1/2|H_{1/2}|1/2,-1/2>$

$= 1/3\ M_{3/2} + 2/3\ M_{1/2}$

Therefore an isospin=3/2 resonance would give cross sections for the 3 processes in the ratio A:B:C=9:2:1, while isospin=1/2 gives A:B:C=0:1:2
Measurement in agreement with isospin 3/2 resonance (the full shape of the resonance could not be observed because of the pion beam energy)

A broad resonance: $\Delta(1232)^{++}$

Cross section values for πp scattering reveals the existence of a broad resonance with I=3/2.

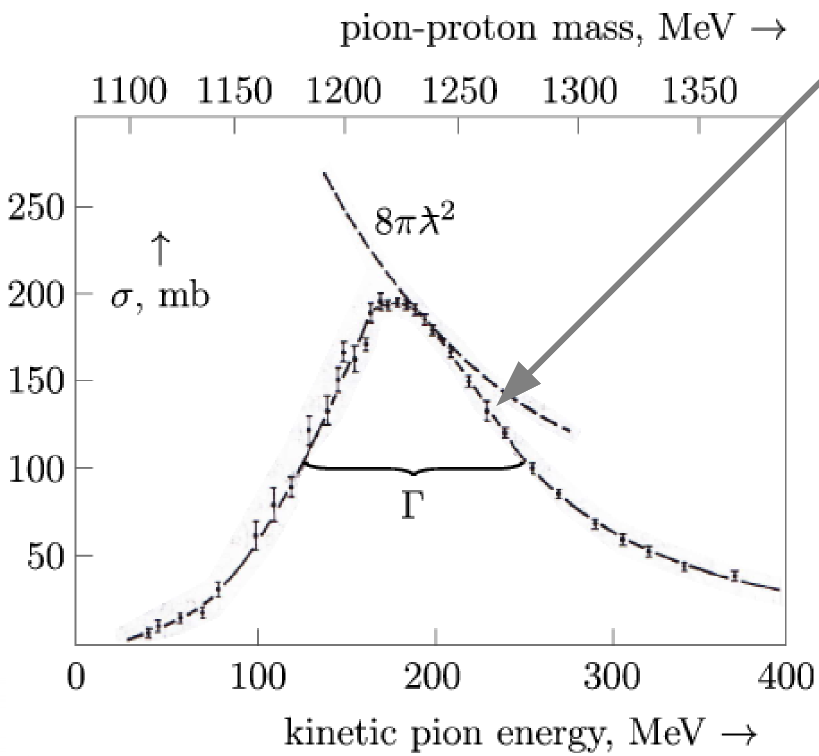
Spin assignment 3/2 from the analysis of the angular distribution:

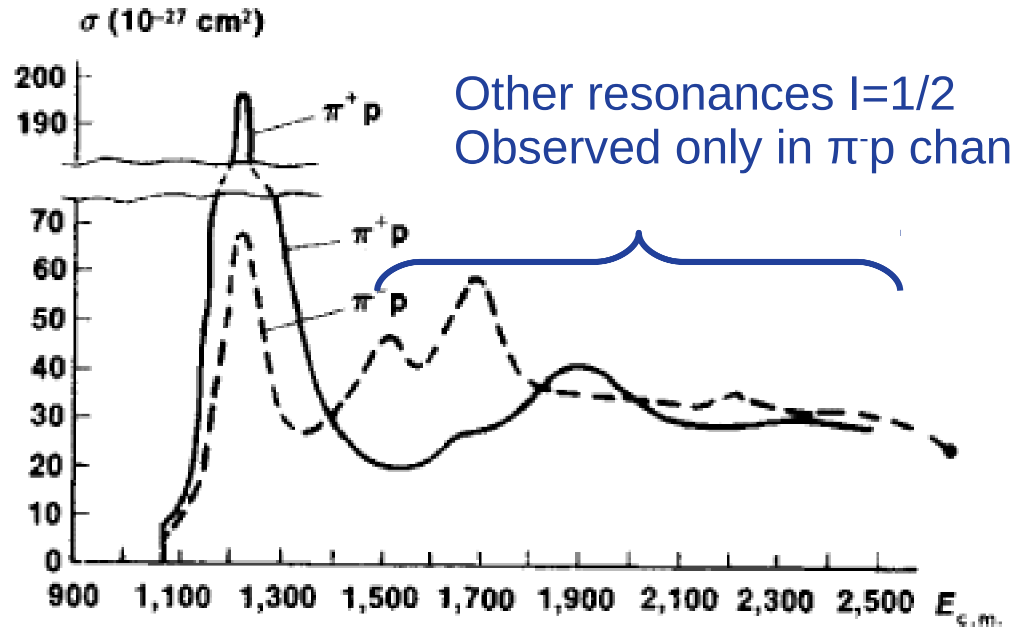
- s=1/2 → isotropic
- s=3/2 → $1+3\cos^2\theta$

$\Gamma \sim 110$ MeV → $\tau \sim 10^{-24}$ s

Non-relativistic Breit-Wigner for particles with spin: a+b→R

$$\sigma_{el}(E)=\frac{4\pi}{k^2}\frac{(2J+1)}{(2s_a+1)(2s_b+1)}\frac{\Gamma^2/4}{(E-E_{res})^2+\Gamma^2/4}$$





Other resonances I=1/2

Observed only in π⁻p channel

A family of resonances: Δ

We observed a peak in the $\pi^- p$ process with approximately the same mass and width of the Δ(1232)

Isospin symmetry implies for I=3/2 the existence of 4 particles in different I_3 state (with different electric charge)

$\Delta^{++} \rightarrow p\, \pi^+$ (I_3=+3/2)
$\Delta^{+} \rightarrow p\, \pi^0$ or $n\, \pi^+$ (I_3=+1/2)
$\Delta^{0} \rightarrow n\, \pi^0$ or $p\, \pi^-$ (I_3=-1/2)
$\Delta^{-} \rightarrow n\, \pi^-$ (I_3=-3/2)

Hadron scattering: wave approach

Consider a beam of particles with momentum *k* represented by a plane wave:

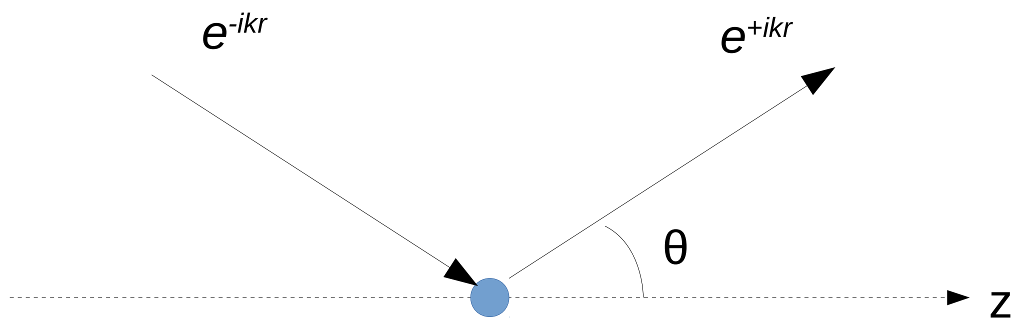
$$\psi_i = e^{ikz}$$

A plane wave can be represented by a superposition of incoming and outgoing spherical waves, whose angular dependence is given by the Legendre polynomials $P_l(\cos\theta)$.

