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Contribution from S and P waves in $p\bar{p}$ annihilation at rest

Obelix collaboration

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Abstract

The annihilation frequencies of 19 $\overline{p}p$ annihilation reactions at rest obtained in different target densities are analysed in order to determine the values of the P-wave annihilation percentage at each target density and the average hadronic branching ratios from P- and S-states. Both the assumptions of linear dependence of the annihilation frequencies on the P-wave annihilation percentage of the protonium state and the approach with the enhancement factors of Batty are considered. Furthermore the cases of incompatible measurements are discussed.

Keywords: $\overline{p}p$ annihilation; Exotic atoms; Annihilation fractions from S and P atomic levels

1. Introduction

Antiprotons slowed down in a H_2 target form $\overline{p}p$ atoms (protonia) in highly excited states with principal quantum number $n \sim 30$ [1]. The highly excited atoms deexcite through a number of processes including:

(i) chemical transitions (in highly excited states) where the antiproton can change the partner proton during a collision,
(ii) radiative transitions with emission of X-rays,
(iii) Auger transitions,
(iv) Stark transitions caused by the noncentral electric field experienced by protonium when crossing hydrogen molecules. The Stark effect mixes the angular momentum states at high-n, allowing the antiprotons to reach high-n S- and P-states (angular momentum $l = 0, 1$). Here they annihilate before reaching the low-n states. The annihilation in states with $l \geqslant 2$ is unimportant because the overlap of the p and $\overline{p}$ wave functions is negligible.

The intensity of the processes (i)–(iv) and consequently the probability of annihilation from S- and P-levels depend on the density of the target. Isolated (or at very low densities) atoms deexcite only through radiative transitions which populate preferentially levels with high l ($\sim n - 1$); therefore $\overline{p}p$ atoms end up in the $2P$ level and P-wave annihilation dominates. In liquid hydrogen the Stark effect mixes rapidly atomic levels with the same n but a different l. The admixture of S-wave levels leads then to a premature S-wave annihilation from high-n levels. As a consequence the S-wave annihilations dominate.

Annihilation from S- and P-wave atomic states can occur in singlet and triplet spin states. Each state is identified by specific values of the quantum numbers I^G and J^{PC} (see Table 1). In isolated antiprotonic atoms the relative weights of the spin states are expected to be statistical (see Table 1) while in dense targets they may be different.

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Table 1
Protonium levels from which the annihilation may occur

	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$
J^{PC}	0^{-+}	1^{--}	1^{+-}	0^{++}	1^{++}	2^{++}
I^G	0^+ 1^-	0^- 1^+	0^- 1^+	0^+ 1^-	0^+ 1^-	0^+ 1^-
Statistical weight	1/4	3/4	3/12	1/12	3/12	5/12

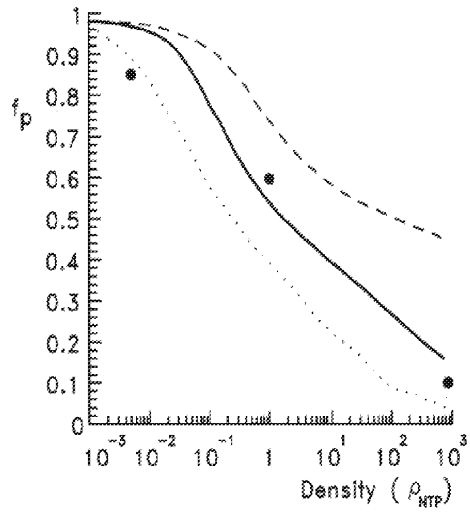

Fig. 1. Different behaviour of the fraction of P-state annihilation as a function of gas density predicted by the Borie–Leon model [2] giving a best fit to the X-ray data with $K_0 = 1$ (dashed line) and $K_0 = 7.6$ (full line) and by [25] (dotted line). The points are derived in this work by the linear fit C-low. The lines are taken from [4].

Several attempts to calculate the S- and P-wave annihilation dependence on the target density in $\overline{p}p$ atomic cascade models have been done [2,3]. A modified version of the model [2] was used by Batty [4]. All these models predict the decrease of the P-wave annihilation percentage from low pressure to liquid, but the decrease differs noticeably in each calculation (look at Fig. 1).

Bearing this in mind, the probability that a specific reaction channel $\overline{p}p \to X$ occurs in a target of density ρ,

$$F_X(\rho) = \left(\frac{N_X}{N_{\text{ann}}}\right)_\rho$$

(we call it *annihilation frequency*), can be written as follows (see, for instance, [5]):

$$F_X(\rho) = \sum_{n,L} f(n, L, \rho)\, \text{BR}_{n,L}(X), \tag{1}$$

where n is the protonium principal quantum number; $L \equiv^{2S+1} L_J$; $f(n, L, \rho)$ is the fraction of the population of the level with quantum numbers (n, L) which annihilates; it includes all the effects due to the protonium cascade and to the target density and satisfies the equation:

$$\sum_{n,L} f(n, L, \rho) = 1.$$

$\mathrm{BR}_{n,L}(X)$ is the branching ratio of the channel X for the pure annihilation process in the level (n, L); it is independent of the target density and of n [5], that is

$$\mathrm{BR}_{n,L}(X) = \mathrm{BR}_L(X).$$

Hence Eq. (1) can be rewritten in the form

$$F_X(\rho) = \sum_L \left(\sum_n f(n, L, \rho) \right) \mathrm{BR}_L(X) = \sum_L f(L, \rho)\, \mathrm{BR}_L(X), \tag{2}$$

where $f(L, \rho)$ is the fraction of the population of levels with quantum number L and any n which annihilates. The quantities $f(L, \rho)$ satisfy the condition

$$\sum_L f(L, \rho) = 1. \tag{3}$$

If we assume that annihilation occurs only from the levels with $L = 0, 1$, only the 6 states shown in Table 1 are effective and Eq. (2) becomes

$$\begin{aligned} F_X(\rho) \;=\; & f\left({}^1S_0, \rho\right) \mathrm{BR}_{{}^1S_0}(X) + f\left({}^3S_1, \rho\right) \mathrm{BR}_{{}^3S_1}(X) \\ & + f\left({}^1P_1, \rho\right) \mathrm{BR}_{{}^1P_1}(X) + f\left({}^3P_0, \rho\right) \mathrm{BR}_{{}^3P_0}(X) \\ & + f\left({}^3P_1, \rho\right) \mathrm{BR}_{{}^3P_1}(X) + f\left({}^3P_2, \rho\right) \mathrm{BR}_{{}^3P_2}(X). \end{aligned} \tag{4}$$

The quantities $f(L, \rho)$ can be combined in order to define the percentages of S- and P-wave annihilations:

$$\begin{aligned} f_S(\rho) \;&=\; f\left({}^1S_0, \rho\right) + f\left({}^3S_1, \rho\right), \\ f_P(\rho) \;&=\; f\left({}^1P_1, \rho\right) + f\left({}^3P_0, \rho\right) + f\left({}^3P_1, \rho\right) + f\left({}^3P_2, \rho\right), \end{aligned}$$

which obey to the condition $f_S(\rho) + f_P(\rho) = 1$.

Beside condition (3), additional constraints must be imposed on the quantities $f(L, \rho)$ due to the electromagnetic nature of the deexcitation process. As it was stressed in [5–7], during the process of the atomic cascade the total spin of the protonium is conserved, therefore the $S = 0$ and $S = 1$ sectors must be considered separately. As a consequence, as the initial population is statistical, the following conditions must be added:

$$\begin{aligned} & f\left({}^1S_0, \rho\right) + f\left({}^1P_1, \rho\right) = \tfrac{1}{4}, \\ & f\left({}^3S_1, \rho\right) + f\left({}^3P_0, \rho\right) + f\left({}^3P_1, \rho\right) + f\left({}^3P_2, \rho\right) = \tfrac{3}{4}. \end{aligned} \tag{5}$$

If two reactions A and B are allowed only from one sublevel S or only from one sublevel P, the ratio $F_A(\rho)/F_B(\rho)$ is independent of the density (of the percentage of P-wave); that is

$$\frac{F_A(\rho)}{F_B(\rho)} = \frac{\mathrm{BR}_S(A)}{\mathrm{BR}_S(B)} \quad \left(\text{or} \quad \frac{F_A(\rho)}{F_B(\rho)} = \frac{\mathrm{BR}_P(A)}{\mathrm{BR}_P(B)}\right). \tag{6}$$

In Eq. (4), $F_X(\rho)$ is the measured quantity and the branching ratios $\mathrm{BR}_i(X)$ are the quantities to be determined. Reaching this goal is not trivial because the unknowns $\mathrm{BR}_i(X)$ are in general more than one and the coefficients $f(L,\rho)$ are unknown too. In principle the problem could be solved considering a system of equations like Eq. (4) obtained by measuring the annihilation frequencies in targets with different densities for a number of reaction channels. In order to reduce the number of unknowns, reactions with a small number of effective branching ratios should be chosen. Such a solution would be independent of any assumptions (model independent). Unfortunately, this goal is too ambitious for the amount of available data and for their accuracy. This problem has been already attacked in the past in different ways by making additional assumptions.

