



fast timing with slow scintillators

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what are the techniques?

- ✍ coincidence
- ✍ delayed coincidence
- ✍ time of flight

what are the applications?

- ✍ massive-gamma-ray detectors
- ✍ neutron time of flight



fast scintillators

plastic and liquid

decay time

1-2 ns

intermediate scintillators

YAP(Ce), LaCl₃(Ce)

20 – 40 ns

slow scintillators

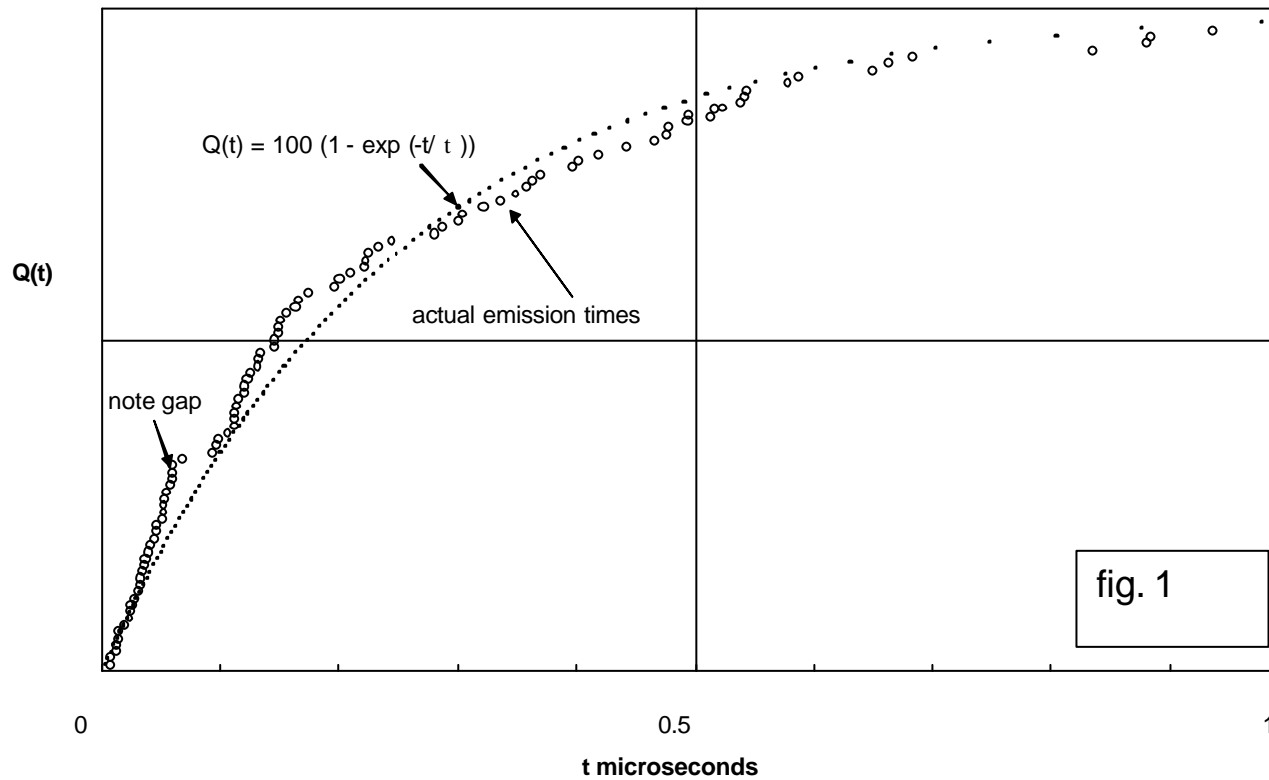
NaI(Tl), CsI, BGO and CdWO₄

0.25 – 5μs

Time distribution of photon emission from scintillators

$$f'(t) = \frac{dQ(t)}{dt} = R \exp(-t/\tau) \quad \dots(1)$$

$$f(t) = Q(t) = R(1 - \exp(-t/\tau)) \quad \dots(2)$$



The probability distribution for the arrival time of the Qth photon, between t and $t + dt$, is a three-part process.

$$P_1(Q, R, ?, t) = p_a \cdot p_b \cdot p_c$$

If R is fixed (no statistical fluctuation)

$$p_a = C(R, Q-1) \cdot \int_0^t e^{-\lambda t} dt^{Q-1}$$

$$p_b = C(R-(Q-1), 1) e^{-\lambda t} dt$$

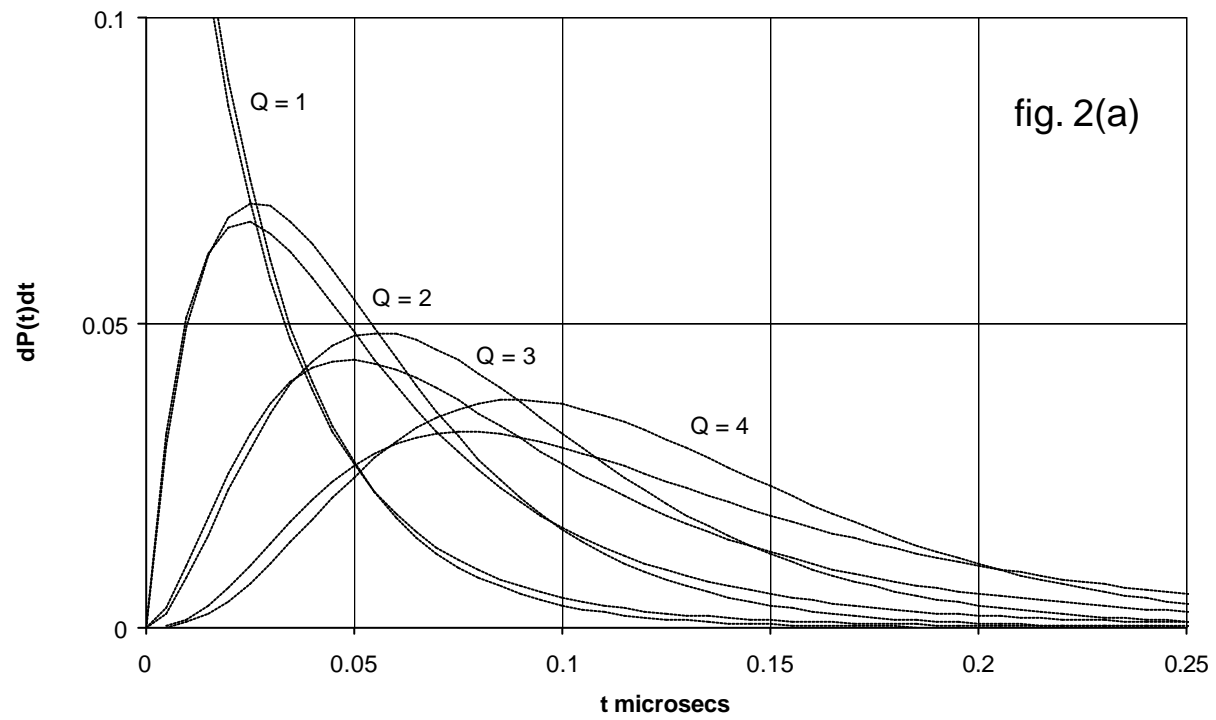
$$p_c = \int_t^{\infty} e^{-\lambda t} dt^{R-Q}$$