At large distances from the scattering centre $kr \gg 1$ and the radial dependence of the spherical wave is $e^{\pm ikr}/kr$.

For elastic scattering $k_i = k_o = k$.

$$\psi_i = \frac{i}{2kr} \sum_l (2l+1)\left[(-1)^l e^{-ikr} - e^{ikr}\right] P_l(\cos\theta)$$



Hadron scattering: wave approach

Adding a scattering potential, affecting the amplitude ($0<\eta_l<1$) and the phase ($2\delta_l$) of the outgoing wave

$$\psi_{tot}=\frac{i}{2kr}\sum_l (2l+1)\left[(-1)^l e^{-ikr}-\eta_l e^{2i\delta_l}e^{ikr}\right]P_l(\cos\theta)$$

$$\psi_{scatt}=\psi_{tot}-\psi_i=\frac{e^{ikr}}{kr}\sum_l (2l+1)\frac{\eta_l e^{2i\delta_l}-1}{2i}P_l(\cos\theta)=\frac{e^{ikr}}{r}F(\theta)$$

$F(\theta)$ is the elastic scattering amplitude. The elastic cross section is:

$$\frac{d\sigma}{d\Omega}=|F(\theta)|^2$$

The total elastic cross section is obtained integrating over the angle:

$$\sigma_{el}=\frac{4\pi}{k^2}\sum_l (2l+1)\left|\frac{\eta_l e^{2i\delta_l}-1}{2i}\right|^2$$

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The optical theorem

The optical theorem relates the total cross section with the imaginary part of the forward scattering amplitude

The reaction cross section, for η_l<*1*, can be obtained as a difference between the total cross section and the elastic one and is:

$$\sigma_r = \frac{\pi}{k^2} \sum_l (2l+1) 2 (1 - \eta_l \cos 2\delta_l)$$

Since the imaginary part of the scattering amplitude in the forward direction ($\theta=0$) is:

$$\Im F(0) = \frac{1}{2k} \sum_l (2l+1)(1 - \eta_l \cos 2\delta_l)$$

The optical theorem follows:

$$\boxed{\Im F(0) = \frac{k}{4\pi} \sigma_{tot}}$$

Hadron scattering: wave approach

The maximum elastic cross section for the l[th] partial wave occurs when $\delta_l = \pi/2$ and $\eta_l = 1$ (pure scattering without absorption) and is:

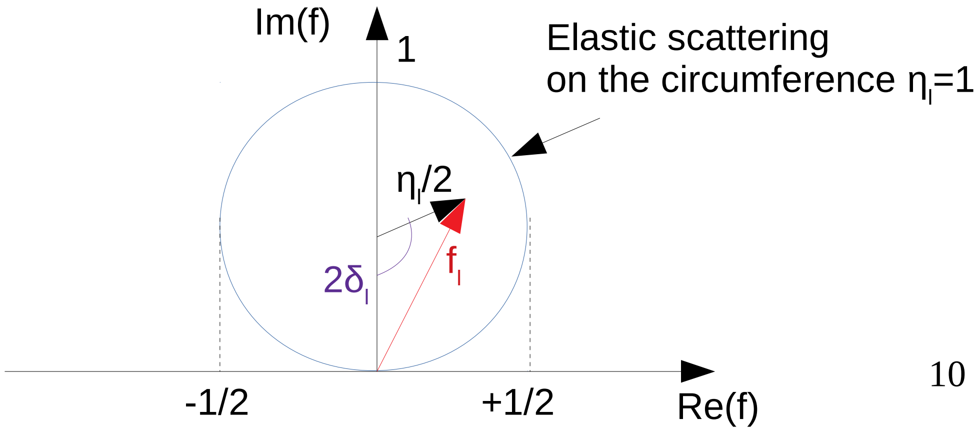
$$\sigma_{el}^{max} = \frac{4\pi}{k^2}(2l+1)$$

Similarly, the maximum reaction cross section is for $\eta_l = 0$:

$$\sigma_{r}^{max} = \frac{\pi}{k^2}(2l+1)$$

The elastic scattering amplitude for the l[th] partial wave *f(l)* is a complex quantity that is included in a circle centered in *i/2* in the complex plane

$$f(l) = \frac{\eta_l e^{2i\delta_l} - 1}{2i} = \frac{i}{2} - \frac{i\,\eta_l}{2} e^{2i\delta_l}$$



The Breit-Wigner formula

If the elastic scattering amplitude *f(l)* is at maximum for some given momentum k and value of l, we have the formation of a resonance, that is therefore characterized by a mass and angular momentum (spin)

In this case the phase shift δ_l should pass through $\pi/2$.

