Tests of the Standard Model in the τ sector at LEP

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The present state of knowledge of the τ sector and its properties is reviewed. Emphasis is placed on the measurements from LEP. Results from SLAC-SLD and b factories are also included where competitive.

1. INTRODUCTION

The LEP accelerator provided electron-positron collisions for four experiments: ALEPH, DELPHI. L3 and OPAL in the period 1989 through 2000. The clean environment provided by LEP for τ lepton production and decay provided an excellent laboratory for precision measurements. The results presented here are a mixture of published and preliminary results, and in general are the most up-to-date results published or presented at conference by the experimental collaborations.

The main production process at LEP is via the weak neutral current, in the process $e^+e^- \to Z \to \tau^+\tau^-$ at centre-of-mass energies near the Z resonance. The τ leptons decay via the weak charged current, to give final states which can include both leptons and hadrons.

Thus, at LEP, couplings of the τ to the Z can be measured via properties such as cross-sections and various asymmetry parameters. These permit evaluation of different properties of the neutral current, such as the vector and axial-vector components. Comparisons can be made with the other leptons produced in dileptonic decays of the Z boson. Limits can also be placed on weak dipole moments which would modify the $\tau\tau Z$ vertex.

For the weak charged current precision tests of the different lepton couplings are possible using various τ decays modes and measurements of the τ lifetime. τ decay spectra in fully leptonic τ decays permit a detailed study of the Lorentz structure in τ decays which can be compared with analogous measurements in muon decay.

In the Standard Model, the Lagrangian of the fermionic current for the fermion fields given by the doublet $\psi \equiv \binom{f}{f'}$ can be written as [1]

$$\begin{aligned}\mathcal{L} = \;& \bar{\psi}\left(i\partial\!\!\!/ - m_f - \frac{e\sin\theta_W H m_f}{2M_W}\right)\psi \\ & - \frac{e}{2\sqrt{2}\sin\theta_W}\bar{\psi}\gamma^\mu(1-\gamma_5)(T^+W_\mu^+ + T^-W_\mu^-)\psi \\ & - eq\bar{\psi}\gamma^\mu\psi A_\mu \\ & + \frac{e}{\sin 2\theta_W}\bar{\psi}\gamma^\mu(g_v - g_a\gamma_5)\psi Z_\mu. \end{aligned} \tag{1}$$

Here e is the electric charge, m_f the relevant fermion mass, H the scalar Higgs field, T^+ and T^- are the weak isospin raising and lowering operators, and the W_μ^+ and W_μ^- fields are linear combinations of the W_μ^1 and W_μ^2 fields with the mixing given by the Weinberg weak mixing angle θ_W. The first term gives the Higgs-fermion Yukawa coupling, the second is the weak isospin raising and lowering part responsible for the charged current while the third term describes the electromagnetic interaction. The fourth term describes the weak neutral current, mediated by the Z boson.

2. NEUTRAL CURRENT

In the neutral current component of Eqn. 1, the coupling constants g_v and g_a regulate the proportions of vector and axial-vector current respectively, and can be related to the weak mixing angle by the relations

$$g_v = I_3 - 2q\sin^2\theta_W, \quad g_a = I_3, \tag{2}$$

where I_3 is the third component of weak isospin; this has a value of $-\frac{1}{2}$ for leptons. These relations apply for point-like interactions and need to be modified to take account of higher order loop corrections which are often larger than the level of experimental precision obtained. These corrections are absorbed into the more experimentally useful quantities, the so-called "effective" couplings $\bar{g}_v$ and $\bar{g}_a$:

$$\bar{g}_v = \sqrt{\rho}(I_3 - 2q\sin^2\theta_{\mathrm{W}}^{eff,\mathrm{f}}); \quad \bar{g}_a = \sqrt{\rho} I_3, \qquad (3)$$

where the parameter $\rho = 1 + \Delta\rho$ contains electroweak corrections and is close to unity.

For $\mathrm{e^+e^-}$ annihilation, the observables which can be measured in Z production and decay fall into two general categories. The first category includes the cross-section for the process $\mathrm{e^+e^-} \rightarrow \mathrm{f\bar{f}}$, which is sensitive to the partial width Γ_f for Z decay fermion-antifermion pairs and hence to the sum of the squares of the vector and axial-vector couplings at the $Z\mathrm{f\bar{f}}$ vertex through the relation