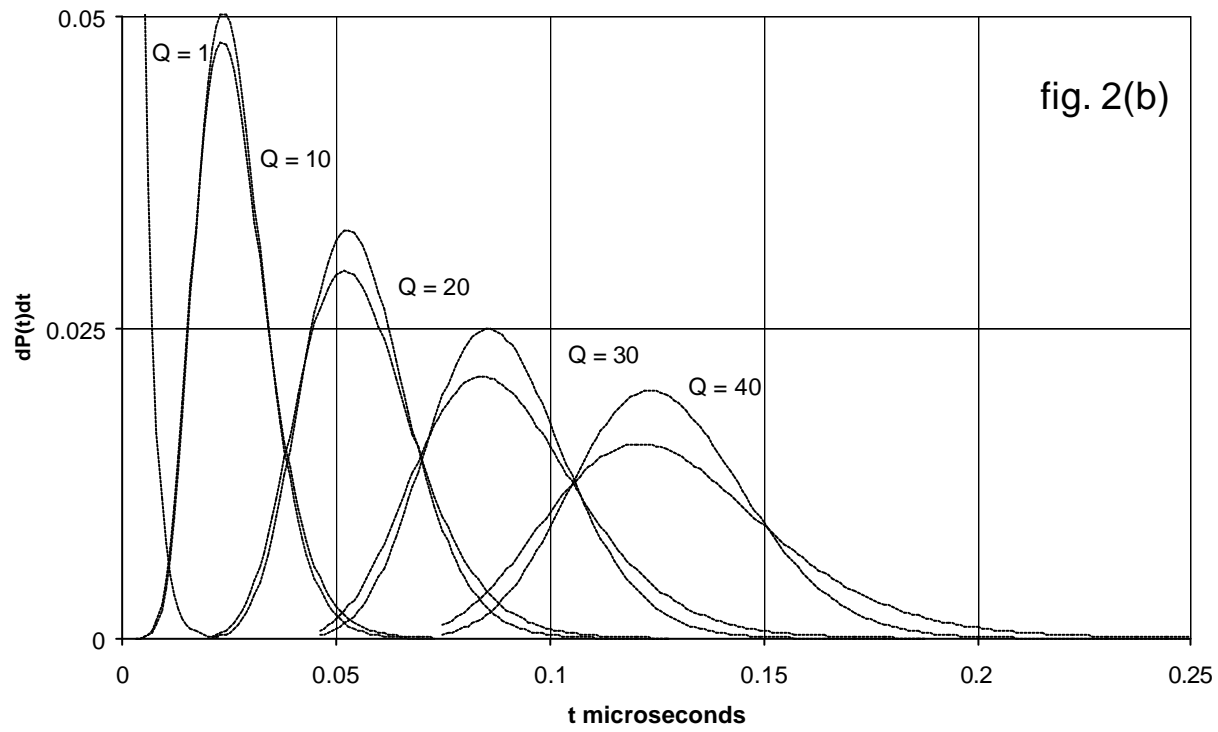
$$P_1(Q, R, ?, t) = \frac{R! (1 - e^{-\lambda t})^{Q-1}}{(Q-1)! (R-(Q-1))!} (R - (Q-1)) e^{-\lambda t} dt (e^{-\lambda t})^{R-Q} \quad \dots(3)$$

$$P_2(Q, R, ?, t) = \frac{R^Q}{(Q-1)!} \lambda \exp(-R(1 - e^{-\lambda t})) \cdot (1 - e^{-\lambda t})^{Q-1} e^{-\lambda t} dt \quad \dots(4)$$