For $\eta=1$ we can write:

$$f(l)=\frac{e^{i\delta_l}\left(e^{i\delta_l}-e^{-i\delta_l}\right)}{2i}=e^{i\delta_l}\sin\delta_l=\frac{1}{\cot\delta_l-i}$$

Taylor expansion around $\delta=\pi/2$ ($\cot\delta=0$), making the dependence on of the center of mass energy E explicit:

$$\cot\delta(E)=\cot\delta(E_{res})+(E-E_{res})\left[\frac{d\cot E}{dE}\right]_{E=E_{res}}+\ldots\simeq-(E-E_{res})\frac{2}{\Gamma}$$

Neglecting further terms justified if $\Gamma \ll E_{res}$

The cross section can be described in terms of width (Γ) or life-time (τ) of the resonance

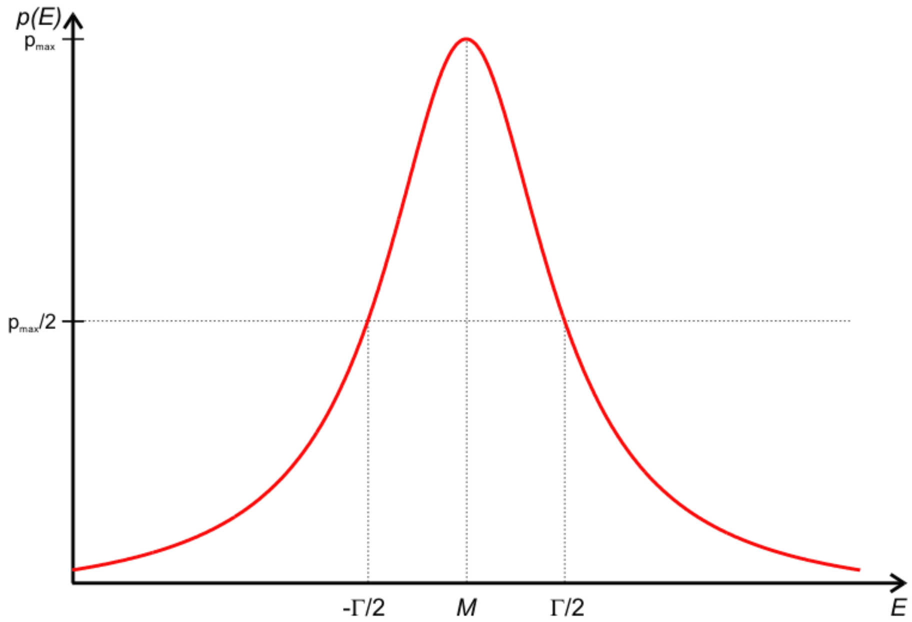
The Breit-Wigner formula

We can now obtain the elastic cross section close to the resonance as a function of E:

$$f(E)=\frac{1}{\cot\delta - i}=\frac{\Gamma/2}{(E_{res}-E)-i\Gamma/2}$$

$$\sigma_{el}(E)=\frac{4\pi}{k^2}(2l+1)\frac{\Gamma^2/4}{(E-E_{res})^2+\Gamma^2/4}$$

Breit-Wigner formula for spinless particles



The amplitude is just the Fourier transform of the exponential decay.

The cross section can be described in terms of width (Γ) or life-time of the resonance (τ=ħ/Γ)

Parity

Parity operator P → inversion of the spatial coordinates: $P\,\psi(x) \rightarrow \psi(-x)$

$P\,\psi(x) = +\,\psi(x)$ Even parity (P=+1)

$P\,\psi(x) = -\,\psi(x)$ Odd parity (P=-1)

$P\,\psi(x) \neq \pm\,\psi(x)$ No definite parity eigenvalue

For the spherical harmonic functions (angular momentum): $P=(-1)^l$

$$P\,Y_l^m(\theta,\phi) = Y_l^m(\pi-\theta,\pi+\phi) = (-1)^l\,Y_l^m(\theta,\phi)$$

Particles have an intrinsic parity +1 or -1

- Fermions: particles and antiparticles have opposite parity
- Bosons: particles and antiparticles have the same parity

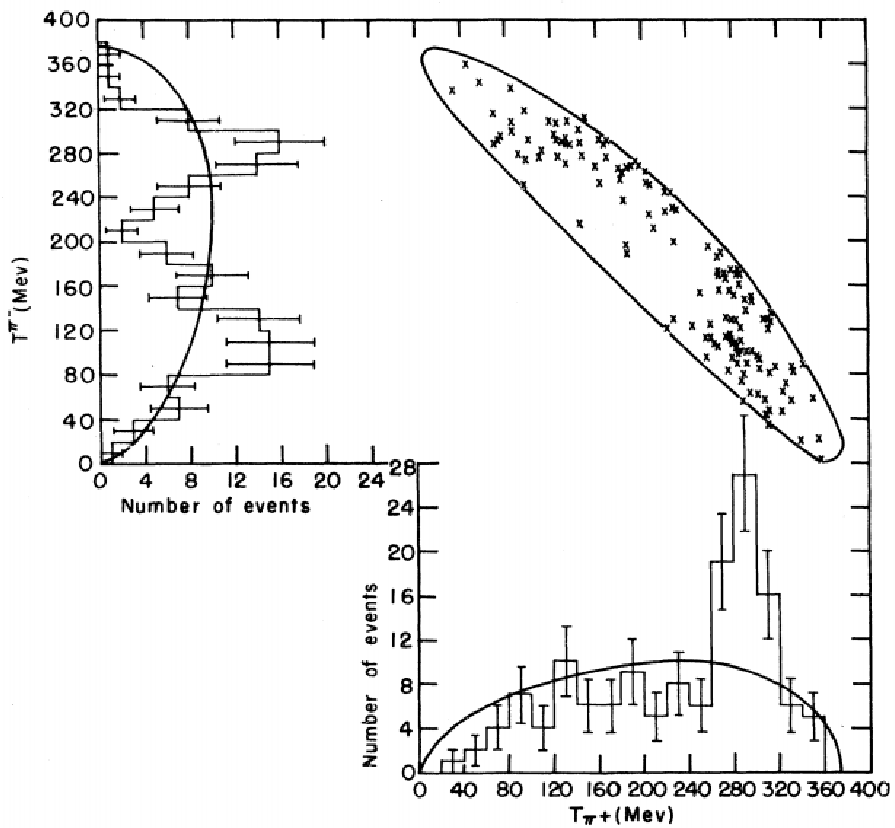
Parity is a multiplicative quantum number (angular and intrinsic parts)
It is conserved by the strong and electromagnetic interactions and this allows to determine the relative parity of particles

$\Lambda\pi$ resonances

Alvarez et al. (1960)
The availability of K beams (Bevatron) allowed a second step forward in the study of barionic resonances.
It was understood that Λ can be produced with π beams in association with other strange particles (K).