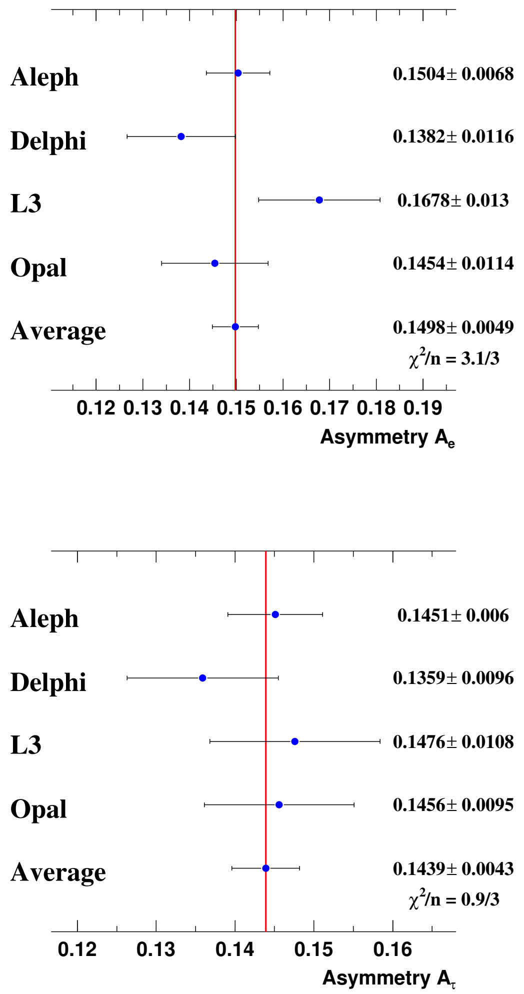
$$\Gamma_f = \frac{G_f m_Z^3}{6\sqrt{2}\pi}\left((\bar{g}_a^f)^2 + (\bar{g}_v^f)^2\right). \qquad (4)$$

The second category of observables contains these quantities which are sensitive to asymmetries in production of Z bosons and their decay. For simplicity, we introduce the asymmetry parameter $\mathcal{A}_f$ for the $Z\mathrm{f\bar{f}}$ vertex defined in terms of the vector and axial-vector couplings as

$$\mathcal{A}_f \equiv \frac{2\bar{g}_v^f/\bar{g}_a^f}{1 + (\bar{g}_v^f/\bar{g}_a^f)^2}. \qquad (5)$$

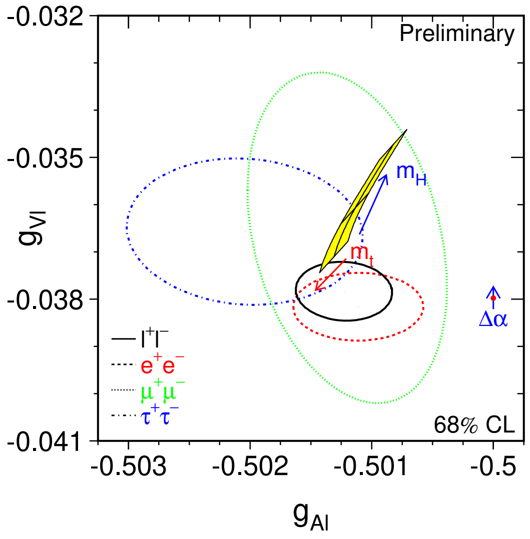
The parameter $\mathcal{A}_f$ is only dependent on the the ratio of the vector and axial-vector coupling constants, the same ratio that appears directly in the definition of the effective weak mixing angle. Measured quantities in dileptonic final states include $A_{FB}^{0,f}$, forward-backward charge asymmetry at the Z pole for fermion flavour f, and $\mathcal{P}_\tau(\cos\theta)$ the polarisation as a function of polar angle of τ leptons in $Z \rightarrow \tau^+\tau^-$ decays. The tau polarisation in particular allows a simultaneous measurement of the electron and τ asymmetry parameters without any sign ambiguity, unlike the case for $A_{FB}^{0,f}$. Figures 1 shows the results for $\mathcal{A}_\mathrm{e}$ (top) and $\mathcal{A}_\tau$ (bottom) obtained from the $\mathcal{P}_\tau(\cos\theta)$ measurement, from all the LEP experiments.

Figure 1. Asymmetry parameters from $\mathcal{P}_\tau$ measurement.



The quoted results [2] on lepton universality tests are derived from the full data samples and include more than data from $\tau^+\tau^-$ final states. Each experiment separately performed fits to its own data. The measured parameters which were used as input included the cross-sections as a function of $\mathrm{e^+e^-}$ centre-of-mass energy for the processes $\mathrm{e^+e^-} \rightarrow \mathrm{q\bar{q}}$, $\mathrm{e^+e^-} \rightarrow \mathrm{e^+e^-}$, $\mathrm{e^+e^-} \rightarrow \mu^+\mu^-$ and $\mathrm{e^+e^-} \rightarrow \tau^+\tau^-$. Included in this fit are the forward-backward asymmetries for $\mathrm{e^+e^-}, \mu^+\mu^-, \tau^+\tau^-$ final states. The forward-

Figure 2. Neutral current vector coupling (y) versus axial-vector coupling (x) for leptons.



backward asymmetry is defined as $A_{FB}(s) = \frac{\sigma_f - \sigma_b}{\sigma_f + \sigma_b}$ where σ_f and σ_b are the forward and backward leptonic production cross-sections.