The time distribution for the arrival of the first photon is

$$P_2(1, R, ?, t) \sim R \lambda \exp(-(R+1)\lambda t) dt \quad \dots(5)$$

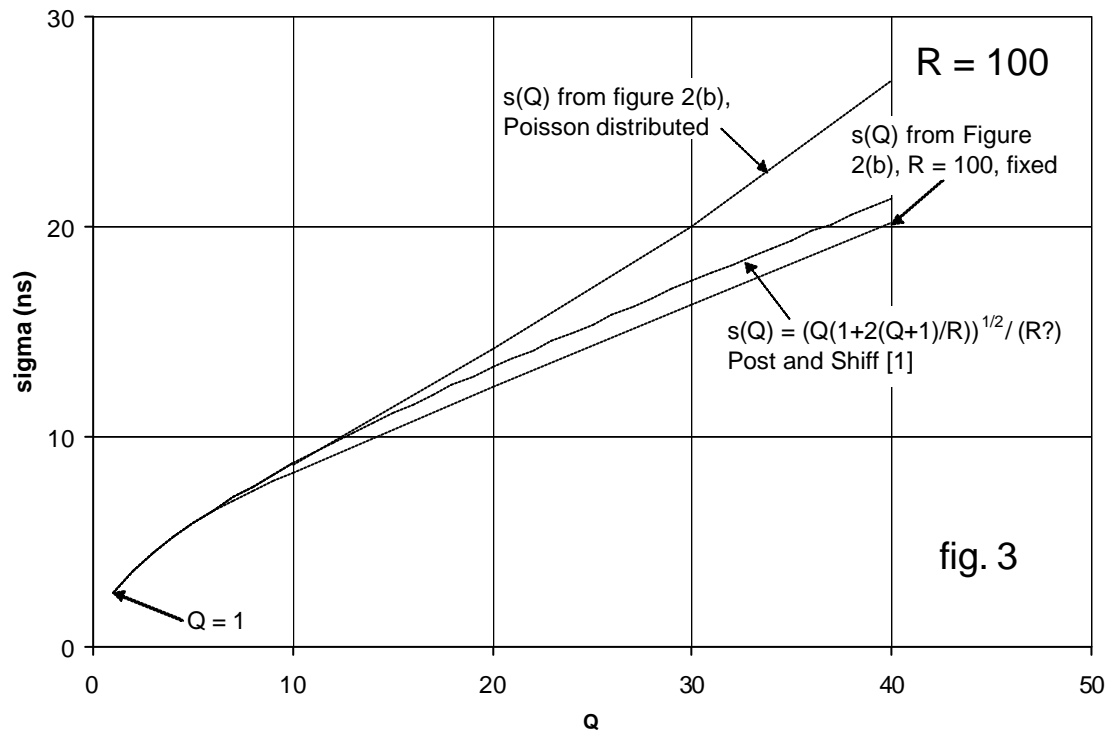




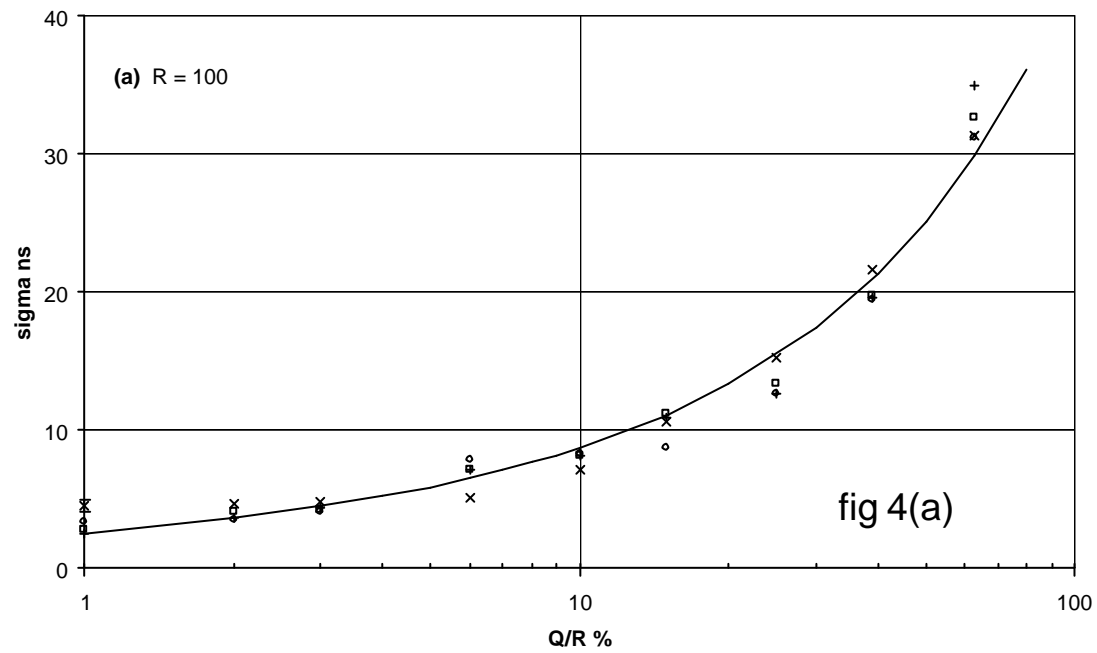
The variance of $P(Q, R, t, t)$

$$\text{var}_2(Q) = \frac{Q t^2}{R^2} (1 + 2(1 + Q)/R + \dots) \quad \dots(6)$$

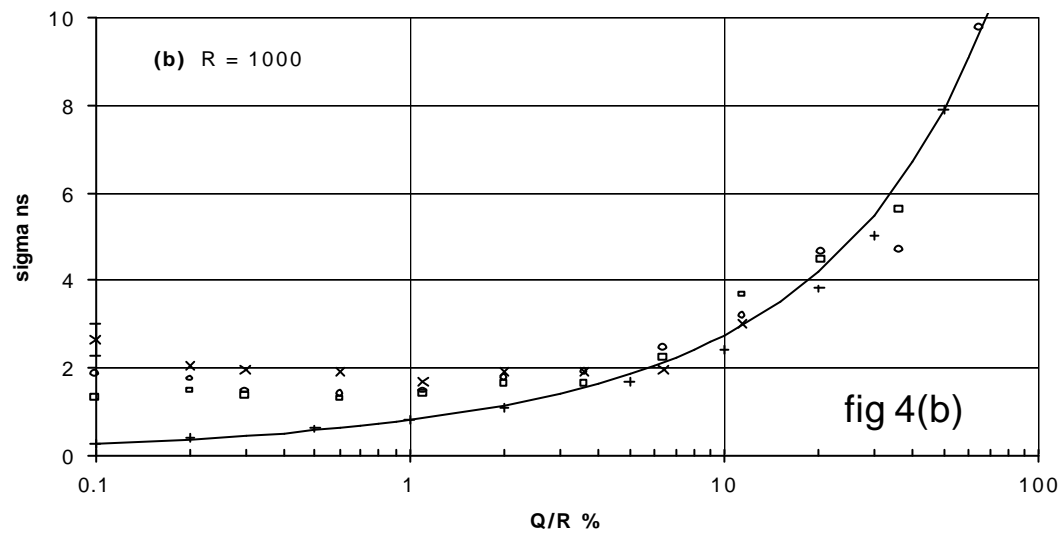
subject to R large and $Q/R \ll 1$, whatever that means



The effect of photomultiplier transit time jitter, e_{ph}



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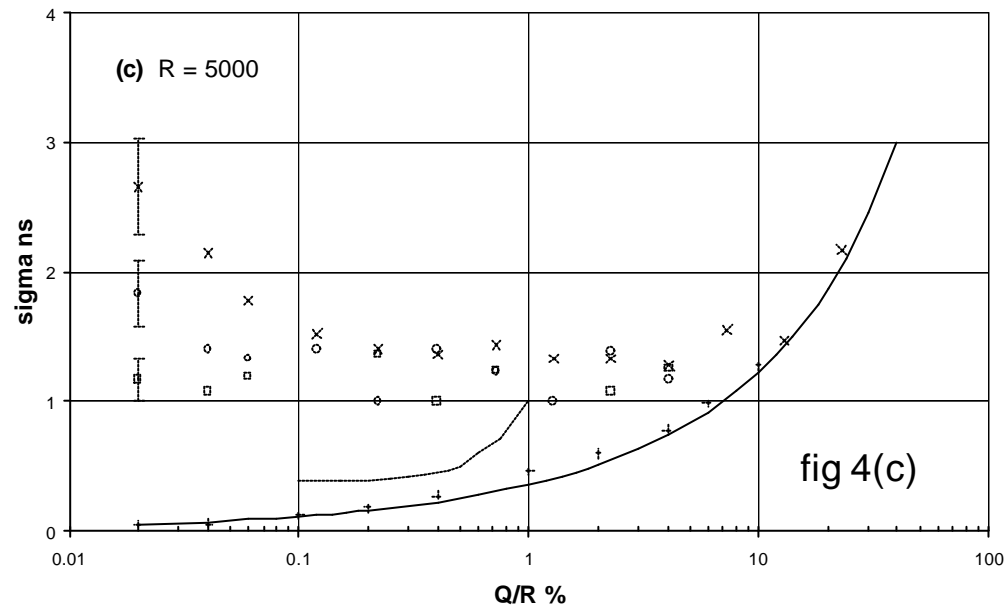
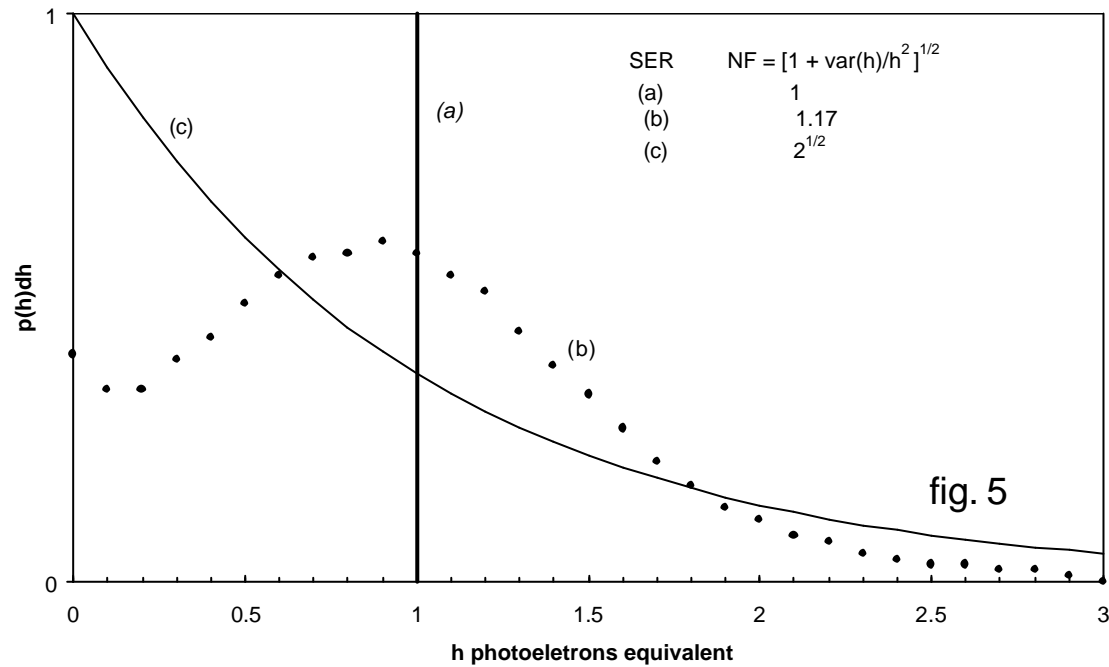