Observation of the process
$K^- p \rightarrow \Lambda\pi^+\pi^-$
in hydrogen bubble chamber

Dalitz plot analysis show center of mass kinetic energy of a particle (equivalent to the invariant mass of the two recoiling particles)
Evidence of resonant $\Lambda\pi$ state
$\Sigma(1385)$ with I=1 (charge triplet)



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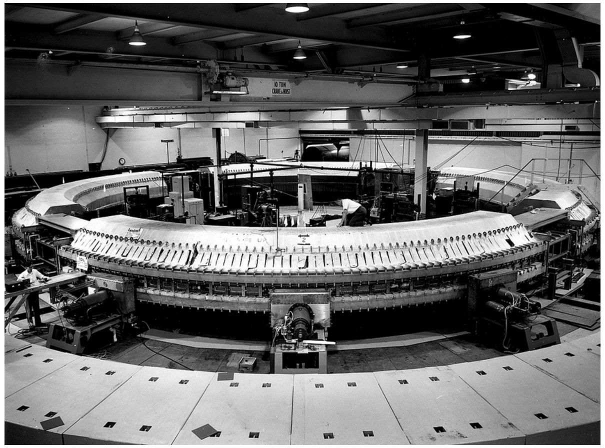
Two π resonances: ρ mesons

Erwin et al. (1961)
Evidence of a $\rho \rightarrow \pi\pi$ resonance using π- beam at Cosmotron (Brookhaven) on hydrogen bubble chamber

Analysis of different final state suggest I=1 resonance (triplet)
(Analysis done for two ranges of Δ, momentum transferred to the nucleon)
J=1: consistent with peak cross section of the Breit Wigner



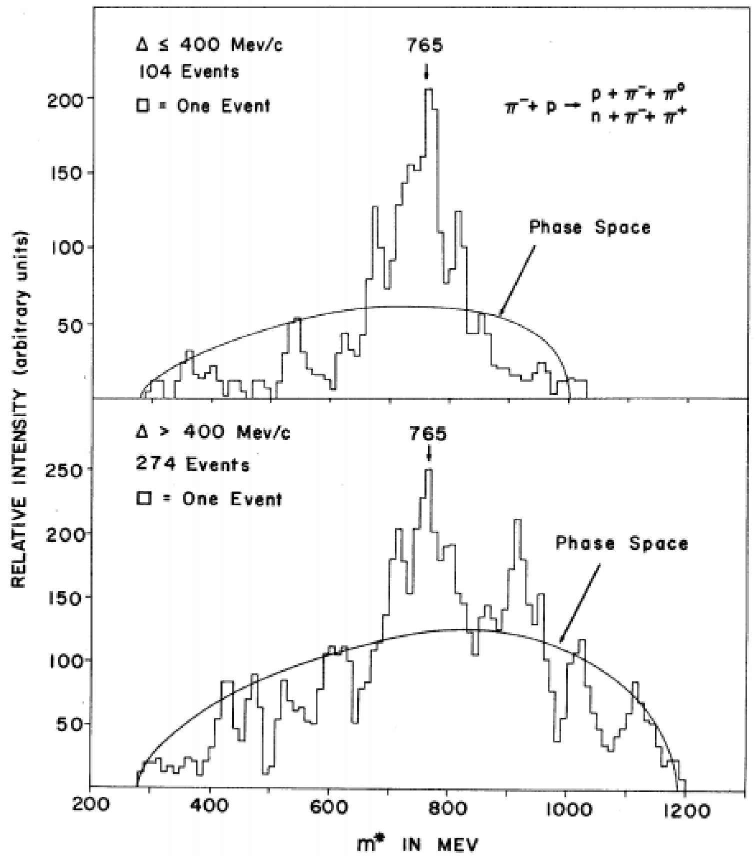


Table I. Ratios of final states.

	$I=0$	$I=1$	$I=2$	Experiment ($\Delta \leq 400$ Mev/c)
$\pi^-\pi^+n$	2	2	2/9	1.7 ±0.3
$\pi^-\pi^0p$	0	1	1	1
$\pi^0\pi^0n$	1	0	4/9	<0.25±0.25

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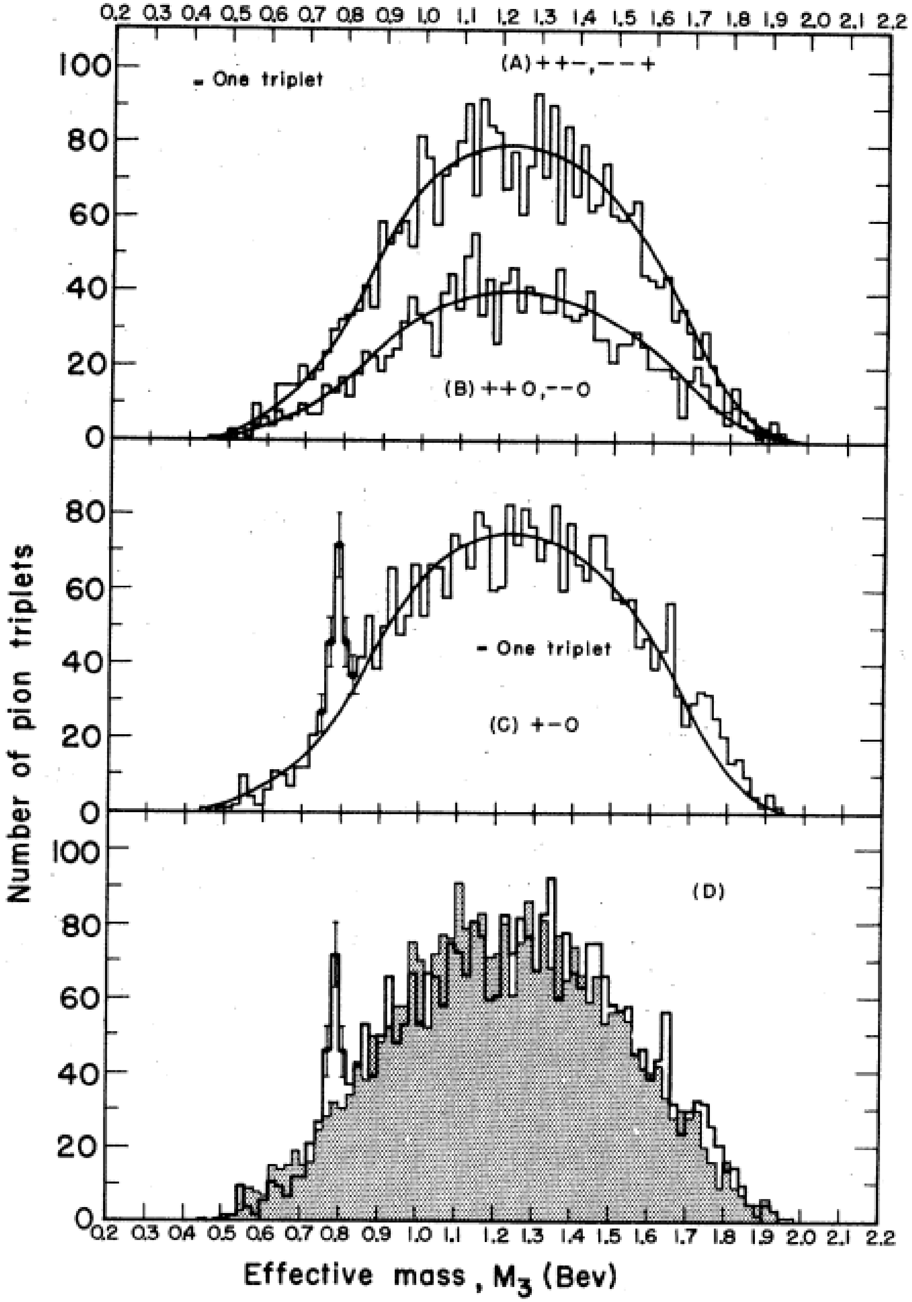
Three π resonance: ω meson