The output of this fit for the different lepton families is shown in Fig. 2 where the 68% C.L. regions for the vector and axial-vector coupling constants are shown. There is clearly good agreement with the hypothesis of lepton universality. The electron parameters are more precisely known as the electron coupling appears in all cross-section and forward-backward asymmetry measurements due to the e^+e^- initial state. The τ vector coupling is much more precisely known than that of the muon as a result of the $\mathcal{P}_\tau$ measurement, which gives better precision than the forward-backward asymmetry measurements. The ratios of the couplings can be summarised:

$$\begin{aligned}
\bar{a}_\tau/\bar{a}_e &= 1.0018 \pm 0.0015 \\
\bar{v}_\tau/\bar{v}_e &= 0.973 \pm 0.041 \\
\bar{a}_\mu/\bar{a}_e &= 1.0001 \pm 0.0014 \\
\bar{v}_\mu/\bar{v}_e &= 0.995 \pm 0.096
\end{aligned}$$

Limits have been put on the τ weak anomalous magnetic dipole moment a_τ^W and weak dipole moment d_τ^W. These modify the Lagrangian with extra terms:

$$\mathcal{L} = \frac{1}{2}\frac{ea_\tau^W}{2m_\tau}\bar{\phi}\sigma^{\mu\nu}\phi Z_{\mu\nu} - \frac{i}{2}ed_\tau^W\bar{\phi}\sigma^{\mu\nu}\phi Z_{\mu\nu}. \quad (6)$$

In the S.M. these are expected to have values of $d_\tau^W \approx 3{\cdot}10^{-37}$ $e{\cdot}$cm and $a_\tau^W = (-2.1{+}0.6i){\cdot}10^{-6}$. The current estimates from LEP [3],

$$\begin{aligned}
Re(d_\tau^W) &= (-0.46 \pm 1.48) \cdot 10^{-18} \quad e \cdot \mathrm{cm} \\
Im(d_\tau^W) &= (-1.78 \pm 3.29) \cdot 10^{-18} \quad e \cdot \mathrm{cm} \\
Re(a_\tau^W) &= (+0.23 \pm 0.47) \cdot 10^{-3} \\
Im(a_\tau^W) &= (-0.42 \pm 1.03) \cdot 10^{-3}
\end{aligned}$$

which, while compatible with the S.M. expectations, are not sufficiently precise to permit stringent limits to be set.

3. CHARGED CURRENT

Assuming the V-A structure of the charged current in Eqn. 1, the partial width of the decay to lepton l of the τ has the form [1]

$$\Gamma(\tau \to l\nu_\tau\bar{\nu}_l) \equiv \frac{B(\tau \to l\nu_\tau\bar{\nu}_l)}{\tau_\tau} \quad (7)$$

$$= \frac{G_{l\tau}^2 m_\tau^5}{192\pi^3} \cdot P(\frac{m_l}{m_\tau}) \cdot r_{RC}^\tau, \quad (8)$$

where the parameter

$$G_{l\tau} = \frac{g_\tau g_l}{4\sqrt{2}m_W^2} \quad (\equiv G_F) \quad (9)$$

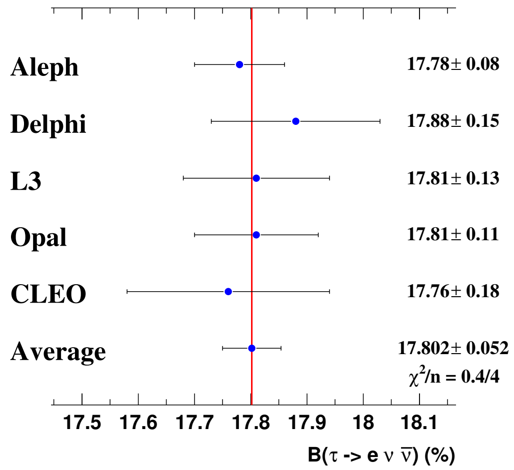
is analogous to the Fermi constant and g_τ and g_l are the coupling constants for the $\tau W \nu_\tau$ and $lW\nu_\tau$ vertices. The phase space factor $P(x)$ is 0.97256 for $\tau \to \mu$ and unity for $\tau \to$e. Radiative corrections are accounted for in the factor