Some analyses [8–10] have assumed that the target density affects the relative populations of the S- and P-levels, but not the relative populations of the S- and P-sublevels. The sublevels are assumed to be populated statistically according to Table 1. After this assumption, Eq. (4) is written in this form:

$$\begin{aligned} F_X(\rho) \;=\; & \big(1 - f_p(\rho)\big) \cdot \big[\tfrac{1}{4}\,\mathrm{BR}_{^1S_0}(X) + \tfrac{3}{4}\,\mathrm{BR}_{^3S_1}(X)\big] \\ & + f_p(\rho) \cdot \big[\tfrac{3}{12}\,\mathrm{BR}_{^1P_1}(X) + \tfrac{1}{12}\,\mathrm{BR}_{^3P_0}(X) + \tfrac{3}{12}\,\mathrm{BR}_{^3P_1}(X) \\ & \qquad + \tfrac{5}{12}\,\mathrm{BR}_{^3P_2}(X)\big], \end{aligned} \tag{7}$$

where the fractions $1/4,\ 3/4$, etc., are the relative statistical weights of the sublevels. According to Eq. (7), $F_X(\rho)$ depends *linearly* on $f_P(\rho)$. By comparing Eqs. (4) and (7) we obtain

$$f\big({}^1S_0,\rho\big) \;=\; \tfrac{1}{4}\big(1 - f_P(\rho)\big), \qquad f\big({}^3S_1,\rho\big) = \tfrac{3}{4}\big(1 - f_P(\rho)\big), \quad \text{etc.},$$

and Eqs. (5) are satisfied automatically.

A recent analysis by Batty [4] has shown that the weights of the P- and S-sublevels could be not statistical. Batty has assumed for Eq. (4) the form

$$\begin{aligned} F_X(\rho) \;=\; & \big(1 - f_p(\rho)\big) \cdot \big[\tfrac{1}{4}E\big({}^1S_0,\rho\big)\,\mathrm{BR}_{^1S_0}(X) + \tfrac{3}{4}E\big({}^3S_1,\rho\big)\,\mathrm{BR}_{^3S_1}(X)\big] \\ & + f_p(\rho) \cdot \big[\tfrac{3}{12}E\big({}^1P_1,\rho\big)\,\mathrm{BR}_{^1P_1}(X) + \tfrac{1}{12}E\big({}^3P_0,\rho\big)\,\mathrm{BR}_{^3P_0}(X) \\ & \qquad + \tfrac{3}{12}E\big({}^3P_1,\rho\big)\,\mathrm{BR}_{^3P_1}(X) + \tfrac{5}{12}E\big({}^3P_2,\rho\big)\,\mathrm{BR}_{^3P_2}(X)\big], \end{aligned} \tag{8}$$

with

$$\begin{aligned} & \tfrac{1}{4}E\big({}^1S_0\big) + \tfrac{3}{4}E\big({}^3S_1\big) = 1, \\ & \tfrac{3}{12}E\big({}^1P_1\big) + \tfrac{1}{12}E\big({}^3P_0\big) + \tfrac{3}{12}E\big({}^3P_1\big) + \tfrac{5}{12}E\big({}^3P_2\big) = 1. \end{aligned}$$

The coefficients E (*enhancement factors*) measure the deviation of the sublevel population from the statistical one. Eqs. (5) take the form

$$(1 - f_P)E\left({}^1S_0, \rho\right) + f_P E\left({}^1P_1, \rho\right) = 1,$$
$$(1 - f_P)E\left({}^3S_1, \rho\right) + \frac{f_P}{9}\left[E\left({}^3P_0, \rho\right) + 3E\left({}^3P_1, \rho\right) + 5E\left({}^3P_2, \rho\right)\right] = 1. \quad (9)$$

Assuming that the coefficients E are equal to 1, Eq. (8) reduces to Eq. (7).

Batty evaluated the factors $E(i, \rho)$ and $f_P(\rho)$ by using cascade calculations including effects due to Stark mixing and strong interaction widths. The widths were obtained from potential models and X-ray yield data. The values $E(i, \rho)$ and $f_P(\rho)$ obtained by the cascade calculation were constrained by the measured yields of L and K protonium X-ray lines. The best estimation of the behaviour of f_P vs. ρ is given by the full line in Fig. 1. The factors $E({}^1S_0)$, $E({}^3S_1)$ and $E({}^3P_2)$, turned out to be close to 1. The value of $E({}^3P_0)$ was found to increase from 1 at low density to 1.25 at NTP conditions and to 2.5 in liquid (NTP means Normal Temperature and Pressure). The values of the factors $E({}^3P_1)$ and $E({}^1P_1)$ were found to be less than one at high density, but the corresponding states do not contribute to the two body reactions considered by Batty.

Subsequently Batty evaluated the values of $f_p(\rho)$ at different densities by using Eq. (8) and fitting the annihilation fractions of two-body reactions in liquid, in NTP gas targets and in coincidence with antiprotonic atom L–X rays. In these fits the $E(i, \rho)$ values were those evaluated by the cascade calculation, while the values of $f_P(\rho)$ and of the partial branching ratios BR were left as free parameters to be optimized by the fit. If the cascade model is adequate and the data are correct, one expects that the f_P values obtained from the fit are equal to those given by the model. It turns out that the fitted f_P values agree satisfactorily with those predicted by the cascade model in liquid target ($(13 \pm 4)\%$ against 16%) and in NTP gas target ($(58 \pm 6)\%$ against 54%), but at very low density the fitted value is quite smaller ($(80 \pm 6)\%$ against 98%), revealing some discrepancy between model and data. Unfortunately the errors on the fitted values of f_P are quite large.

It is also worth noticing that the good fit on the annihilation frequencies with high values of $E({}^3P_0)$ found by Batty is a direct consequence of the high value of the annihilation frequency $F_{\pi^0\pi^0}$(liquid) $(= 6.93 \times 10^{-4})$ used in the analysis. As a matter of fact, there are 7 experimental values of $F_{\pi^0\pi^0}$(liquid), which are not all consistent within the errors (see Table 2). The use of a lower value of $F_{\pi^0\pi^0}$(liquid) in the analysis could lead to the conclusion that also the value of $E({}^3P_0)$ is compatible with (1). The problem of the data consistency concerns also other reactions, in particular $\pi^0\eta$ and $\eta\eta$, produced from the same states as well as $\pi^0\pi^0$ (see Table 3), and $\pi^+\pi^-$.

We notice that the high value of $E({}^3P_0, \text{liquid})$ predicted by Batty turns out to be consistent with a recent analysis of $\overline{p}p \to \eta\pi^0\pi^0\pi^0$ data in liquid and 12 bar gas targets [11], where the ratio

$$r = \frac{E({}^3P_0, \text{liquid})/E({}^3P_0, \text{gas})}{E({}^1P_1, \text{liquid})/E({}^1P_1, \text{gas})}$$

is found to be 2.46 ± 0.15 to be compared to the value $\simeq 1.9$ according to Batty.

In the following we will reconsider this subject considering that the number of annihilation frequency measurements in different target densities is larger than in the past.

Table 2
Annihilation frequency for nonstrange channels. In the table P indicates the reactions produced mainly from P-wave while S, P means that both S- and P-waves are effective. *AV* is the average of the values written above

Final states	$F \times 10^4$ liquid	Ref.	$F \times 10^4$ NTP	Ref.	$F \times 10^4$ 5 mbar/LX	Ref.
P						
$\pi^0\pi^0$	2.5 ± 0.3	[20]	12.7 ± 2.1	[19]		
	2.8 ± 0.4 ††	[21]				
	2.61 ± 0.24	*AV*				
	2.06 ± 0.14	[22]				
	1.4 ± 0.3	[41]				
	4.8 ± 1.0	[42]				
	6.0 ± 4.0	[43]				
	6.93 ± 0.4 †	[23]				
$\pi^0\eta$	4.6 ± 1.3	[20]	3.4 ± 0.7	[44]		
	1.33 ± 0.25	[47]				
	2.12 ± 0.12	[35]				
	0.90 ± 0.20	[21]				
$\eta\eta$	1.6 ± 0.8	[48]	2.7 ± 0.7	[44]		
	0.81 ± 0.31	[47]				
	1.64 ± 0.10	[49]				
S, P						
$\pi^+\pi^-$	30.7 ± 1.3	[35]	43.0 ± 1.4	[36]	42.6 ± 1.1	[15]
	33 ± 4	[38]	42.7 ± 2.1	[39]	$48.1 \pm 4.9 *$	[36]
	32 ± 3	[40]	42.9 ± 1.2	*AV*		
	31 ± 3	[37]				
	31.06 ± 1.06	*AV*				
$\pi^+\pi^-\pi^0$	536 ± 27	[27]	516 ± 26	[27]	489 ± 28	[27]
	582 ± 43	[50]	520 ± 35	[9]	$485 \pm 50 \diamond$	[9]
	549 ± 23	*AV*	517 ± 21	*AV*		
	690 ± 35	[32]				
$\omega\pi^+\pi^-$	719 ± 74	[45]	628 ± 34	[45]		
	660 ± 60	[33]	682 ± 74	[46]	$703 \pm 116 \circ$	[46]
	683 ± 47	*AV*	637 ± 31	*AV*		

L–X-ray coincidence: $* \ f_P = 100\%$, $\diamond \ f_P = (91.8 \pm 1.0)\%$, $\circ \ f_P = (86 \pm 6)\%$.