Alvarez group (1961)
Antiproton beam on hydrogen bubble chamber.
Study of: $\bar{p}\, p \rightarrow \pi^+ \pi^+ \pi^- \pi^- \pi^0$

Computing the invariant mass of 3π system in different combinations

Observation of a peak only in the channel $\omega \rightarrow \pi^+ \pi^- \pi^0$
due to I=0, $J^P=1^-$ resonance with mass 780 MeV



The static quark model of hadrons

In the accumulation of data on baryon and meson resonances regularities and patterns started to emerge.

Successful model by Gell-Mann and Ne'eman ("the eightfold way").
Extension of isospin SU(2) → SU(3) flavor symmetry
Lead to the development of the quark model: three fundamental constituents (quarks) u,d,s with s=1/2 and fractional electric charge.

u,d are I=1/2 doublet and S=0, s have I=0 and S=-1.

- Baryons are made of 3 quarks
- Mesons are made of 1 quark and 1 anti-quark

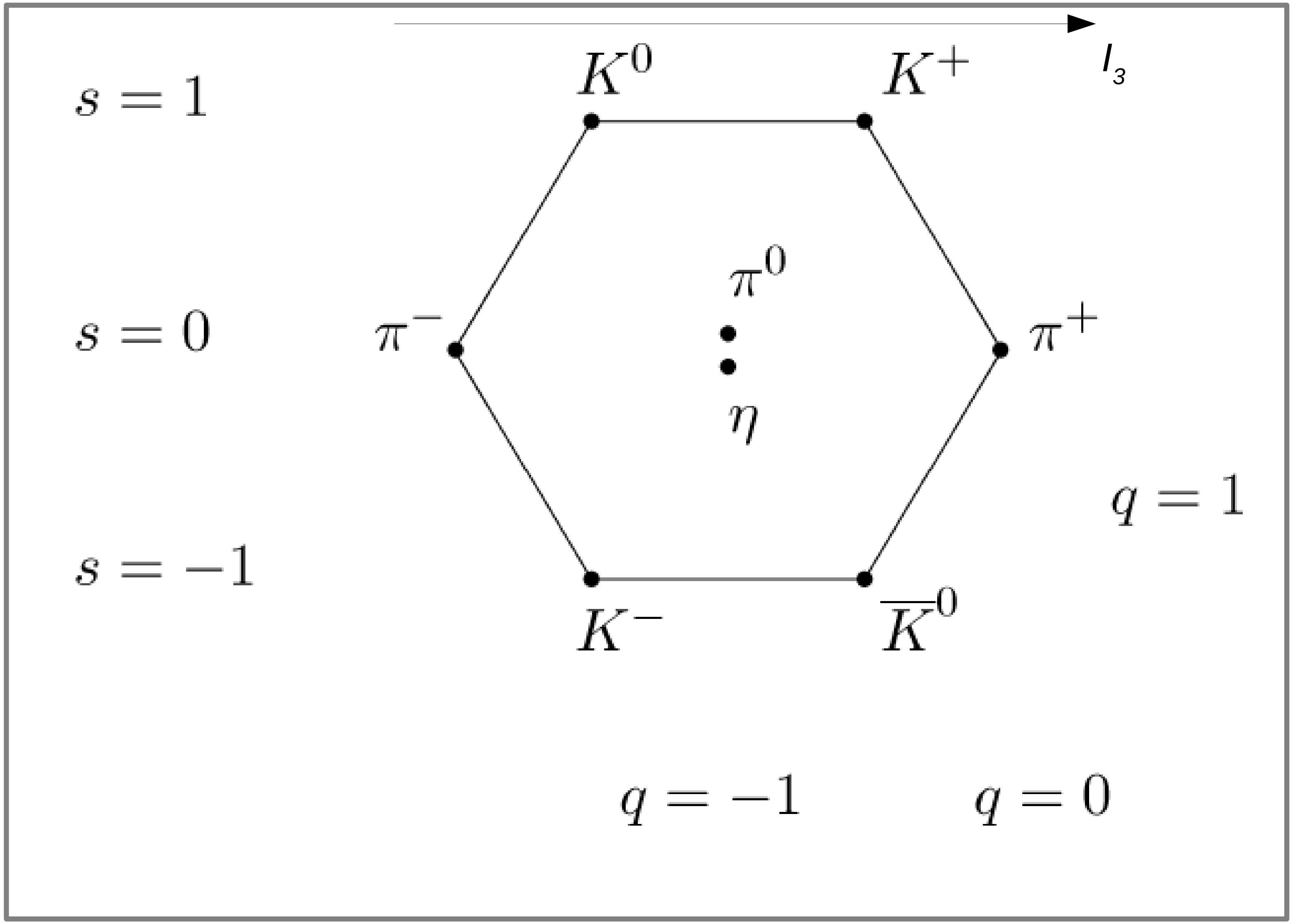
Under the SU(3) hypothesis we have the following multiplets:

- Baryons: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$ (1 singlet, 2 octets, 1 decuplet)
- Mesons: $3 \otimes \bar{3} = 1 \oplus 8$ (1 singlet, 1 octet)

