Mass and width of the W boson at e^+e^- colliders

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Studying the properties of the W boson plays a key role in precision tests of the Standard Model. In this paper, the measurements performed at LEP are reviewed. The current world knowledge of the W boson mass and width are presented.

1. Introduction

At tree level, the electroweak sector of the Standard Model is determined by three parameters, which are the gauge couplings to the $SU(2)\otimes U(1)$ fields, and the vacuum expectation value of the Higgs field. Due to the presence of higher-order radiative corrections, the simple tree-level predictions of the Standard Model are modified, predominantly through loop corrections, such that additional parameters are needed to fully describe the theory.

The mass of the W boson is predicted to be 80.373 ± 0.023GeV from precision electroweak measurements [1], and provides indirect knowledge on the Higgs mass through the formula:

$$M_{\mathrm{W}}^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_{\mathrm{W}} G_F}(1+\Delta r),$$

where Δr are higher-order radiative corrections related to the Higgs mass. Hence, a precise measurement of M_{W} is a powerful test of the Standard Model and allows predictions of the Higgs mass.

The first precision measurements of the mass and width of the W boson were performed at $\mathrm{p\bar{p}}$ colliders. The Tevatron collider at Fermilab ran at 1.8 TeV during 1992 – 1995 (Run I) delivering colliding beams to the D0 and CDF collaborations. With a very large production rate of $\sigma_{WX} \cdot \mathrm{Br}(\mathrm{W} \to \ell\nu_\ell) \approx 2.4$ nb, the Run I total W sample exceeded 200,000 events. Using this sample, CDF and D0 combined achieve [2]: $M_{\mathrm{W}} = 80.454 \pm 0.060$ GeV.

Between 1996 and 2000, the LEP collider was operated at centre-of-mass energies above the $\mathrm{W^+W^-}$ production threshold (LEP2), allowing for the first time direct measurements of the W boson mass. The process $\mathrm{e^+e^-} \to \mathrm{W^+W^-}$ can be identified with high efficiency in all decay modes of the W boson. The four LEP collaborations (ALEPH, DELPHI, L3, and OPAL) collected a total sample of around 35,000 W pairs in the 2.7 inverse fb of data recorded.

The sample of W bosons collected at LEP2 is significantly smaller than that collected at the Tevatron, but one benefits of a clean $\mathrm{e^+e^-}$ collision environment allow more information to be extracted from each recorded event. The well defined initial state and a good knowledge of the beam energy allows the use of constrained kinematic fitting to greatly improve the mass resolution on the reconstructed W boson.

The final goal of LEP2 is to measure M_{W} with a precision of about 30-35 MeV. An uncertainty below 30 MeV will not improve the knowledge on the Higgs mass, as the limiting factor is the current experimental precision of the top mass (M_{t}).

There are two main methods to measure M_{W} at an e^+e^- collider. The mass of the W can either be obtained directly from the reconstruction of the W decay products or derived from the $\mathrm{W^+W^-}$ threshold cross section.

2. Direct reconstruction of W mass

For most of the time, LEP2 has operated at energies significantly above the $\mathrm{W^+W^-}$ threshold, where the $\mathrm{e^+e^-} \to \mathrm{W^+W^-}$ cross section has little sensitivity to M_{W}. For these higher energy data

M_W was measured through the direct reconstruction of the W boson's invariant mass from the observed W decay products on an event-by-event basis.

$\mathrm{W^+W^-}$ events are classified into three final states, according to their decay modes. $\mathrm{W^+W^-} \rightarrow \mathrm{q\overline{q}q\overline{q}}$ events comprise 45% of the total $\mathrm{W^+W^-}$ cross-section and are characterised by four energetic jets of hadrons with little or no missing energy. Semi-leptonic $\mathrm{W^+W^-} \rightarrow \mathrm{q\overline{q}}\ell^\pm\nu_\ell$ decays comprise 44% of the total $\mathrm{W^+W^-}$ cross-section and are characterised by two distinct hadronic jets, a high-momentum lepton and missing momentum due to the prompt neutrino from the leptonic W decay. The $\mathrm{W^+W^-} \rightarrow \ell^+\nu_\ell\ell'^-\overline{\nu}_{\ell'}$ channel, with at least two unobserved neutrinos and a relatively low branching fraction, has limited M_W sensitivity and is not discussed herein.