figure 4 $s(Q)$ determined directly from (4), +, with the predictions of (6), solid line. The timing fidelity is impaired once transit time jitter is introduced. \circ : $e_{ph} = 1 \text{ ns}$; \square : $e_{ph} = 2.5 \text{ ns}$; \times : $e_{ph} = 5 \text{ ns}$. The sporadic error bars give an indication of the precision

The effect of noisy gain



$$NF = [1 + e_A]^{1/2} \quad \dots(7)$$

where $e_A = \text{var}(h)/\langle h \rangle^2$, represents the gain dispersion, which can be calculated directly from the SER

The signal forming process at the anode

Each photoelectron produces a signal at the anode of

$$i(t) = q/t_s \exp(-t/t_s) \quad \dots(7)$$

where $q = e\langle g \rangle$

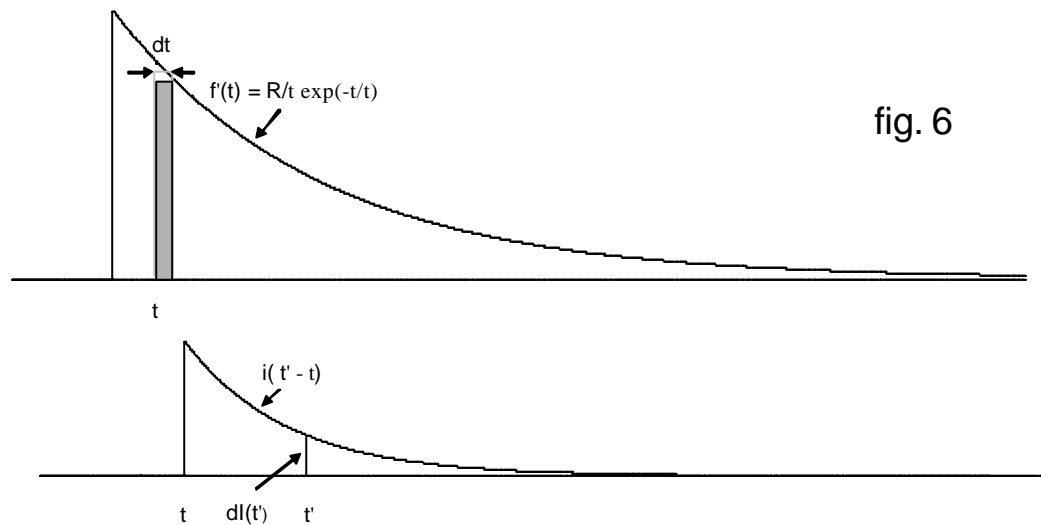
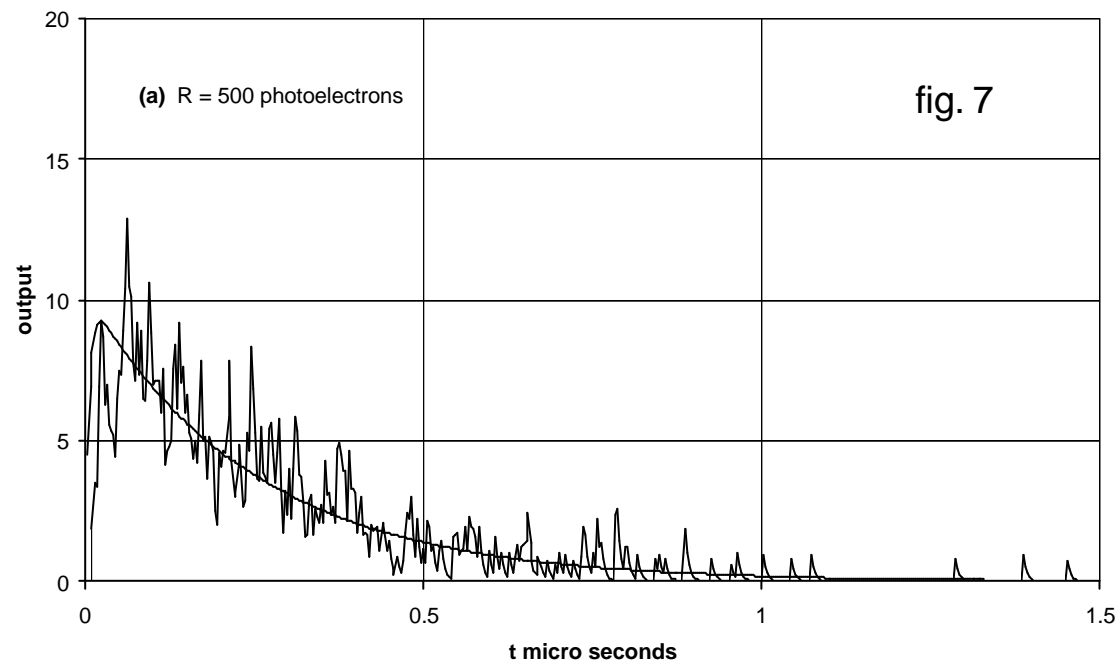