† New analyses from the same experiment [53] confirm this value.

†† This value agrees with the result of a new analysis made on different data by the same group [55].

Firstly we will investigate whether the data are compatible with the more traditional assumption of statistical annihilation from the S- and P-sublevels, and then we will consider them following the treatment suggested by Batty.

We have used the 110 values of $F_X(\rho)$ given in Tables 2 and 4. The data are different for age, experimental technique, precision and accuracy. How to be sure of their reliability, particularly when they are obtained in only one experiment?

Table 3
Selection rules for nonstrange channels

Reaction channels	$^{2S+1}L_J$	$^1S_0$		$^3S_1$		$^1P_1$		$^3P_0$		$^3P_1$		$^3P_2$	
	J^{PC}	0^{-+}		1^{--}		1^{+-}		0^{++}		1^{++}		2^{++}	
	I^G	0^+	1^-	0^-	1^+	0^-	1^+	0^+	1^-	0^+	1^-	0^+	1^-
$\pi^+\pi^-$		–	–	–	*	–	–	*	–	–	–	*	–
$\pi^0\pi^0$		–	–	–	–	–	–	*	–	–	–	*	–
$\pi^0\eta$		–	–	–	–	–	–	–	*	–	–	–	*
$\eta\eta$		–	–	–	–	–	–	*	–	–	–	*	–
$\pi^+\pi^-\pi^0$		–	*	*	–	*	–	–	–	–	*	–	*
$\omega\pi^+\pi^-$		–	*	*	–	*	–	–	*	–	*	–	*

– Channels forbidden by selection rules.
* Channels permitted by selection rules.

When there are more than one measurement of F_X for a f_P value, and they are consistent within errors, we considered their consistency a sufficient guarantee of reliability and we used their average value (given in the tables).

In the case of inconsistent measurements of F_X for a f_P value, we looked for which f_P value the fit is better performed with the linear or the nonlinear hypothesis. For the reliability we assumed a criteria of collective consistency: a set of values is considered to be acceptable within the frame of a particular hypothesis when all together they fit well that hypothesis.

As K_SK_L is produced only in S-wave, hence $F_{K_SK_L}(f_P = 1) = 0$, we have neglected in the fits the X-ray coincidence value (0.73 ± 0.56) measured by [13].

2. Assumption of a statistical population of S- and P-sublevels

2.1. Data analysis

We consider separately the reactions where either only S-wave channels (a) or only P-wave channels (b) or both (c) are effective due to the conservation of J^{PC} or to some dynamics effect (see Tables 3 and 5).

(a) If only the S-wave interaction is effective, then $\mathrm{BR}(^1P_1) = \mathrm{BR}(^3P_J) = 0$, and Eq. (7) becomes

$$F_X(f_p) = (1 - f_P) \cdot \left[\tfrac{1}{4}\,\mathrm{BR}_{^1S_0}(X) + \tfrac{3}{4}\,\mathrm{BR}_{^3S_1}(X)\right] = (1 - f_P)B_S(X). \quad (10)$$

In this equation we have stressed the dependence on the percentage of P-wave f_P instead of that on the density ρ. $B_S(X)$ is the *average* value of the branching ratios over the different S-sublevels; it is independent of the density. F_X is 0 for $f_P = 1$ and $B_X(S) = F_X(f_P = 0)$.

(b) Similarly, if only the P-wave interaction is effective, then $\mathrm{BR}(^1S_0) = \mathrm{BR}(^3S_1) = 0$ and Eq. (7) becomes

Table 4
Annihilation frequency for strangeness production

Final states	$F \times 10^4$ liquid	Ref.	$F \times 10^4$ NTP	Ref.	$F \times 10^4$ 5 mbar/LX	Ref.
S						
$\phi\pi^0$	4.88 ± 0.32	[27]	2.47 ± 0.21	[27]	0.92 ± 0.10	[27]
	3.0 ± 1.5	[20]	1.9 ± 0.5	[10]	$0.3 \pm 0.3\diamond$	[10]
	$5.5 \pm 0.70^\circ$	[17]	2.46 ± 0.23	[18]		
	5.02 ± 0.27	*AV*	2.33 ± 0.13	*AV*		
$K_S K_L$	6.1 ± 0.9	[29]	3.6 ± 0.6	[13]	1.00 ± 0.32	[26]
	8.0 ± 0.5	[30]	3.50 ± 0.54	[26]	$0.73 \pm 0.56*$	[13]
	$9.0 \pm 0.6\dagger$	[17]	3.55 ± 0.42	*AV*		
	7.80 ± 0.76	[26]				
	7.84 ± 0.32	*AV*				
$\eta(1440)\pi^+\pi^-$	6.0 ± 0.5	[28]	2.9 ± 0.4	[28]	1.0 ± 0.2	[28]
	7.1 ± 0.7	[30]	3.0 ± 0.9	[31]		
	6.37 ± 0.41	*AV*	2.91 ± 0.36	*AV*		
$K^\pm K^0_{\rm mis} 3\pi^\mp$	8.5 ± 0.5	[28]	4.8 ± 0.2	[28]	2.0 ± 0.2	[28]
$K^+K^-\eta$	7.00 ± 0.29	[27]	3.89 ± 0.25	[27]	1.93 ± 0.27	[27]
			4.68 ± 0.35	[18]		
			4.15 ± 0.20	*AV*		
P						
$K_S K_S$	0.04 ± 0.03	[32]	0.3 ± 0.1	[13]	$0.37 \pm 0.14*$	[13]
	0.044 ± 0.050	[29]				
	0.041 ± 0.026	*AV*				
S, *P*						
$K^{*\pm}K^\mp$	5.27 ± 0.52	[27]	13.2 ± 0.86	[27]	18.4 ± 1.1	[27]
$\phi\pi^+\pi^-$	4.6 ± 0.9	[33]	5.4 ± 1.0	[10]	$7.7 \pm 1.7\times$	[10]
$K^\pm K^0_S \pi^\mp$	31.6 ± 4.8	[34]	36.4 ± 5.5	[34]	43.2 ± 6.2	[34]
	28.2 ± 1.1	[51]				
	21.3 ± 2.8	[52]				
	27.0 ± 1.0	*AV*				
$K^+K^-\pi^0$	23.7 ± 1.6	[27]	30.3 ± 2.0	[27]	31.5 ± 2.2	[27]
K^+K^-	9.9 ± 0.5	[35]	6.92 ± 0.41	[36]	4.6 ± 0.3	[15]
	9.6 ± 0.8	[30]			$2.87 \pm 0.51*$	[36]
	11 ± 1	[29]				
	9.9 ± 0.2	[37]				
	9.92 ± 0.17	*AV*				
$\phi\omega$	6.63 ± 2.30	[27]	3.0 ± 1.1	[10]	$4.2 \pm 1.4\times$	[10]

L–X-ray coincidence: $*$ $f_P = 100\%$, $\diamond$ $f_P = (92.5 \pm 1)\%$, $\times$ $f_P = (86 \pm 6)\%$.
† New analyses from the same experiment [53] confirm this value.
∘ An updated value is given in [54]: 6.5 ± 0.6.

Table 5
Selection rules for strange channels

Reaction channels	$^{2S+1}L_J$ J^{PC}	$^1S_0$ 0^{-+}	$^3S_1$ 1^{--}	$^1P_1$ 1^{+-}	$^3P_0$ 0^{++}	$^3P_1$ 1^{++}	$^3P_2$ 2^{++}	References for dynamic effects
$K^+K^-\pi^0$		$*$	$*$	$*$	$-$	$*$	$*$	
$K^{*\pm}K^{\mp}$		$*$	$*$	$*$	$-$	$*$	$*$	
$\phi\omega$		$*$	$-$	$-$	$*$	$*$	$*$	
$\phi\pi^+\pi^-$		$*$	$*$	$*$	$*$	$*$	$*$	
K^+K^-		$-$	$*$	$-$	$*$	$-$	$*$	
$K^{\pm}K^0_S\pi^{\mp}$		$*$	$*$	$*$	$-$	$*$	$*$	
$\phi\pi^0$		$-$	$*$	x	$-$	$-$	$-$	[10,18]
$\phi\eta$		$-$	$*$	$*$	$-$	$-$	$-$	
K_SK_L		$-$	$*$	$-$	$-$	$-$	$-$	
$\eta(1440)\pi^+\pi^-$		$*$	$-$	$-$	$-$	x	$-$	[26]
$K^{\pm}K^0_{\rm mis}\pi^{\mp}\pi^+\pi^-$		$*$	$*$	$*$	$*$	$*$	$*$	
$K^+K^-\eta$		$*$	$*$	$*$	$-$	$*$	$*$	
K_SK_S		$-$	$-$	$-$	$*$	$-$	$*$	

$-$ Channels forbidden by selection rules.
$*$ Channels permitted by selection rules.
x Channels permitted by selection rules but suppressed by dynamics.

$$F_X(f_P) = f_P \cdot \left[\tfrac{3}{12}\,\mathrm{BR}_{^1P_1}(X) + \tfrac{1}{12}\,\mathrm{BR}_{^3P_0}(X) + \tfrac{3}{12}\,\mathrm{BR}_{^3P_1}(X) + \tfrac{5}{12}\,\mathrm{BR}_{^3P_2}(X)\right] = f_P B_P(X) \tag{11}$$

with $B_P(X) = F_X(f_P = 1)$.