Spin-parity must be determined to assign resonances to the appropriate multiplet

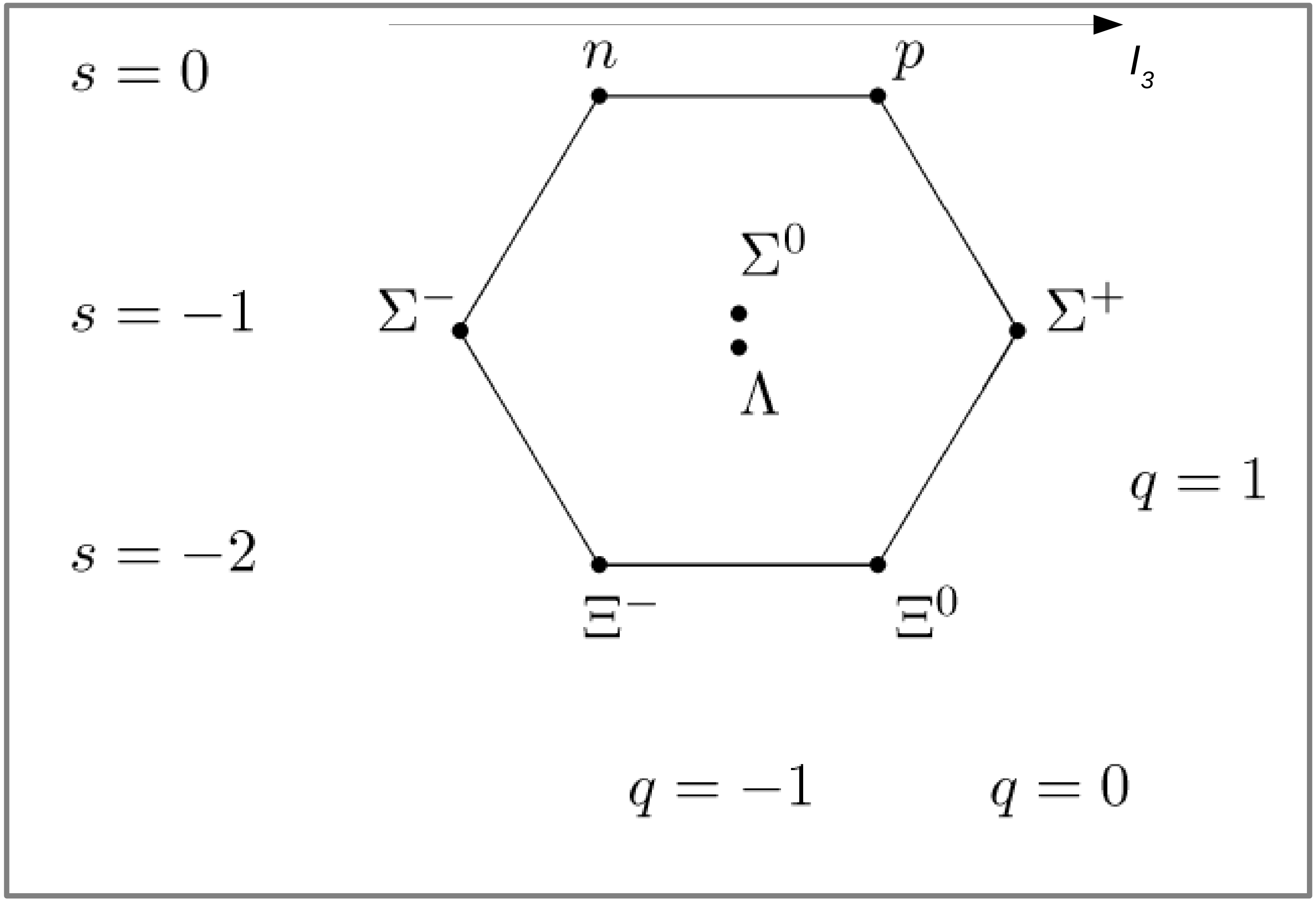
The meson octet

The known $J^P=0^-$ (pseudoscalar) meson resonances form an octet



The baryon octet

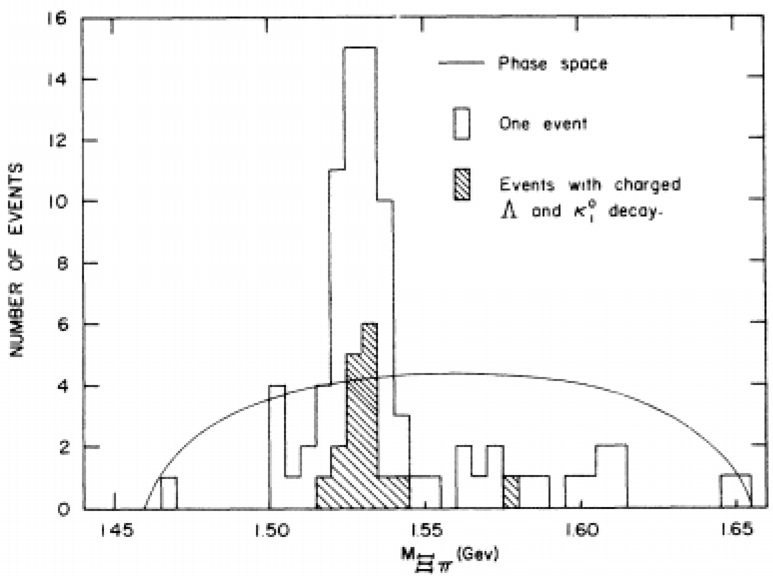
The known 8 baryon resonances with $J^P=1/2^+$ form an octet
The Δ quadruplet has $J^P=3/2^+$ cannot be part of this multiplet