The invariant masses of the two W bosons are determined directly from the reconstructed momenta of observed decay products. Hadrons are grouped together into jets using clustering algorithms such as $k_\perp$. In $\mathrm{qq}\ell\nu$ events, charged leptons are identified and neutrinos are inferred from the missing energy and momentum.

Experimentally, the limiting factor in the mass resolution is the uncertainty in the jet energy measurement, which is poor in contrast to the measured jet directions. As the centre-of-mass energy is well known, the mass resolution can be improved significantly (factor ∼2–3) by use of a constrained kinematic fit imposing the four constraints of energy and momentum conservation. In some analyses, an additional fifth constraint forcing the two W boson masses to be equal is also imposed.

For the semi-leptonic $\mathrm{q\overline{q}}\ell^\pm\nu_\ell$ final states, there is a neutrino which is not measured, leaving a two constraint fit after the equal mass constraint is applied. In these events, most of the mass information is given by the hadronically decaying W.

For the fully hadronic qqqq final states, there is no missing momentum and a full four or five constraint fit is possible depending upon whether the equal mass constraint is used. The fully hadronic qqqq final state has the additional complication of a three-fold ambiguity in pairing the observed jets to the underlying W bosons.

In addition, as quarks may radiate energetic gluons leading to a distinct five-jet topology, several analyses force some or all events to be reconstructed as five-jets to improve the overall mass resolution.

The reconstructed mass spectrum looks rather different from a pure relativistic Breit–Wigner distribution for several reasons. For example, the presence of initial state radiation (ISR) means that the energy producing the W pairs is always less than twice the incoming beam energy. This causes a tail towards higher invariant masses, as the collision energy assumed in the kinematic fit is overestimated. The detector resolution also tend to significantly smear out the lineshape as the experimental resolution is not significantly better than the W boson width for most channels. As a result, the W boson mass cannot be extracted by simply fitting an analytic Breit–Wigner shape, but all extraction methods need to be calibrated on a Monte Carlo simulation which includes all the various effects to model the dependence of the spectrum on M_W. Most of the systematic errors associated to M_W account for effects which may be missing in this Monte Carlo. Figure 1 shows some mass spectra compared to the Monte Carlo predictions.

As extraction methods, DELPHI and OPAL use a convolution technique to construct a likelihood $\mathcal{L}(\mathcal{M}_\mathrm{W})$ for each event. The other two collaborations use re-weighting techniques to construct Monte Carlo templates at arbitrary M_W to be fit to the data distributions.

The most important uncertainties for the LEP W mass measurement are related to the modelling of the detector response, modelling the hadronization of quarks into jets, understanding the LEP beam energy, and the final state interactions. To limit the uncertainty from detector modelling, the LEP collider was run from time to time throughout each running year at the $\mathrm{Z^0}$ resonance to take large samples of $\mathrm{Z^0}$ decays. These data, as well as similar samples at high energy, are used to study the detector response to leptons and jets and limit the deficiencies in the detector models.

Hadronization uncertainties are estimated by

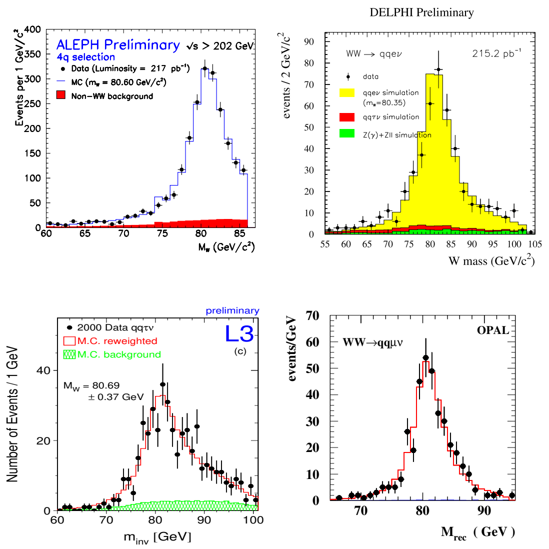

Figure 1. Reconstructed mass spectra from the LEP collaborations.