(c) If both S- and P-wave interactions are effective, Eq. (7) can be written in the form:

$$F_X(f_P) = (1 - f_P)B_S(X) + f_P B_P(X) = B_S(X) + f_P\big[B_P(X) - B_S(X)\big]. \tag{12}$$

We have fitted by Eqs. (10)–(12) a number of annihilation data in order to determine both the percentage f_P in liquid hydrogen, in NTP — and 5 mbar — hydrogen gas and the average branching ratios $B_S(X)$ and $B_P(X)$. We have also fitted annihilation data obtained in coincidence with L–X-rays from the deexcitation cascade of the antiprotonic atom.

We have taken into account that some reactions occur only in S-wave or only in P-wave: for these reactions we have used Eqs. (10) and (11) and required that the annihilation frequencies were measured at least at two different P-wave percentages for each reaction. In the other cases we have used Eq. (12) and required that the annihilation frequencies were measured at least at 3 different P-wave percentages for each reaction.

We have applied best fit procedures by the code MINUIT [12] on a number of equations equal to the measured values of $F_X(f_P)$ with f_P(liquid), f_P(NTP), f_P(5 mbar), $B_S(X)$ and $B_P(X)$ as free parameters. The value of f_P of the LX-coincidence data were those

given in the original papers, where the hypothesis of statistical annihilation was used in the data analysis. Finally, we have considered 4 sets of reactions according to the effective protonium levels:

Set A: 4 reactions (K_SK_L, $K^{*\pm}K^{\mp}$, $\eta(1440)\pi^+\pi^-$, $\phi\pi^0$) without contribution from the $^3P_0$ level plus K_SK_S (other reactions could be considered in addition or in substitution without significant change in the results).
14 values of F_X were used.

Set B: 5 reactions ($K^+K^-, \pi^+\pi^-, \pi^0\pi^0, \pi^0\eta, \eta\eta$) produced mainly from P-levels with possible contribution from the $^3P_0$ level.
13 values of F_X were used.

Set C: 18 reactions: set A + set B + $K^+K^-\pi^0$, $K^+K^-\eta$, $\phi\pi^+\pi^-$, $K^{\pm}K^0_S\pi^{\mp}$, $K^{\pm}K^0_{\text{mis}}\pi^{\mp}\pi^+\pi^-$, $\phi\omega$, $\omega\pi^+\pi^-$, $\pi^+\pi^-\pi^0$.
56 values of F_X were used.

Set D: set C less $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$.

The relation due to the charge symmetry between the branching ratios for $\pi^0\pi^0$ and $\pi^+\pi^-$ (i.e. $\text{BR}_P(\pi^0\pi^0) = \frac{1}{2}\text{BR}_P(\pi^+\pi^-)$ has been used. For example, the equation system for set B has the form:

$$
\begin{aligned}
F_{K^+K^-}(f_P) &= (1-f_P)B_S\big(K^+K^-\big) + f_P B_P\big(K^+K^-\big),\\
F_{\pi^+\pi^-}(f_P) &= (1-f_P)B_S\big(\pi^+\pi^-\big) + f_P B_P\big(\pi^+\pi^-\big),\\
F_{\pi^0\pi^0}(f_P) &= \tfrac{1}{2} f_P B_P\big(\pi^+\pi^-\big),\\
F_{\pi^0\eta}(f_P) &= f_P B_P\big(\pi^0\eta\big),\\
F_{\eta\eta}(f_P) &= f_P B_P(\eta\eta),\\
F_{K^+K^-}\big(f_P^X\big) &= B_P\big(K^+K^-\big),\\
F_{\pi^+\pi^-}\big(f_P^X\big) &= B_P\big(\pi^+\pi^-\big),\\
F_{\pi^0\pi^0}\big(f_P^X\big) &= B_P\big(\pi^0\pi^0\big),
\end{aligned}
$$

where f_P^X is the percentage of P-wave for the measurements with X-rays coincidence; for the reactions in set B, $f_P^X = 1$.

The reactions $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$, which are affected by inconsistent data as mentioned in Section 1, affect heavily set B and are excluded from sets A and D.

2.2. Results

2.2.1. Percentage of P-wave

Our fit procedures were conditioned by two problems: one concerning the mentioned inconsistency among the F_x(liquid) values for $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ and the other related to the data on the $\pi^+\pi^-$ production.

The former problem concerns sets B and C. We made different fits using "low" values of the annihilation frequencies in liquid for $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ (2.61×10^{-4}, 0.9×10^{-4} and 0.81×10^{-4}, respectively; see Table 2) and "high" values (6.93×10^{-4}, 2.12×10^{-4} and 1.64×10^{-4}, respectively). The two sets of results are labelled B-low and B-high (C-low and C-high).

The $\pi^+\pi^-$ data affect the sets B, C and D. For this reaction in liquid target there are 4 measurements, which are consistent, so that their average may be considered very reliable. Moreover there are two consistent measurements at NTP and only one at 5 mbar, which have a low probability to belong to the same straight line with that in liquid. The $\pi^+\pi^-$ data give a large contribution to the χ^2 value of the fits. We made different fits considering all the data and neglecting the $F_{\pi^+\pi^-}$ value at NTP or at 5 mbar alternatively: the χ^2 values reduce strongly in both cases, with a higher reduction in the B-low and C-low fits than in the B-high and C-high fits. Anyway the minimum values are obtained neglecting the $F_{\pi^+\pi^-}$ value at NTP. In the D fits the χ^2 reduction is the same neglecting the $\pi^+\pi^-$ value at NTP or at 5 mbar, alternatively.

Our results are summarized in Table 6 and the fit C-low is illustrated in Figs. 2 and 3.

(a) The fit on set A (which is not affected by the above described problems) is very good, but the value of f_P in liquid has a large error. The very small χ_r^2 value indicates that the measurement errors are overestimated. In a first fit on set A we have neglected K_SK_S obtaining values of f_P with very large errors. To reduce these, we have resorted to the constraints given by a reaction produced only from P levels, which satisfies the condition $F_X(f_P = 0) = 0$. We have chosen the reaction K_SK_S as its data are not ambiguous and turn out to agree very well with the linear dependence assumption. We notice that the central f_P values obtained with and without the K_SK_S data differ negligibly.

(b) Concerning fits B, a large difference is found in the f_P(liquid) values obtained from the fits B-low and B-high: $(10.9 \pm 1.1)\%$ against $(29.0 \pm 2.3)\%$. The f_P(NTP) and f_P(5 mbar) values from both B-low and B-high are in satisfactorily agreement with those from set A.
The B-low value of χ^2 is significantly smaller than the B-high one. The χ^2 values decrease strongly if the value of $F_{\pi^+\pi^-}$(NTP) or of $F_{\pi^+\pi^-}$(5 mbar) is neglected.

(c) The fits C display features similar to those of fits B but the χ^2 values are much better and, in the case of fit C-low, the agreement with the f_P values at the different densities from fit A is very good.

(d) The fit on set D, where all the reactions excepted $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ are included, give results in very good agreement with those from fit A.

We can summarize the above analysis by saying that, if "low" values of the $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ production rates in liquid are used, the hypothesis of a linear dependence of F_X on f_P works well and is compatible with all the experimental data (excepted, perhaps, the reaction $\pi^+\pi^-$) .