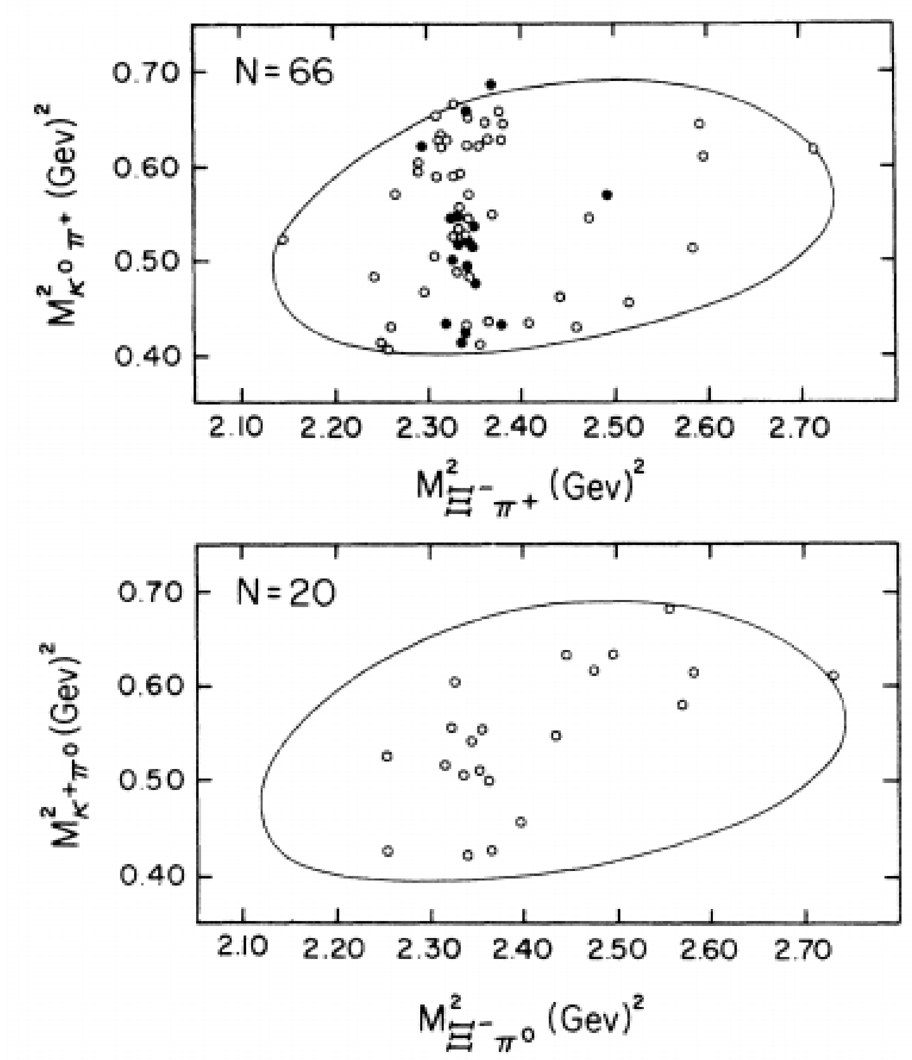


Other baryons: Σ^* and Ξ^*

Other resonances with spin 3/2 and S≠0 observed with K beams in hydrogen bubble chamber: $\Sigma^* \rightarrow S=-1$ and $\Xi^* \rightarrow S=-2$

Pjerrou et al. (UCLA group, 1962)
$\Xi^* \rightarrow \Xi\pi$ resonance, m=1.53 GeV
S=-2 conserved in strong decay
I=1/2 from cross sections ratios of different isospin states
Ξ^* spin later determined to be 3/2