comparing different Monte Carlo implementations of the hadronization process. Other approaches include re-weighting key variables in Monte Carlo to correspond to data and propagating the effect to M_{W}. DELPHI also use the so-called “mixed Lorentz boosted Z^0”.

The relative uncertainty in the LEP beam energy enters directly into the uncertainty in M_{W}, due to the use of kinematic fits which include the constraint of the centre-of-mass energy. Uncertainties in beam energy are taken from the extrapolation to hight energy the result of the resonant depolarisation measurement, which is done for beam energies below 60 GeV [3].

A significant bias in the apparent W mass measured in the $\mathrm{q\bar{q}q\bar{q}}$ channel could arise if the hadronisation of the two W bosons is not independent and correctly modelled. Standard Monte Carlo models assume that the two systems decay independently. However, interactions that can exchange momentum between the two W systems could distort the final W lineshape.

Two specific phenomena are known to exist, but with rather uncertain strengths: colour reconnection (CR) [4] and Bose–Einstein correlations (BEC) [5]. Effects on the mass of such interactions are estimated by using phenomenological models and direct searches for these effects may limit the viable set of such models.

In $\mathrm{q\bar{q}q\bar{q}}$ events, the products of the W decays in general have a significant space-time overlap as the separation of their decay vertices is small compared to characteristic hadronic distance scales. Colour reconnection refers to a rearrangement of the colour flow between the two W bosons. The effects of interactions between the colour singlets during the perturbative phase are expected to be small. The situation is less clear in the non-perturbative phase, where phenomenological models are implemented in hadronic Monte Carlos. A higher susceptibility to CR (and more $\mathrm{Z}^0 \rightarrow \mathrm{q\bar{q}}$ background) is expected when W^+ and W^- hadronisation regions overlap, so the space-time picture of the QCD shower development is important.

The predicted (barely) observable effects of CR include changes to the charged particle multiplicity, momentum distributions and the particle flow relative to the 4-jet topology.

Colour reconnection effects tend to enhance or suppress particle production in the regions between the main jets. Currently, all four LEP collaborations are pursuing analyses aimed at measuring the particle flow distribution in qqqq final states with the ultimate aim of discriminating between various CR models. With all four LEP experiments combined, it is likely that the uncertainty from CR can be significantly reduced from its current value.

Bose-Einstein correlation leads to the enhanced production of identical boson pairs, such as $\pi^+\pi^+$ or $\pi^-\pi^-$, at small 4-momentum difference, $Q^2 = -(\mathbf{p}_1 - \mathbf{p}_2)^2$. This phenomena is firmly established in hadronic Z^0 at LEP1 and between the particles of a single W boson at LEP2, among others. Since the W boson decay length (0.1 fm) is significantly shorter than the hadronization scale (1 fm), it is entirely plausible that there can be additional BE effects between particles originating from different W bosons in qqqq events.

Traditionally, BEC is studied using a 2-particle correlation function:

$$R(\mathbf{p}_1, \mathbf{p}_2) = \rho_2(\mathbf{p}_1, \mathbf{p}_2)/\rho_0(\mathbf{p}_1, \mathbf{p}_2),$$

where ρ_2 and ρ_0 are 2-particle densities with and without BEC, respectively. One serious problem in this area is the construction of the reference sample, ρ_0, in a model independent way. Using the technique described in [6] it is possible to construct such a sample (apart from background subtraction). This method involves mixing pairs of data events, such as the hadronically decaying W in qq$\ell\nu$. Data from two semi-leptonic qq$\ell\nu$ events are mixed (without the lepton) and compared to data from genuine qqqq events. In a rigorously model–independent test, these two samples should look identical if there is no BEC present between the decay products of different W bosons.