fig. 6

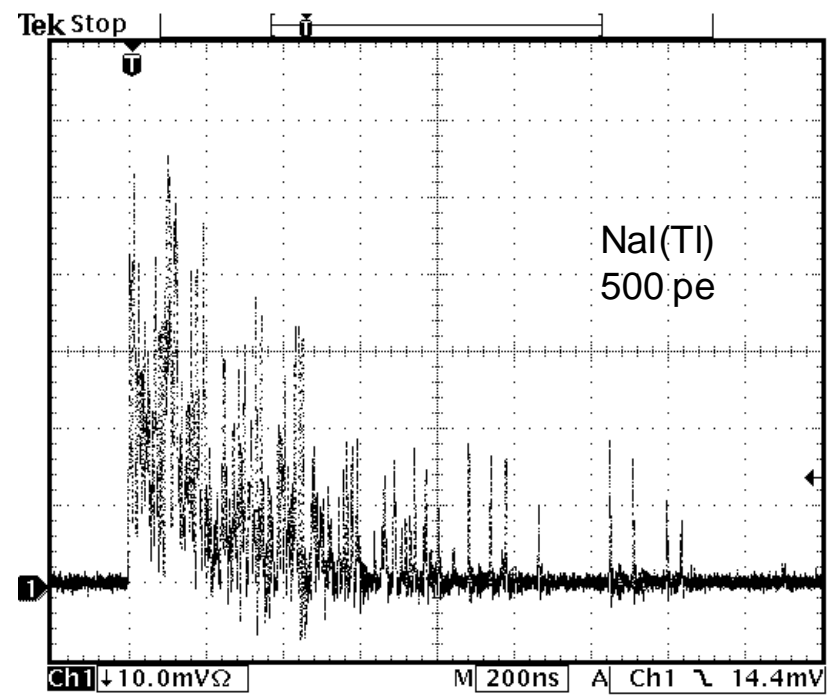
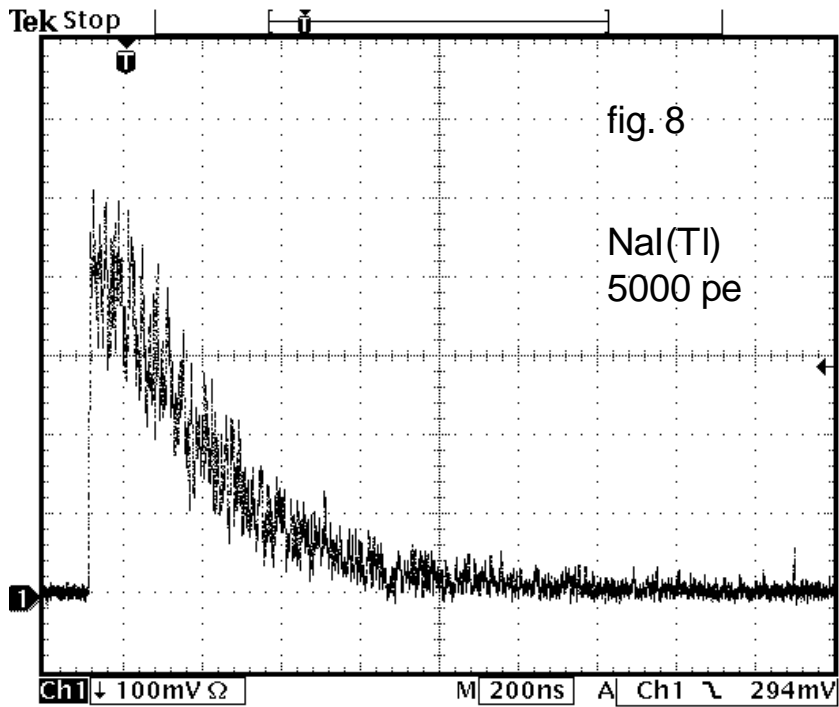
$$I(t') = \int_0^{t'} R \exp(-t/t_s) \cdot q \exp(-t_s(t'-t)) dt$$

$$I(t) = \frac{Rq}{(t - t_s)} [\exp(-t/t) - \exp(-t/t_s)] \quad \dots(8)$$

Monte Carlo simulation of output pulses allowing for noisy gain and jitter e_{ph}



Actual NaI(Tl) pulses



The signal beyond the anode with time constant $R'C'$

$$I(t) = \frac{Rqt}{(t - t_s)} \frac{1}{t} \exp(-t/t) - \frac{Rqt_s}{(t - t_s)} \frac{1}{t_s} \exp(-t/t_s)$$

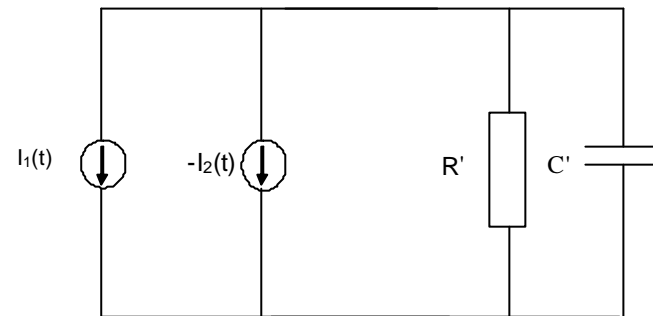
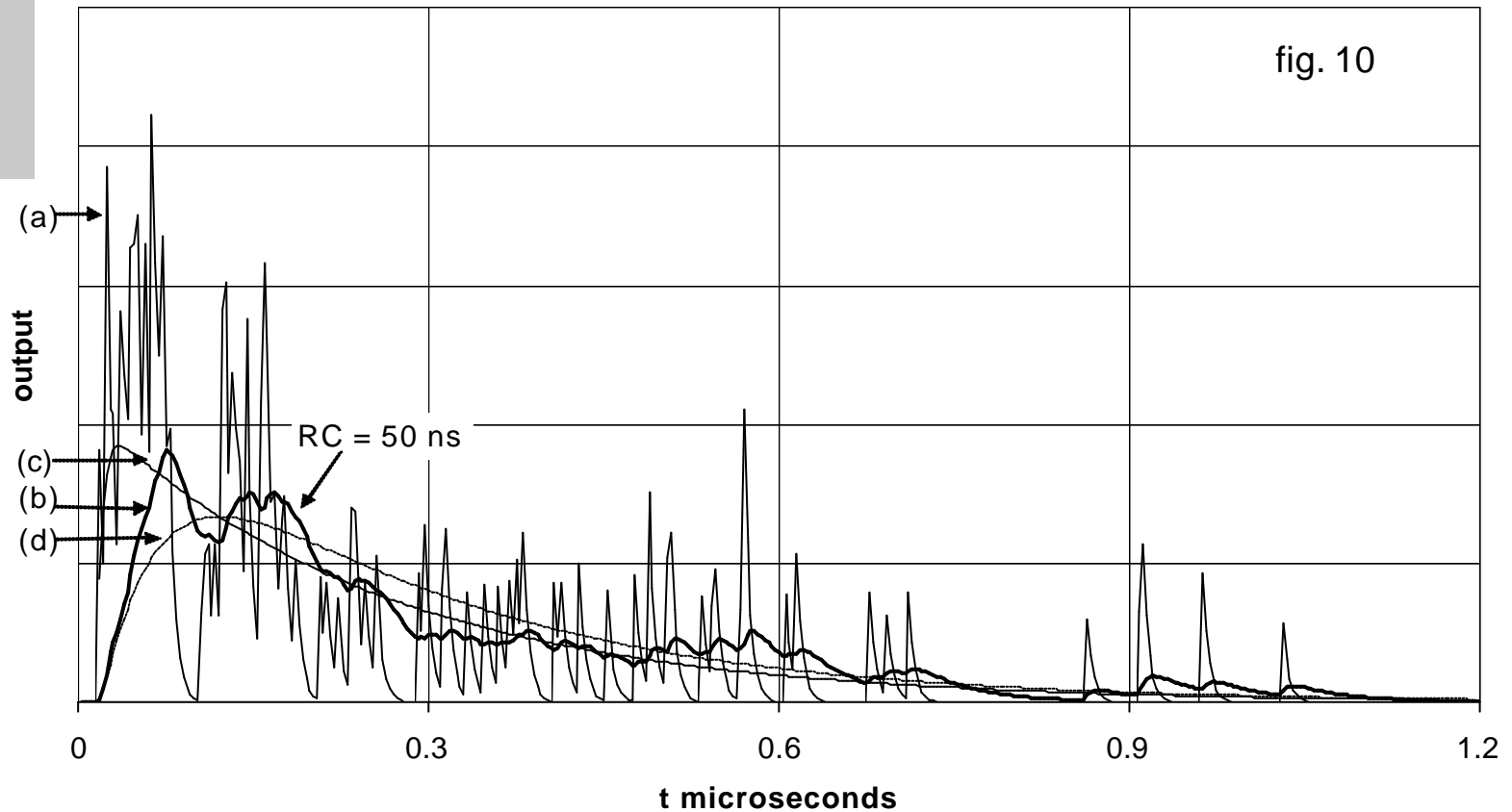