Independently of the values of the annihilation fractions, the f_P values at different densities do not agree with the predictions of cascade models. In particular, the predicted f_P values at low densities are somewhat higher than the fitted ones. In Fig. 1 our values

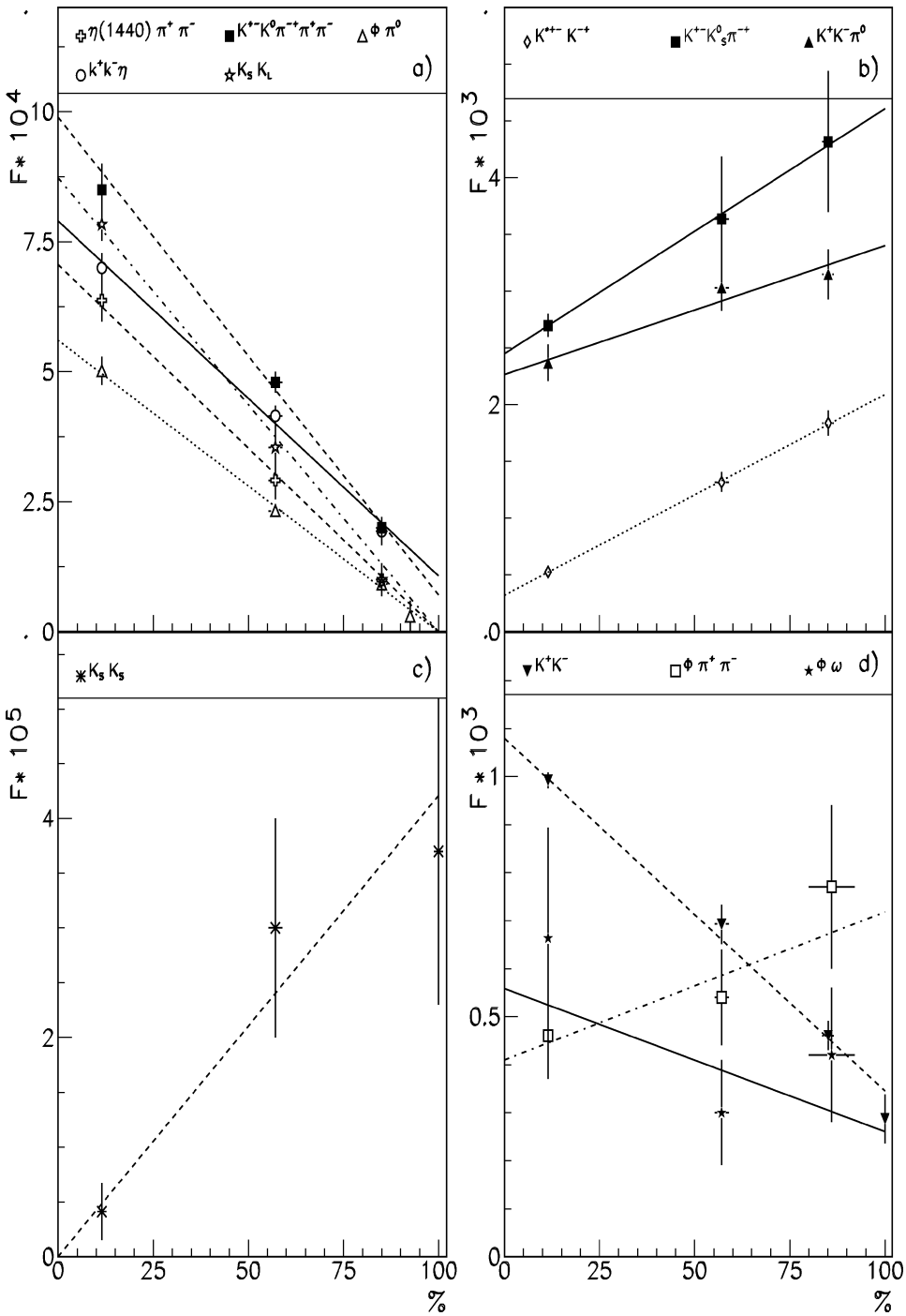

Fig. 2. Annihilation fraction vs. P-wave percentage according to fit C-low.

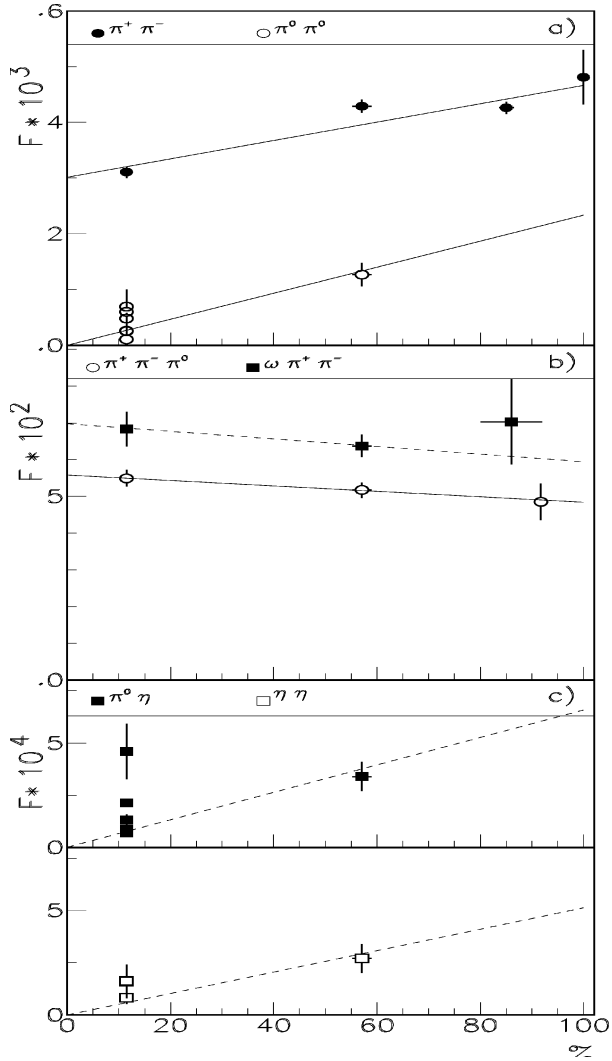

Fig. 3. Annihilation fraction vs. P-wave percentage according to fit C-low.

of f_P from fit C-low are compared with theoretical predictions; one can see that our experimental points follow roughly the full line, but with significant deviations.

Recently also the reaction $\overline{p}p \to \phi\eta$ was experimentally studied at three target densities (liquid, NTP and 5 mbar gas pressure) [14]. $\phi\eta$ is produced only from the ${}^3S_1$ and ${}^1P_1$ levels. Ref. [14] (see Table 7) gives upper and lower limits of the annihilation frequencies. The upper limit represents the maximum annihilation frequency for production from a pure ${}^3S_1$ level ($F({}^3S_1, \rho)$), while the lower limit is the maximum annihilation frequency for production from a pure ${}^1P_1$ level ($F({}^1P_1, \rho)$). The annihilation frequency for $\phi\eta$ production from an admixture of ${}^3S_1$ and ${}^1P_1$ levels can be written as

Table 6
P-wave percentage. Results of the fits according to the *linear-in-f_P* assumption and results from previous analyses. The labels (A), (B-low), (B-high), (C-low), (C-high) and (D) are explained in Section 2.2.1

		f_P(%)			
$\chi^2/N = \chi^2_r$	$P(\chi^2)$	Liquid	NTP	5 mbar	Ref.
$2.23/7 = 0.3$	95%	9.8 ± 6.3	58.5 ± 3.7	84.9 ± 1.8	(A)
$10.9/5 = 2.2$	7%	10.9 ± 1.1	54.5 ± 5.3	73.7 ± 6.8	(B-low)
$0.88/4 = 0.2$	92%	11.5 ± 1.1	49.6 ± 5.3	78.2 ± 7.2	(B-low)**
$22.1/23 = 1.0$	50%	11.5 ± 1.0	57.1 ± 1.8	85.1 ± 1.5	(C-low)
$10.9/22 = 0.5$	98%	11.9 ± 1.1	56.4 ± 1.9	85.2 ± 1.5	(C-low)**
$20.4/19 = 1.1$	40%	10.1 ± 6.9	56.7 ± 3.8	84.9 ± 1.9	(D)
$14.9/5 = 3.0$	1%	29.0 ± 2.3	60.7 ± 5.1	77.8 ± 6.7	(B-high)
$29.7/23 = 1.3$	20%	30.6 ± 1.9	66.0 ± 1.8	88.2 ± 1.2	(C-high)
		27 ± 2	69 ± 4	77 ± 5	[4]a
		13 ± 4	58 ± 6	80 ± 6	[4]b
		16	54	98	[4]c
			59 ± 5		[10]
		11.9	55.5	86.2	[44]
		8.6 ± 1.1 (*)	50.3 ± 6.4		[36]
		20.0 ± 4.6 (△)			[36]
		28.8			[23]
			53 ± 8		[19]

* Obtained with $F_{\pi^0\pi^0}(\text{liquid}) = 2.06 \times 10^{-4}$.
** The value $F_{\pi^+\pi^-}$(NTP) is excluded from the fit.
△ Obtained with $F_{\pi^0\pi^0}(\text{liquid}) = 4.8 \times 10^{-4}$.
a Obtained with $E = 1$ in Eq. (8).
b Obtained using E factors from cascade models.
c Cascade models (see Fig. 1, full line).