All four LEP collaborations are currently using this mixing method and see no evidence for inter-W BEC effects at the limits of their statistical power to observe them. By combining these results and evaluating a range of BEC models, the systematic uncertainty from this source should also be reduced for the final LEP results.

The big problem in combining the LEP results is the ignorance of non-perturbative QCD amplitudes. Because of the way Monte Carlo generators operate, modelling purely quantum mechanical effects is extremely difficult, and imperfect knowledge of the space-time evolution of the hadronizing system adds to the difficulty[5].

The strategy of the LEP experiments to address these difficult questions is to use the various models as a guide for the types of effects which can be expected, but to constrain the allowed magnitude of any final state interaction effect using the data directly.

The four LEP collaborations combine their results taking into account systematic uncertainties which are correlated between channels, experiments, and years of LEP running. This combination procedure is still evolving, as better information about the nature of these various correlations becomes available.

One way to crudely check that the CR and BEC effects are not underestimates in the M_{W} analyses is to compare the W mass measured in the qqqq channel to that measured in the qq$\ell\nu$ channel. Including all uncertainties with their correlations aside from those related to CR and BEC directly, the LEP collaborations find [7]

$$M_{\mathrm{W}}(\mathrm{qqqq}) - M_{\mathrm{W}}(\mathrm{qq}\ell\nu) = +9 \pm 40\,\mathrm{MeV},$$

which is consistent with the quoted final state interactions systematics.

At present, the preliminary combined LEP W mass result from direct reconstruction for is [7]:

$$M_{\mathrm{W}}(\mathrm{LEP}) = 80.447 \pm 0.039, \mathrm{GeV}.$$

As can be seen in the detailed breakdown of the direct measurements uncertainties shown in Table 1, the qqqq channel has rather large uncertainties associated with Bose–Einstein correlations and colour reconnection. Due to these uncertainties, the qqqq channel carries a weight of only 27% in the combined result, even though it is statistically more precise. If there were no sources of systematic uncertainty, the combined statistical error would be 22 MeV.

Table 1
Uncertainties on the LEP combined W mass.

Uncertainty	qq$\ell\nu$ (MeV)	qqqq (MeV)	Combined (MeV)
ISR/FSR	8	9	8
Hadronisation	19	17	17
Detector	12	8	10
LEP Beam Energy	17	17	17
Colour Recon.	–	40	11
Bose-Einstein	–	25	7
Other	4	4	3
Total Syst.	29	54	30
Statistical	33	30	26
Total	44	62	40

3. W Mass from the WW cross section

Near the WW threshold the WW production cross section is sensitive to the value of M_{W}. The

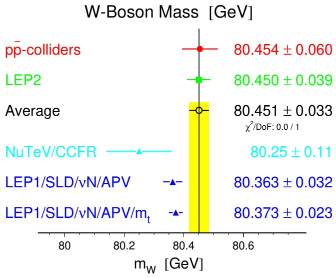

Figure 2. World results on the mass of the W boson.

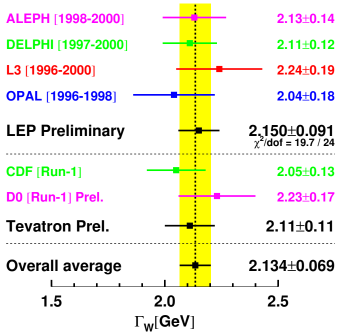

Figure 3. World results on the width of the W boson.

optimal point is at 161 GeV centre-of-mass energy, which was chosen as one of the point for running LEP2 for this reason. The advantages of this method are that all W decay modes can be used and that the systematics from final state interactions, as colour reconnection or Bose-Einstein correlations, are not important for this measurement. At LEP2 only 10 pb^{-1} of data per experiment have been recorded at threshold for W pair production. Combining the four LEP experiments [8]-[11] we get:

$$M_{\mathrm{W}}(\mathrm{threshold}) = 80.40 \pm 0.21 \text{ GeV}.$$

3.1. W mass world average

The current world knowledge of the W mass from LEP2 and the Tevatron is shown in Figure 2, along with the indirect value inferred from other precision electroweak measurements. While the current world average is dominated by LEP2, with the data expected at the Tevatron during Run IIa, the combined error on direct W mass measurements should approach 20 MeV by around 2006.