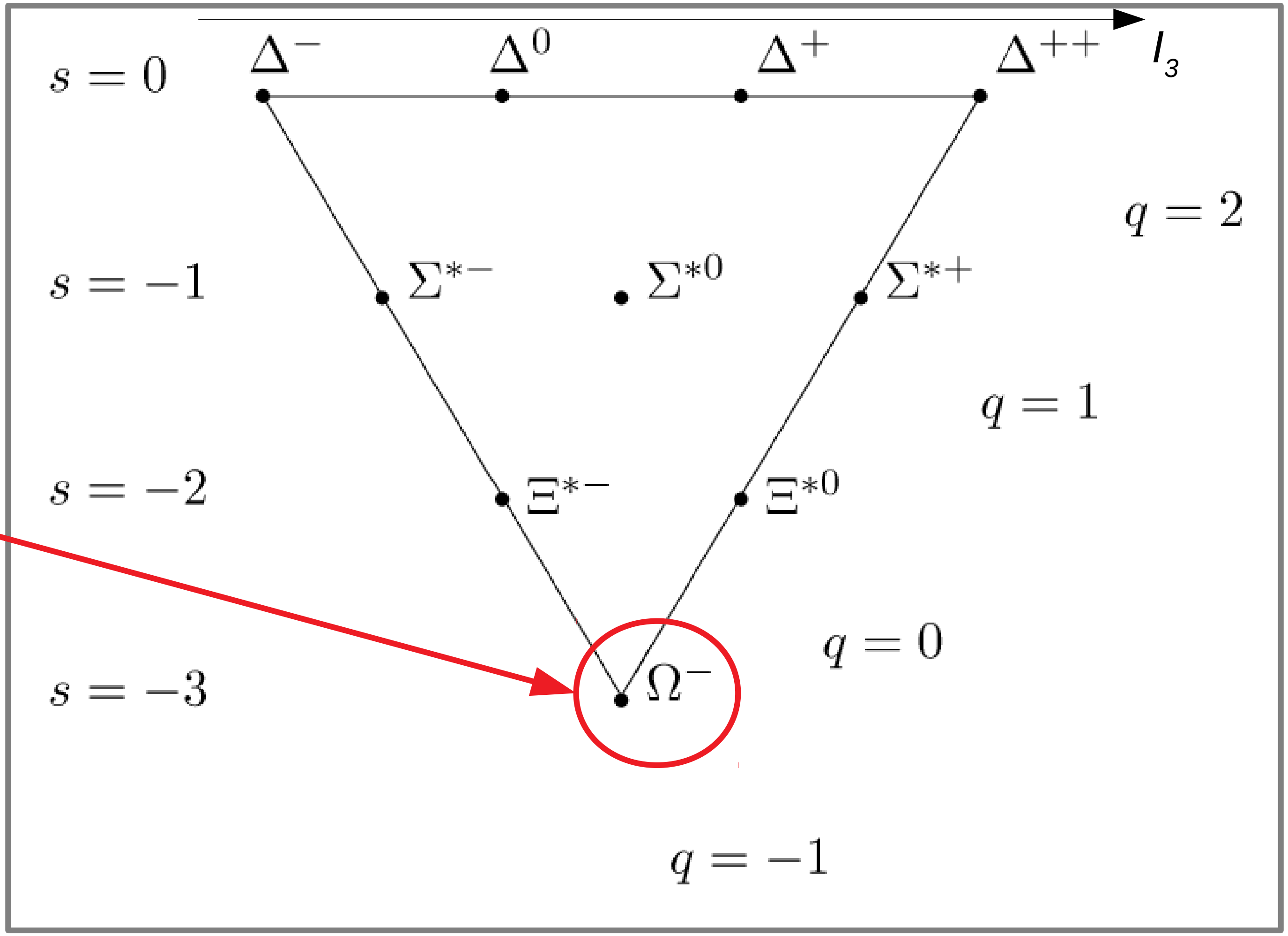




The baryon decuplet

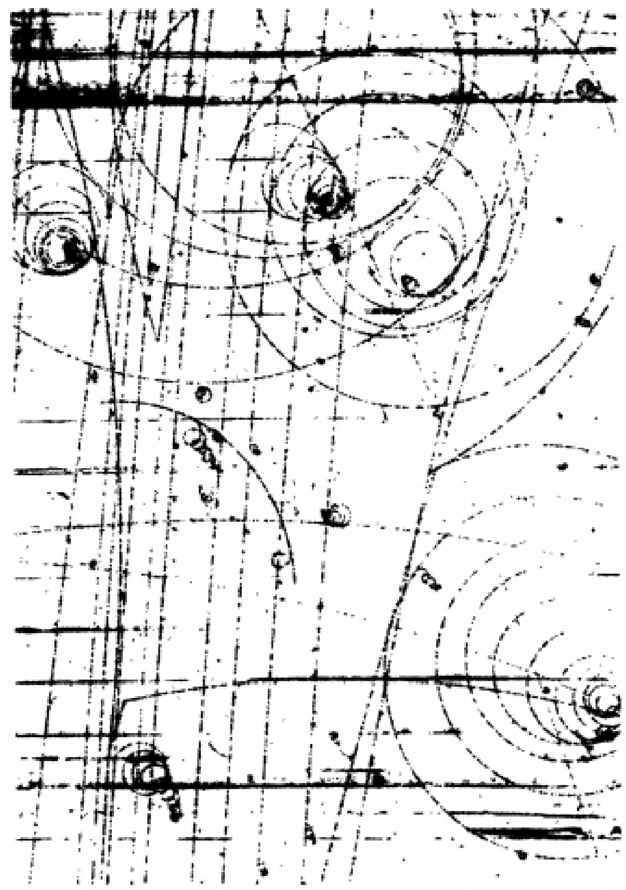
Only 9 baryon resonances with $J^P=3/2^+$ known at the time
Gell-Mann in 1962 at Rochester Conference assessed that, based on SU(3) symmetry, a state with $J^P=3/2^+$, S=-3, m~1680 should exist MeV

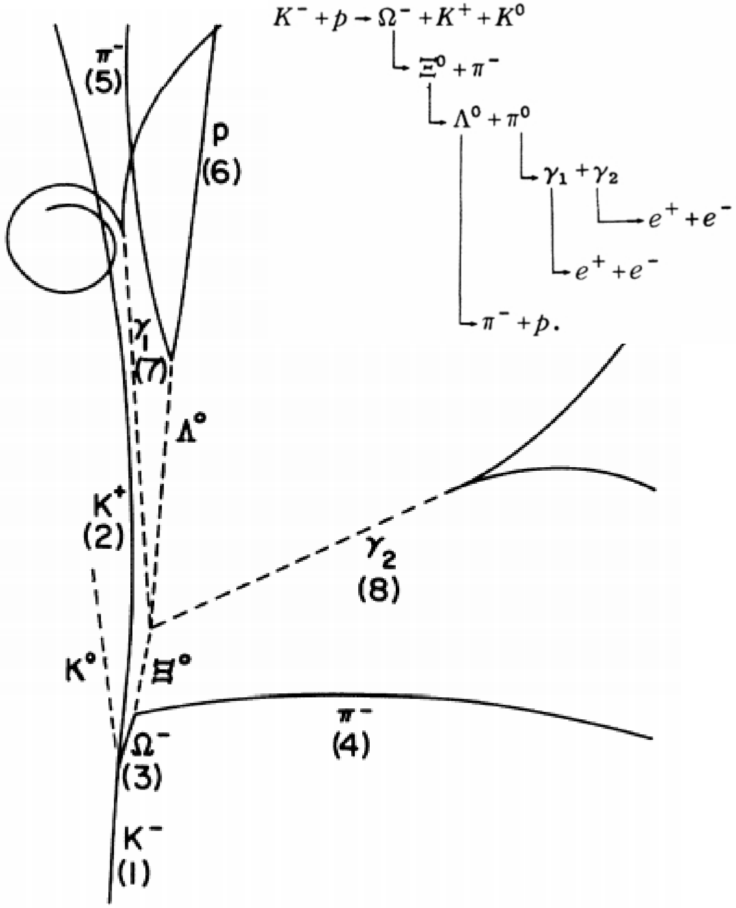
Expected mass of the new state derived from the mas differences of the known states

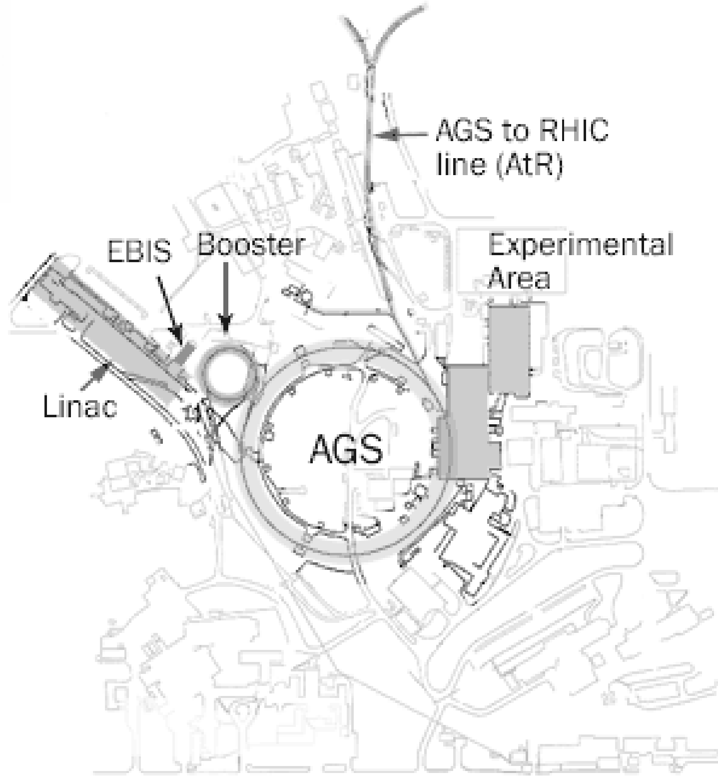


Observation of the Ω^-

Brookhaven group (1964)
BNL hydrogen bubble chamber exposed to 5.0 GeV K^- beam at BNL Alterating Gradient Synchrotron (AGS, today injector of RHIC)







Triumph for theory and experiment!