Table 7
Upper and lower limits of the annihilation frequencies of the $\phi\eta$ production [14] ($F \times 10^4$)

	Liquid	NTP	5 mbar
$F(^3S_1, \rho)$	0.60 ± 0.20	1.04 ± 0.20	1.05 ± 0.26
$F(^1P_1, \rho)$	1.01 ± 0.33	1.53 ± 0.29	1.62 ± 0.40

$$F(\rho) = \big(1 - f_P(\rho)\big)F\big(^3S_1, \rho\big) + f_P(\rho)F\big(^1P_1, \rho\big). \tag{13}$$

Assuming the f_P values at the three densities according to the C-low fit, the corresponding annihilation fractions are given in Table 8 and Fig. 4. Our result in liquid target agrees very well with that deduced from Refs. [16,17]. The values at NTP and 5 mbar gas targets are

Table 8
$\phi\eta$ annihilation fractions

	Liquid	NTP	5 mbar
f_P	11.5	57.1	85.1
$F(\rho)$	0.65 ± 0.18	1.32 ± 0.19	1.53 ± 0.34
$F(\rho)$	0.66 ± 0.19*	0.37 ± 0.09 [10]	0.41 ± 0.16 [10]†

∗ Deduced from the ratio $(\overline{p}p \to \phi\eta)/(\overline{p}p \to \phi\pi^0)$ [16] and the annihilation fraction of $\overline{p}p \to \phi\pi^0$ [17].
† X-ray coincidence.

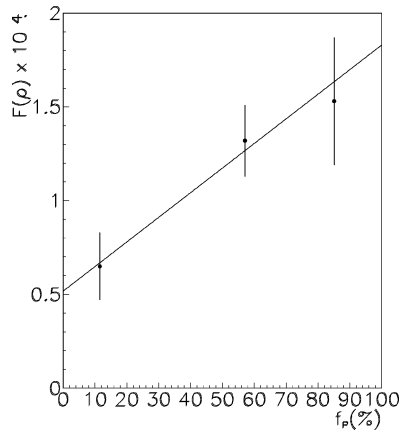

Fig. 4. $\phi\eta$ annihilation fractions vs. P-wave percentage. Values are from Table 8.

about three times larger than those given by Ref. [10]. The discrepancy between the gas target data was stressed already in Refs. [10,18].

In the following we will comment the results of other analyses presented in Table 6. Ref. [4] considered data on the two body reactions $\pi^0\pi^0$, $\pi^+\pi^-$, K_SK_S, K_SK_L, K^+K^- (produced only in $^3S_1$, $^3P_0$ and $^3P_2$ levels) mainly in liquid, NTP gas target and in X-ray coincidence; the relation between the branching ratios in P-wave for $\pi^+\pi^-$ and $\pi^0\pi^0$ due to charge symmetry was taken into account. Two sets (a and b) of values from [4] are shown in Table 6. Set a was obtained by setting $E = 1$ in Eq. (8) and set b by using E values given by the cascade calculations. The two sets are not compatible in liquid, where $f_P = 0.27 \pm 0.02$ (set a) against 0.13 ± 0.04 (set b).

Subsets of the above two body reactions were used also in the other analyses according to the assumption of statistical annihilation. They show a tight correlation between the $F_{\pi^0\pi^0}$(liquid) values and the used f_P(liquid) values.

2.2.2. Branching ratios

The values of the average branching ratios according to the fit C-low are summarized and compared to previous evaluations in Tables 9 and 10. Our results agree very well with the previous ones.

Table 9
Best fit values of average branching ratios for nonstrange channels from the linear fit C-low

Final state	$\overline{p}p$ state	$B_S = F(0)$	$B_P = F(1)$	Ref.
$\pi^0\pi^0$	$^3P_0,\ ^3P_2$	0	$(2.33 \pm 0.06) \times 10^{-3}$	(*)
		0	$(2.06 \pm 0.14) \times 10^{-3}$	[22]
$\pi^+\pi^-$	$^3S_1,\ ^3P_0,\ ^3P_2$	$(3.02 \pm 0.12) \times 10^{-3}$	$(4.67 \pm 0.12) \times 10^{-3}$	(*)
		$(3.19 \pm 0.20) \times 10^{-3}$	$(4.81 \pm 0.49) \times 10^{-3}$	[8]
$\eta\pi^0$	$^3P_0,\ ^3P_2$	0	$(6.58 \pm 1.04) \times 10^{-4}$	(*)
$\eta\eta$	$^3P_0,\ ^3P_2$	0	$(5.13 \pm 1.13) \times 10^{-4}$	(*)
$\pi^+\pi^-\pi^0$	$^1S_0,\ ^3S_1,\ ^1P_1,\ ^3P_1,\ ^3P_2$	$(5.58 \pm 0.27) \times 10^{-2}$	$(4.84 \pm 0.38) \times 10^{-2}$	(*)
		$(6.6 \pm 0.8) \times 10^{-2}$	$(4.5 \pm 0.6) \times 10^{-2}$	[9]
$\omega\pi^+\pi^-$	$^1S_0,\ ^3S_1,\ ^1P_1,\ ^3P_J$	$(6.98 \pm 0.60) \times 10^{-2}$	$(5.95 \pm 0.80) \times 10^{-2}$	(*)
		$(6.55 \pm 0.68) \times 10^{-2}$	$(7.05 \pm 1.05) \times 10^{-2}$	[46]

(*) This work.

Table 10
Best fit values of average branching ratios ($\times 10^4$) for strange channels from the linear fit C-low

Final state	$\overline{p}p$ state	$B_S = F(0)$	$B_P = F(1)$	Ref.
$\phi\pi^0$	$^3S_1$	5.61 ± 0.26	0	(*)
		4.0 ± 0.8	0	[10]
		5.5 ± 0.6	0	[27]
$K_S K_L$	$^3S_1$	8.74 ± 0.36	0	(*)
		8.20 ± 0.89	0	[8]
		10.05 ± 1.12	0	[44]
		9.0 ± 0.6	0	[17]
$\eta(1440)\pi^+\pi^-$	$^1S_0$	7.07 ± 0.41	0	(*)
$K^+K^-\eta$	$^1P_1$	7.90 ± 0.33	1.07 ± 0.31	(*)
$K_S K_S$	$^3P_2$	0	0.42 ± 0.10	(*)
		0	0.48 ± 0.50	[8]
$K_S^0 K^\pm \pi^\mp$	$^1S_0,\ ^3S_1,\ ^1P_1,\ ^3P_{1,2}$	2.45 ± 0.14	4.61 ± 0.61	(*)
$K^{*\pm} K^\mp$	$^1S_0,\ ^1P_1$	3.22 ± 0.66	20.9 ± 1.1	(*)
$K^\pm K^0_{\rm mis} \pi^\mp \pi^+ \pi^-$	S, P (all)	9.90 ± 0.50	0.72 ± 0.30	(*)
$K^+K^-\pi^0$	$^1S_0,\ ^3S_1,\ ^1P_1,\ ^3P_{1,2}$	2.27 ± 0.19	3.41 ± 0.23	(*)
K^+K^-	$^3S_1,\ ^3P_{0,2}$	10.79 ± 0.21	3.45 ± 0.31	(*)
		10.8 ± 0.5	2.87 ± 0.51	[13]
		11.33 ± 1.10	2.0 ± 0.6	[8]
$\phi\pi^+\pi^-$	S, P (all)	4.09 ± 1.07	7.18 ± 1.51	(*)
		4.7 ± 1.1	6.6 ± 1.5	[10]
$\phi\omega$	$^1S_0,\ ^3P_{0,1,2}$	5.6 ± 2.6	2.6 ± 2.0	(*)
		5.3 ± 2.2	2.9 ± 1.4	[10]

(*) This work.

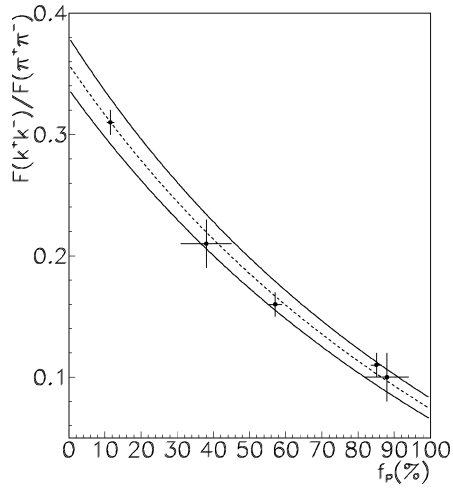

Fig. 5. Ratio between the annihilation frequencies of K^+K^- and of $\pi^+\pi^-$ vs. f_P. The band limited by the full lines are according to the B_P and B_S values in Tables 9 and 10. The data at $f_P = 38\%$ (16 atm), $f_P = 86\%$ (5 mbar) and $f_P \sim 88\%$ (2 mbar) were not used in the fit procedures.

Table 11
Direct measurement of the ratio between the annihilation frequencies of the K^+K^- and the $\pi^+\pi^-$ reactions at different target densities

	15 bar*	5 mbar	2 mbar**
$\frac{K^+K^-}{\pi^+\pi^-}$	0.205 ± 0.016 [24]	0.108 ± 0007 [15]	0.102 ± 0.015 [15]

* $f_P = 0.38 \pm 0.07$.
** $f_P \sim 0.88$.