3.2. W width measurements

The direct reconstruction method employed at LEP2, as well as the transverse mass method employed at the Tevatron, are sensitive to the W width as well as the W mass. In the standard mass analyses, the width is typically fixed to the Standard Model expectation for a given mass value, but it can equally well be allowed to be a second free parameter in the fits. World measurements of Γ_{W} are shown in Figure 3. The world average of the W width agreed with the the Standard Model expectation. Not all LEP data has been analysed, however already now, the precision on the direct width measurements is competitive with the indirect width from the Tevatron.

4. Conclusions and Perspectives

The combined electroweak data is often summarized as shown in Figure 4. The first plot in this figure shows M_{W} versus M_{t} : the direct measurements, the indirect electroweak data, and the Standard Model prediction as a function of the Higgs mass. It can be seen that the precise input data predicts values of M_{W} and M_{t} consistent with those observed, demonstrating that electroweak correction can correctly predict the mass of heavy particles. It is observed that both input data and direct measurement of M_{W} and M_{t} favour a light Higgs. The direct measurements are starting to stray into a region more favourable for Supersymmetric theories [12]. It can also be seen from this plot that significant improvements in the uncertainty on M_{W} will not be very useful if they are not accompanied by comparable improvements in M_{t}. The second plot shows the variation of the minimum value of the χ^2 as a function of $\mathrm{M_H}$ for the full electroweak fit. The best-fit value of the Higgs mass is $\mathrm{M_H} = 88^{+53}_{-35}$GeV, where the error is asymmetric as the leading corrections depends on $\log \mathrm{M_H}$,

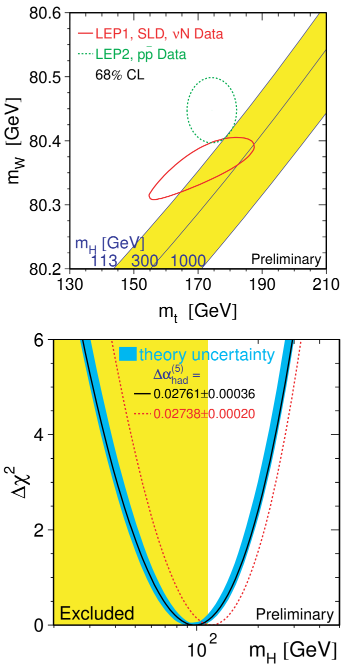

Figure 4. World combined electroweak results.

from which the constraint $M_H < 222$GeV at 95% C.L. can be derived.

In summary, LEP has produced remarkable improvements in the knowledge of the W boson mass and width. With around 650 pb^{-1} recorded per LEP experiment, the world's best measurements of the W boson mass and width are performed.

The next five years will see measurements of similar precision performed at the Tevatron with the advent of Run II. Further substantial improvement in precision will have to wait for the Large Hadron Collider and the future Linear Collider.

From LEP2 , we learn that to measure M_W with an error below 15-10 MeV at a Linear Collider, it is essential to control the detector systematics and to know the beam energy to few MeV. Preliminary studies indicate that a precision of 6 MeV can be reached using the threshold method. The possibility of direct measurement is under investigation, however the qqqq channel may not be used if final state interaction effects will not be modelled/known with a precision of few MeV [13].

The M_W value will continue to be an important input for the electro weak fit, and improving its knowledge is a crucial step to the eventual goal of fully understanding the process of electroweak symmetry breaking. However, further elucidation of the electroweak symmetry breaking mechanism will likely have to await an improved discovery reach for a Higgs boson.

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