fig. 9

$$v_o(t) = \frac{R'qR}{(t - t_s)} \frac{t}{(t_0 - t)} (\exp(-t/t) - \exp(-t/t_0))$$

$$- \frac{R'qR}{(t - t_s)} \frac{t_s}{(t_0 - t_s)} (\exp(-t/t_s) - \exp(-t/t_0)) \dots(9)$$



- a) $R = 100, t = 250 \text{ ns}, t_s = 5 \text{ ns}$ and $e_{ph} = 5 \text{ ns}$.
- b) is the result of smoothing (a) with time-constant $R'C'$.
- c) is the average of many repeat traces of type (a)
- d) is a plot of equation (9) with $R'C' = 50 \text{ ns}$.

Note that the area under all curves is the same.

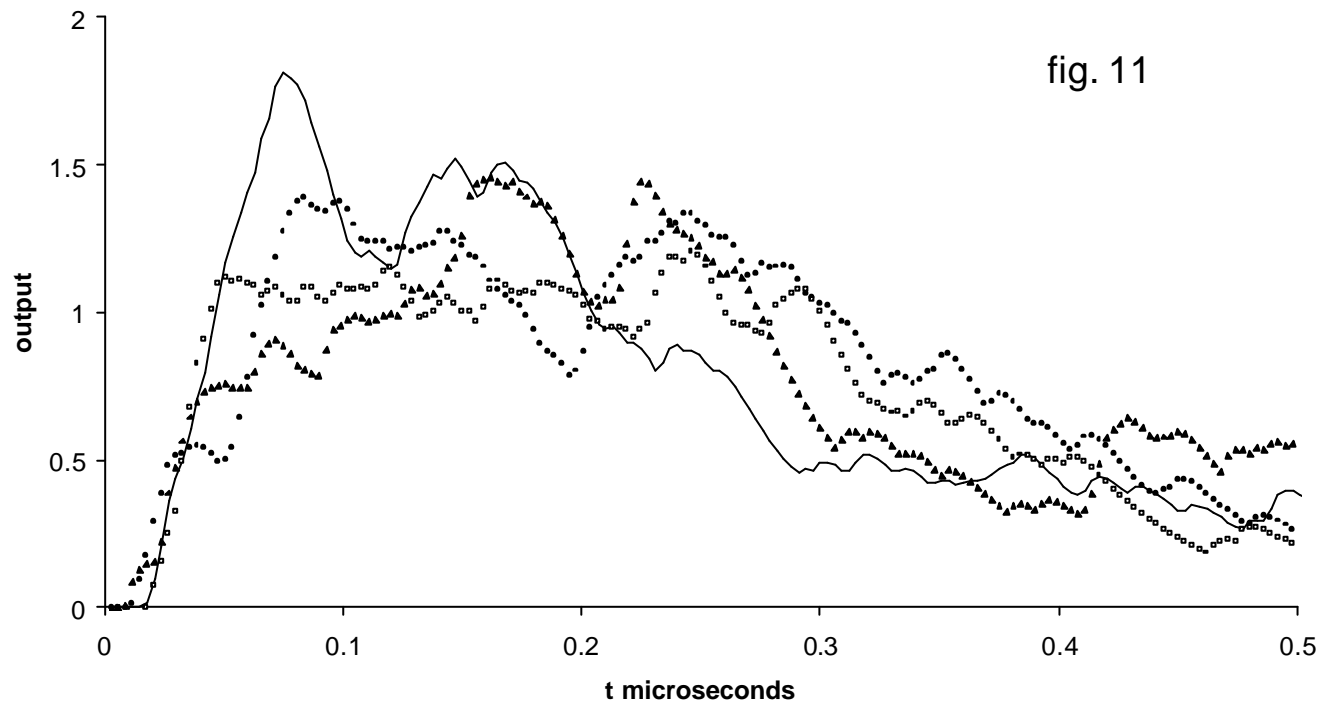
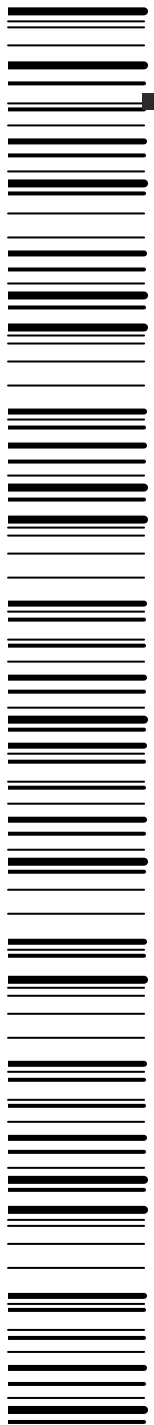
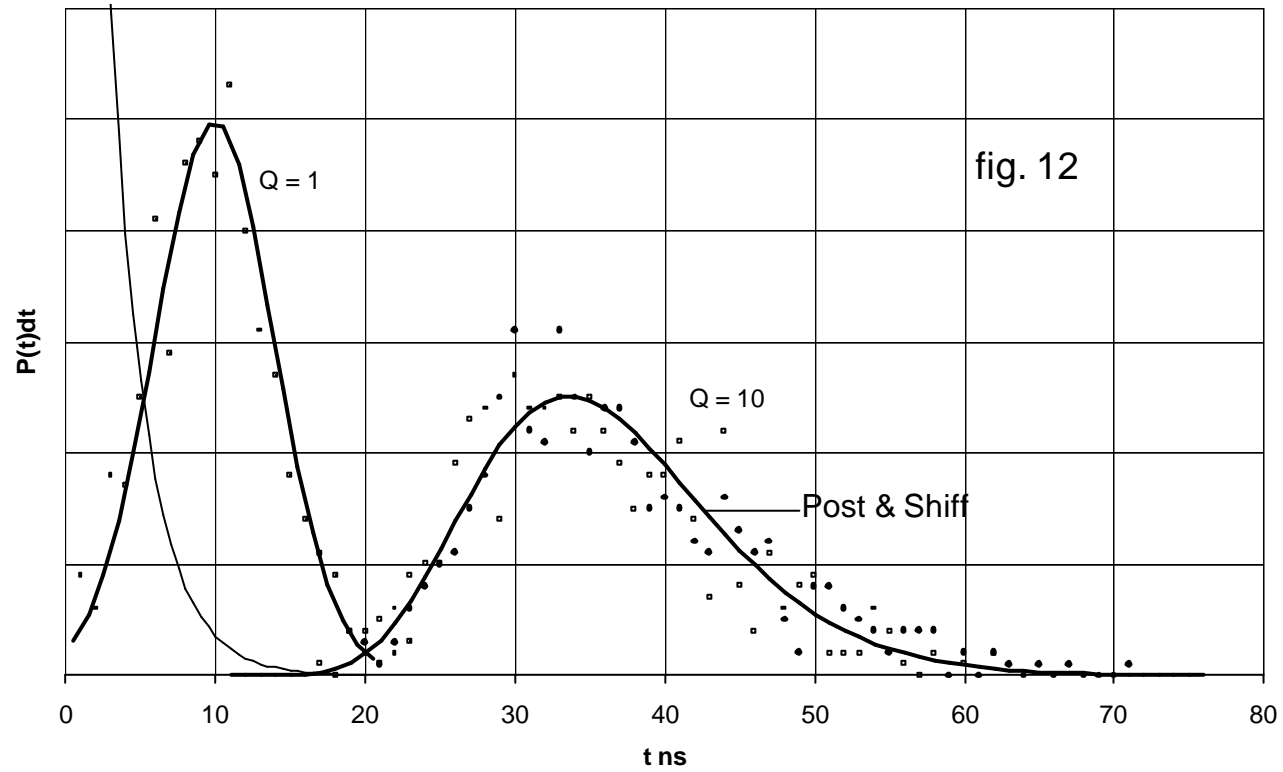
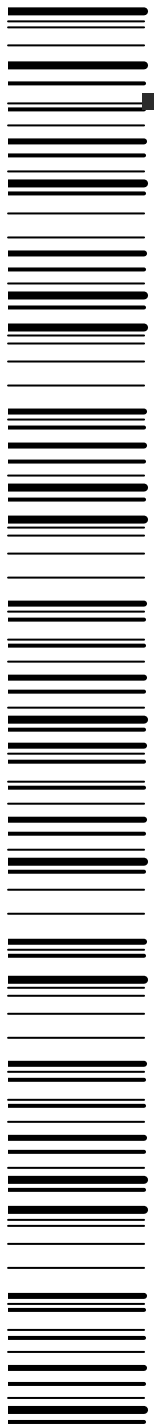
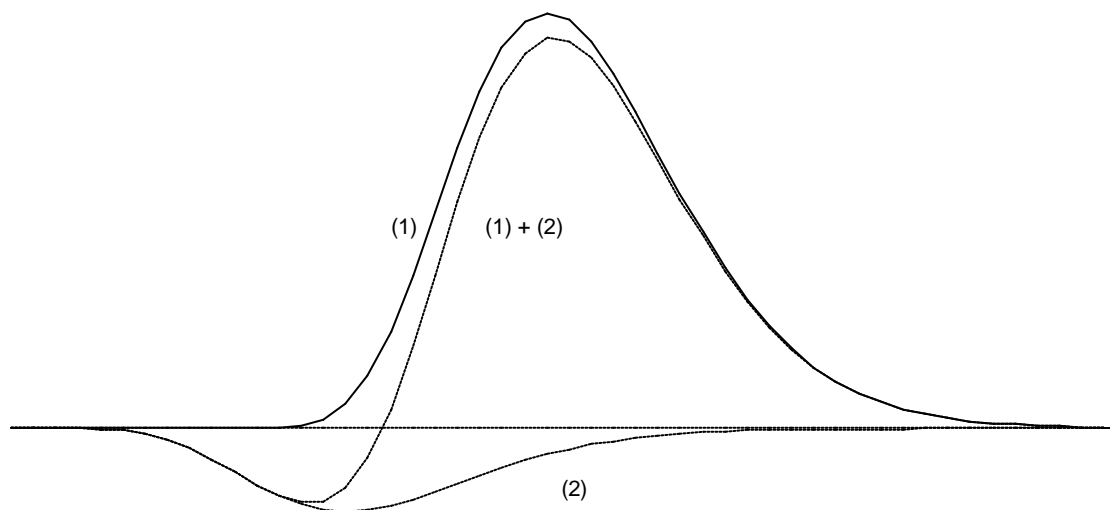
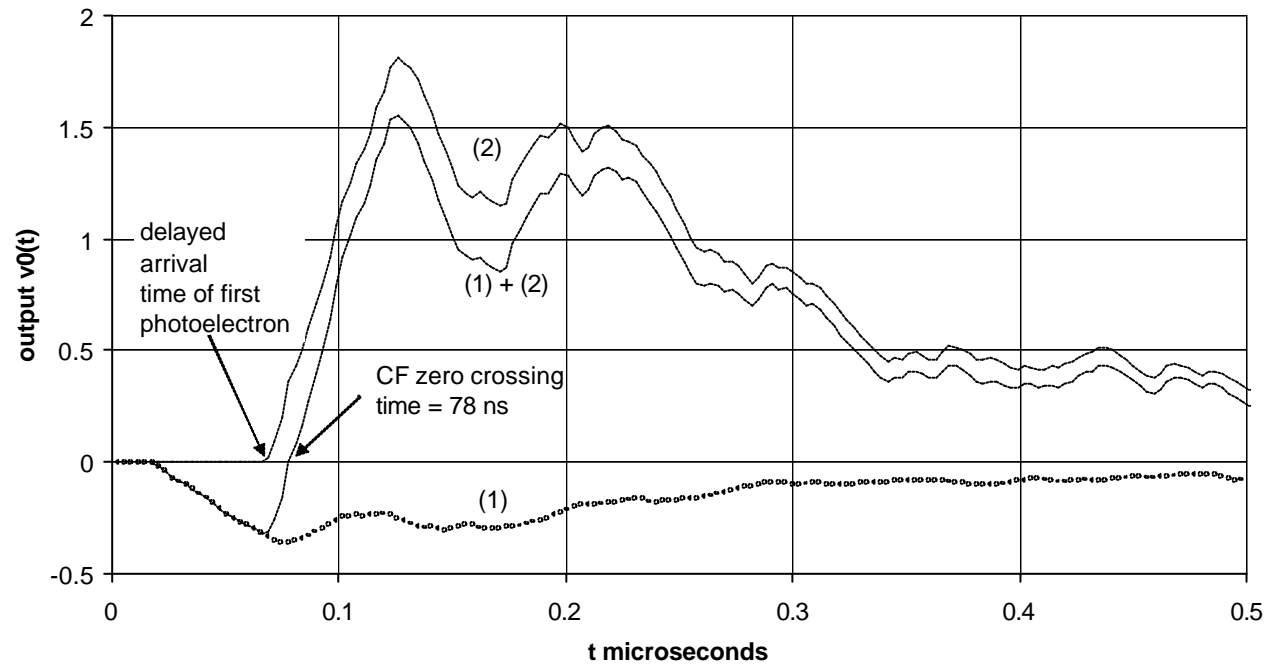