We stress that Tables 9 and 10 give *average* branching ratios. Branching ratios BR(L) are related to average branching ratios $B_{S(P)}$ by the statistical weights in Eqs. (10)–(12). In the cases where only one S-sublevel and/or only one P-sublevel are effective, branching ratios can be carried out immediately. For instance:

$$\phi\pi^0\text{:}\quad \mathrm{BR}\left({}^3S_1\right) = \tfrac{4}{3}B_S, \qquad \phi\eta\text{:}\quad \mathrm{BR}\left({}^1P_1\right) = \tfrac{12}{3}B_P.$$

Fig. 5 shows a comparison between the behaviour of the ratio $F_{K^+K^-}/F_{\pi^+\pi^-}$ vs. f_P deduced by the best fit values of B_S and B_P and three independent measurements in 2 mbar and 5 mbar [15] and in 16 atm [24] gas targets (see Table 11). As shown, the agreement is good.

3. Analysis with enhancement factors

We have also fitted the available data set of the annihilation fractions using Eq. (8) with values of the enhancement factors according to Table 4 of [4] (DR1 model). Since the

Table 12
P-wave percentage. Results of the fits with enhancement factors from [4]. Set A and B include the same reactions as in the linear fits

		f_P(%)			
$\chi^2/N = \chi^2_r$	$P(\chi^2)$	Liquid	NTP	5 mbar	Data sample
$1.81/5 = 0.36$	87%	4.1 ± 2.8	56.2 ± 2.9	83.9 ± 1.6	A
$1.89/4 = 0.47$	76%	5.5 ± 0.9	49.5 ± 5.3	74.4 ± 6.1	B-low
$6.54/13 = 0.54$	92%	6.1 ± 1.1	56.4 ± 2.8	83.8 ± 1.6	C-low
$5.58/4 = 1.47$	23%	15.5 ± 2.5	55.8 ± 5.3	75.4 ± 5.4	B-high
$16.8/13 = 1.29$	22%	15.4 ± 2.1	60.7 ± 2.7	85.3 ± 1.5	C-high

Batty's cascade model fixes both the enhancement factors and the values of f_P (16% in liquid, 54% at NTP and 98% at 5 mbar), our fit allows to verify the consistence of the model with the experimental data. The results on f_P as free parameter are presented in Table 12. We comment them considering firstly the results on f_P(liquid). The data set A includes reactions where the $^3P_0$ level is not effective ($K_S K_L$, $\eta(1440)\pi^+\pi^-$, $\phi\pi^0$) plus $K_S K_S$. 13 experimental F_X values are used. The fit on these data gives a value of f_P(liquid) smaller than that given by the cascade model of Batty (4.1 ± 2.8% against 16%).

The reactions of set B are allowed from the $^3P_0$ initial state where one can expect the biggest effect of the enhancement factors. The reactions are the same as in the linear fit and the experimental values of F_X are 14. The $\pi^0\eta$ and $\eta\eta$ production is allowed both from $^3P_0$ and $^3P_2$ initial states, but all the fits give a negligible contribution of the $^3P_2$ state. For this reason the branching ratio in the state $^3P_2$ was finally set equal to zero.

The data set B has been fitted in two ways. In the first one (B-low) we have used the low values for the $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ production rates in liquid, in the second one (B-high), the high values. The quality of the fit is acceptable in both cases, but the χ^2 value is lower in the case B-low; the high χ^2 value in the case B-high is due to the measurements of $\pi^+\pi^-$ at NTP and $\pi^0\pi^0$ at NTP. If the reaction $K_S K_S$ is included into set B, f_P values are obtained which are very close to those without $K_S K_S$. The main difference between B-low and B-high fits is the P-wave annihilation percentage in liquid (5.5 ± 0.9 against 15.5 ± 2.5). The two values have a low probability to be consistent reflecting the incompatibility of the data. The low value is consistent with that obtained by the fit on set A and the high value is equal to that given by the cascade model of Batty.

The data set C includes all the 9 reactions from set A and set B, so that the total number of F_X values used in the fit is 27. The results are very similar to those obtained in fits B.

It is worth remarking that the $\pi^+\pi^-$ data are fitted better with the enhancement factors than with the linear hypothesis (compare fits B-low and B-high in Tables 6 and 12). The same holds for the fit D (not given in Table 12).

Summarizing, we have an agreement between the fits on different sets of data (A and B or C) if we use low values of F_X(liquid) for the reactions $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$, but we find a value of f_P smaller than that given by the Batty model (16%). If we use the high values

of F_X(liquid) we find consistency between the f_P values from the set B (or C) and the Batty value, but a disagreement between the values from the fit on set A. χ^2 is better with the low values. The f_P values in NTP gas target from the different fits are compatible each other and with the value from the cascade model. Instead, there is a problem with the P-wave percentage of the $\overline{p}p$ annihilation at low target density. The cascade model within which the enhancement factors are calculated predicts about 98% of P-wave annihilation at 5 mbar (see Fig. 1), while the fits give significantly lower values (about 85% in the fits A and C and about 75% in the fits B).

4. Remarks on the inconsistent data

In principle, it is possible to check the reliability of some data in a model-independent way; unfortunately, large errors forbid us to be conclusive.

The reactions $K_S K_L$ and $\phi\pi^0$ occur only in the ${}^3S_1$ state; therefore their annihilation frequencies must satisfy Eq. (6). Considering the values in Tables 2 and 4, the ratios

$$\left(\frac{F_{\phi\pi^0}}{F_{K_S K_L}}\right)_{\text{liquid}} = 0.64 \pm 0.04, \qquad \left(\frac{F_{\phi\pi^0}}{F_{K_S K_L}}\right)_{\text{NTP}} = 0.65 \pm 0.08$$

are really independent of the target density within the errors. We can therefore rely on the above annihilation frequencies. Considering the isospin symmetry, we can carry out the annihilation frequency in the ${}^3S_1$ state for $\pi^+\pi^-$ and compare it to the $K_S K_L$ annihilation frequency. In order to do this, we used the following equations:

$$\begin{aligned}
F_{\pi^0\pi^0} &= f_P\left[E\left({}^3P_0\right)B_{\pi^0\pi^0}\left({}^3P_0\right) + E\left({}^3P_2\right)B_{\pi^0\pi^0}\left({}^3P_2\right)\right],\\
F_{\pi^+\pi^-} &= (1-f_P)E\left({}^3S_1\right)B_{\pi^+\pi^-}\left({}^3S_1\right) + 2F_{\pi^0\pi^0} = F_{\pi^+\pi^-}\left({}^3S_1\right) + 2F_{\pi^0\pi^0},\\
F_{\pi^+\pi^-}\left({}^3S_1\right) &= (1-f_P)E\left({}^3S_1\right)B_{\pi^+\pi^-}\left({}^3S_1\right) = F_{\pi^+\pi^-} - 2F_{\pi^0\pi^0}.
\end{aligned}$$

We can estimate $F_{\pi^+\pi^-}({}^3S_1)$ from the experimental values of the annihilation fractions for $\pi^+\pi^-$ and $\pi^0\pi^0$; we expect the ratio

$$\frac{F_{\pi_+\pi^-}({}^3S_1)}{F_{K_S K_L}({}^3S_1)}$$

to be independent of the target density. By using for $F_{\pi^0\pi^0}$(liquid) the "low" value 2.61, we have

$$\left(\frac{F_{\pi^+\pi^-}({}^3S_1)}{F_{K_S K_L}({}^3S_1)}\right)_{\text{liquid}} = 3.29 \pm 0.20$$

and, by using the "high" value 6.93,

$$\left(\frac{F_{\pi^+\pi^-}({}^3S_1)}{F_{K_S K_L}({}^3S_1)}\right)_{\text{liquid}} = 2.19 \pm 0.17.$$

At NTP we have

$$\left(\frac{F_{\pi^+\pi^-}(^3S_1)}{F_{K_SK_L}(^3S_1)}\right)_{\mathrm{NTP}} = 4.93 \pm 1.37.$$

Obviously, the two values in liquid are incompatible. It would be nice to understand which one is more reliable by comparing them with the value at NTP. Unfortunately, this value is higher than the others and has a larger error. It is more compatible with the first value in liquid (within 1.2σ) than with the second one (within 2σ), but we cannot decide firmly which one should be rejected. The high value at NTP could reflect a too high value of the $\pi^+\pi^-$ annihilation fraction and/or a too low value of the $\pi^0\pi^0$ one.