fig. 11



Threshold and constant fraction discrimination





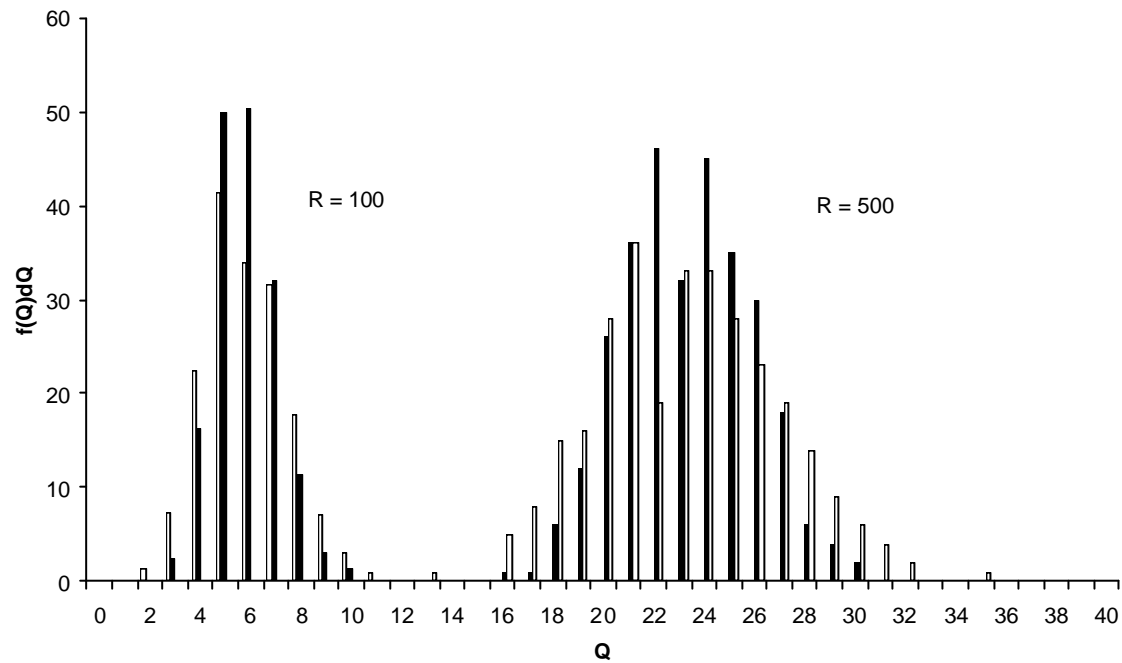
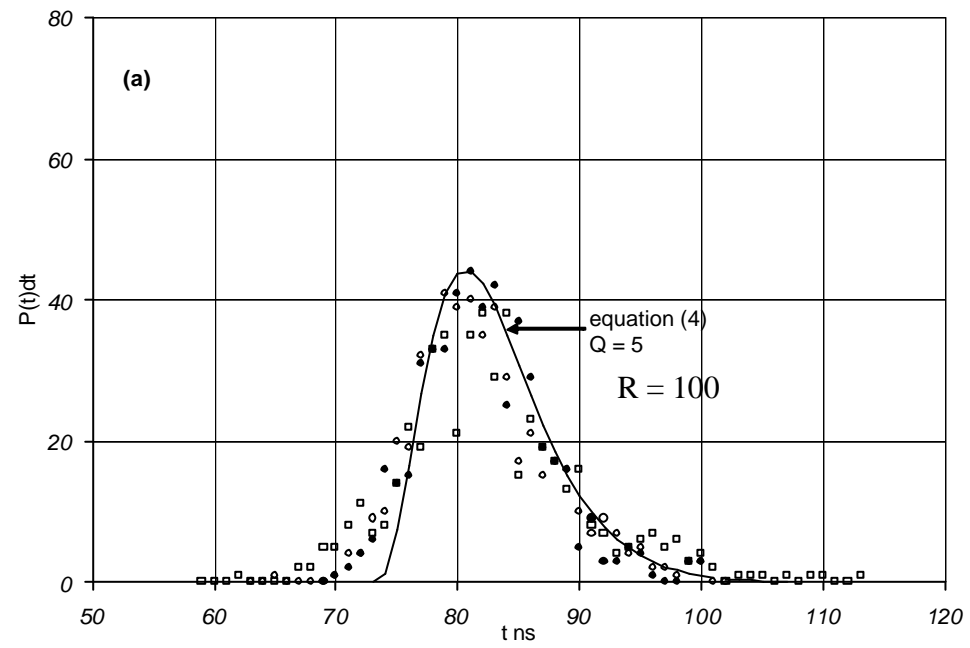


Figure 18 The number of photoelectrons, Q , required to trigger a CFD (20% , 50 ns delay). ; noiseless gain ($e_A = 0$) ; ? SER taken as Figure 7(b) ($e_A = 0.37$)



Scintillator timing characteristics and other features

scintillator	?	t, ns	ph/keV	R/t (FOM)
BC418	1.03	1.4	8	5.7
BC400	1.03	2.4	8	3.3
LaCl ₃	3.79	28	49	1.8
LuAP	8.30	18	20	1.1
YAP(Ce)	5.55	28	7	0.25
NaI(Tl)	3.67	250	40	0.16
CsI(Tl)	4.51	1000	60	0.06
BGO	7.13	300	8	0.03
CdWO ₄	8.00	5000	14	0.003



Conclusions

- 1) fast timing with slow scintillators is possible if R/τ sufficiently large
- 2) although $t \gg e_{ph}$ pmt jitter is important at low Q values
- 3) a poor SER degrades timing under all conditions, directly as NF
- 4) smoothing is necessary to prevent multiple triggering
- 5) with CF technique smoothing under certain conditions does not degrade performance.
- 6) ultimately resort to experimentation with regard to smoothing, threshold, delay and trigger fraction



photomultiplier requirements

high QE
collection efficiency
SER
low jitter

catch the light



solutions in light detection