The consistency between the data can be explored in a different way, which is unfortunately not model independent. The fit D under the approximation of statistical annihilation is affected by the $\pi^+\pi^-$ frequencies but not by the controversial data on $\pi^0\pi^0$. Hence it is possible to compare the values of the measured annihilation frequency of the reaction $\pi^0\pi^0$ with those which can be deduced from the average P-wave branching ratio for $\pi^+\pi^-$. The best fit D gives the value

$$B_P\left(\pi^+\pi^-\right) = (4.68 \pm 0.12) \times 10^{-3}$$

which is the same as that given by fit C-low (see Table 9). Owing to the charge symmetry, the annihilation frequency for $\pi^0\pi^0$ is

$$F_{\pi^0\pi^0}(f_P) = f_P \tfrac{1}{2} B_P\left(\pi^+\pi^-\right) = f_P\ (2.34 \pm 0.06) \times 10^{-3}.$$

Consequently we obtain:

$$F_{\pi^0\pi^0}(\text{liquid}) \sim 2.30 \times 10^{-4}, \qquad F_{\pi^0\pi^0}(\text{NTP}) \sim 13.5 \times 10^{-4}.$$

These values must be compared with the experimental ones shown in Table 2. Clearly, the deduced value at NTP is equal to the value measured from [19] (12.7×10^{-4}) and the deduced value in liquid is closer to the "low" value according to [20–22] (2.61×10^{-4}) than to the "high" value from [23] (6.93×10^{-4}). It is worthwhile noting that the Crystal Barrel Collaboration, which measured the high $F_{\pi^0\pi^0}$(liquid) value, has recently confirmed that result analyzing with different criteria different data sets [53]. Additional comments on this intricate situation can be found in [53].

5. Conclusions

We have analyzed the annihilation frequencies of 19 $\overline{p}p$ annihilation reactions in three target densities and in coincidence with LX-ray emitted by the deexcitation of antiprotonic atoms.

Our goal was to determine the fraction of P-wave annihilation f_P at different target densities (in particular in liquid) and the hadronic branching ratios for the different annihilation channels.

The full achievement of this goal is impeded by the large experimental errors, by the inconsistency of some data on the same channel from different experiments, namely those in liquid on $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$, and by inconsistency on the dependence on the density

of the annihilation frequencies of different channels (for instance, $\pi^+\pi^-$ and $\pi^0\pi^0$). In particular, $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ channels are characterized by "low" and "high" values of the annihilation frequency F_X(liquid), which are not consistent within the errors.

In spite of this, we try to come to some conclusions recalling the main features of the results of our analyses.

We have analyzed the data on two assumptions.

(a) *Assumption of statistical annihilation from S- and P-sublevels.*
In this case the annihilation frequency F_X depends linearly on the P-wave annihilation percentage f_P (see Eq. (6)). If, for F_X(liquid) of the reactions $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$, the "low" values are used, the linear dependence works very well on all reactions (a little worse for $\pi^+\pi^-$) and appears to be quite natural. In particular, the fits on different sets of reactions give f_P values which are consistent at each target density. According to the C-low fit the more reliable estimations are f_P(liquid) $\sim 11.5\%$, f_P(NTP) $\sim 57.1\%$ and f_P(5 mbar) $\sim 85.1\%$. The previous general consistency is lacking, if "high" values of F_X(liquid) are used, because the fits on the sets of reactions including $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ give very high values of f_P(liquid). Finally, it is worth noting that the ratios

$$f_S(\text{liquid}) : f_S(\text{NTP}) : f_S(5\text{ mbar}) = 1 : 0.49 : 0.17$$

coincide with the values obtained from the analysis of η and $K_S^0K_L^0$ production in $\overline{p}p$ annihilation [28].

(b) *Assumption of a deviation from a statistical annihilation according to Batty* [4].
This assumption is based on a cascade model which evaluates the deviation from the statistical annihilation through the enhancement factors. The model gives also the values of the S- and P-wave annihilation percentages. We have fitted F_X data by using Eq. (8), where the values of the enhancement factors were taken from [4] and the f_P values were free parameters. By using low F_X(liquid) values of the reactions $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ we have obtained good fits on all the data ($\pi^+\pi^-$ included), but the value of f_P(liquid) is much smaller than the value predicted by the cascade model. By using the high values of F_X(liquid), the fits on the sets of reactions including $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ give f_P values higher than previously but in agreement with the value predicted by the cascade model.

The above points (a) and (b) show that, independently of the assumptions, the f_P values at each density obtained from the reactions $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ are consistent with those obtained from the other reactions only if the "low" values of F_X(liquid) for $\pi^0\pi^0$, $\pi^0\eta$ and $\eta\eta$ are used. Therefore, if consistency is an indication of reliability, the "low" values appear to be more reliable than the "high" ones.

Finally, independently of the data and the model used, the value of f_P at low pressure turns out to be quite lower than that predicted by the cascade model. So the cascade models which match well the X-ray yields at densities below $10\rho_{\text{NTP}}$ are not consistent with the low-density annihilation data.

References

[1] C. Batty, Rep. Prog. Phys. 52 (1989) 1165.
[2] E. Borie, M. Leon, Phys. Rev. A 21 (1980) 1460.
[3] J. Reifenroether, E. Klempt, Phys. Lett. B 245 (1990) 129.
[4] C. Batty, Nucl. Phys. A 601 (1996) 425.
[5] U. Gastaldi et al., Phys. Lett. B 320 (1994) 193.
[6] G. Snow, Phys. Lett. 1 (1962) 213.
[7] R. Bizzarri, Nuovo Cimento A 53 (1968) 956.
[8] M. Doser, Thesis, Zurich (1988).
[9] B. May et al., Z. Phys. C 46 (1990) 191, Z. Phys. C 46 (1990) 203.
[10] J. Reifenroether et al., Phys. Lett. B 267 (1991) 299.
[11] A. Abele et al., Nucl. Phys. B 514 (1998) 45.
[12] F. James et al., MINUIT Function Minimization and Error Analysis, CERN Program Library D506, v.94.1, 1994.
[13] M. Doser et al., Phys. Lett. B 215 (1988) 792.
[14] V. Alberico et al., Phys. Lett. B 432 (1998) 427.
[15] V.G. Ableev et al., Phys. Lett. B 329 (1994) 407.
[16] C. Amsler et al., Phys. Lett. B 319 (1993) 373.
[17] C. Amsler et al., Phys. Lett. B 346 (1995) 363.
[18] V.G. Ableev et al., Nucl. Phys. A 594 (1995) 375.
[19] M. Agnello et al., Phys. Lett. B 337 (1994) 2.
[20] M. Chiba et al., Phys. Rev. D 38 (1988) 2021.
[21] A. Zoccoli, Hadron Spectroscopy, in: S.U. Chung, H. Willutzki (Eds.), AIP Conference Proc., Vol. 432, 1997, p. 347.
[22] L. Adiels et al., Z. Phys. C 35 (1987) 15.
[23] C. Amsler et al., Phys. Lett. B 297 (1992) 214.
[24] R. Adler et al., Phys. Lett. B 267 (1991) 154.
[25] J. Reifenroether, E. Klempt, Nucl. Phys. A 503 (1989) 885.
[26] A. Bertin et al., Phys. Lett. B 386 (1996) 486.
[27] A. Alberico et al., Phys. Lett. B 438 (1998) 430.
[28] A. Bertin et al., Phys. Lett. B 385 (1996) 493.
[29] C. Baltay et al., Phys. Rev. Lett. 15 (1965) 532.
[30] R. Armenteros, B. French, in: E.H.S. Burhop (Ed.), High Energy Physics, Vol. 4, Academic Press, New York, 1969.
[31] K.D. Duch et al., Z. Phys. C 45 (1989) 223.
[32] M. Foster et al., Nucl. Phys. B 6 (1968) 107.
[33] R. Bizzarri et al., Nucl. Phys. B 14 (1969) 169.
[34] A. Bertin et al., Phys. Lett. B 434 (1998) 180.
[35] C. Amsler et al., Z. Phys. C 58 (1993) 175.
[36] M. Doser et al., Nucl. Phys. A 486 (1988) 493.
[37] G. Smith, in: Proc. Workshop on the Elementary Structure of Matter, Les Houches, 1987.
[38] M. Cresti et al., in: Proc. Int. Conf. Elementary Particles, Sienna, Vol. 1, Soc. It. Fis., Bologna, 1963, p. 263.
[39] V.G. Ableev et al., Nuovo Cimento A 107 (1994) 2837.
[40] C. Baltay et al., Phys. Rev. 145 (1966) 1103.
[41] G. Bassompierre et al., in: Proc. IV European Antiproton Symposium, Barr, 1978.
[42] S. Devons et al., Phys. Rev. Lett. 27 (1971) 1614.
[43] G. Backenstoss et al., Nucl. Phys. B 228 (1983) 424.
[44] A. Zoccoli, Yad. Fiz. 59 (1996) 1448.
[45] A. Bertin et al., Phys. Lett. B 388 (1996) 45.

[46] W. Weidenauer et al., Z. Phys. C 59 (1993) 387.
[47] L. Adiels et al., Z. Phys. C 42 (1989) 49.
[48] M. Chiba et al., Phys. Rev. D 39 (1989) 3227.
[49] C. Amsler et al., Z. Phys. C 58 (1987) 175.
[50] A. Abele et al., Phys. Lett. B 411 (1997) 354.
[51] R. Armenteros et al., Phys. Lett. 17 (1965) 170.
[52] N. Barash et al., Phys. Rev. 139 (1965) B1659.
[53] C. Batty, Nucl. Phys. A 655 (1999) 305c.
[54] C. Amsler, Rev. Mod. Phys. 70 (1998) 1293.
[55] Obelix collaboration, work in progress.