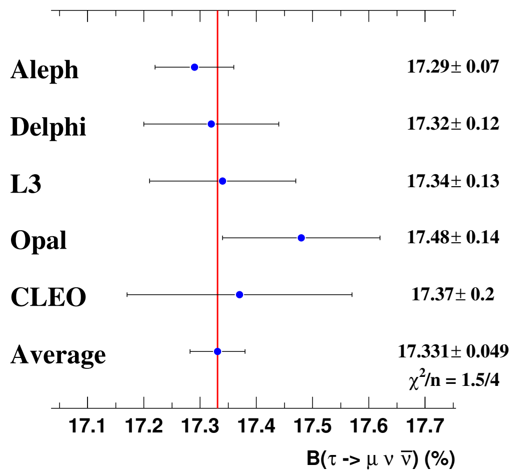
$$r_{RC}^\tau = \left[1 + \frac{3m_\tau^2}{5m_W^2}\right]\left[1 + \frac{\alpha(m_\tau)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right] (10)$$

which has a value of 0.9960 for τ decay.

Combinations of measurements of the τ leptonic branching ratios and of the tau lifetime permit the calculation of ratios of the coupling constants and thus a test of lepton universality in the charged current.

Figure 3. Most precise measurements and combinations of leptonic BRs.





The most precise experimental measurements of the $\tau^- \to e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-\bar{\nu}_\tau\nu_\mu$ branching ratios are shown in Fig. 3. The measurements are dominated by the LEP experiments. This can be attributed to low backgrounds in the τ sample and very low systematics in the particle identification, in part due to the relatively high momentum of the τ decay products (compared with a b-factory) which makes lepton/hadron separation easier than at lower energies. Each combination has a 0.3% relative precision. These can be used to perform a test of e-μ universality via the relation

$$\frac{g_\mu^2}{g_e^2} = \frac{P(m_e/m_\tau)}{P(m_\mu/m_\tau)} \cdot \frac{B_\mu}{B_e} \qquad (11)$$

which can be obtained from Eqn. 8 with the assumption $G_{e\tau} = G_{\mu\tau}$. This gives the result

$$\frac{g_\mu}{g_e} = 1.0005 \pm 0.0022,$$

which is comparable to the precision obtained from the pion branching ratios $B(\pi^- \to \mu^-\bar{\nu}_\mu)$ and $B(\pi^- \to e^-\bar{\nu}_e)$,

$$\frac{g_\mu}{g_e} = 1.0017 \pm 0.0015.$$

although they are not exactly equivalent as in the former case transversely polarised W's are involved whereas in the latter case the W's are longitudinal. One can combine the muon and electronic branching ratios under the assumption of lepton universality to obtain the branching ratio for τ decay to a single family of (massless) leptons:

$$B_{e,\mu} = (17.811 \pm 0.033)\%$$

This takes into account multinomial statistical correlations between the electronic and muonic measurements.

The tau lifetime is dominated by measurements from LEP, as can be seen in Fig. 4. Any one of these measurements would dominate the world average. The relative precision on the combination is 0.35%, comparable which that for the leptonic branching ratios. With inputs of the μ and τ masses and lifetimes and τ leptonic BRs, the relation (again obtainable from Eqn.8 with suitable assumptions,)

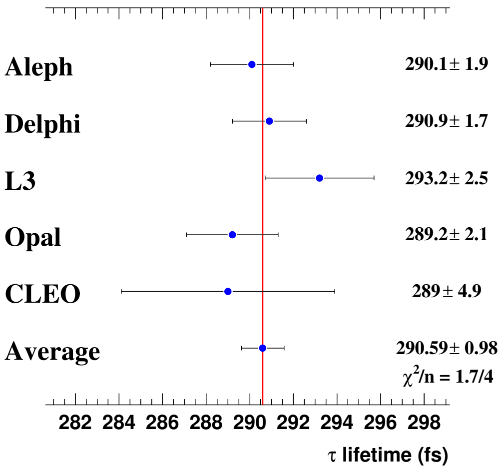
$$\frac{g_\tau^2}{g_\mu^2} = \frac{P(m_e/m_\mu)}{P(m_e/m_\tau)} \cdot \frac{r_{RC}^\mu}{r_{RC}^\tau} \cdot \frac{m_\mu^5}{m_\tau^5} \cdot \frac{\tau_\mu}{\tau_\tau} \cdot B_e \qquad (12)$$

can be used to obtain the result

$$\frac{g_\tau}{g_\mu} = 0.9999 \pm 0.0023,$$

in good agreement with unity and hence with the hypothesis of lepton universality. By interchange

Figure 4. Measurements of the tau lifetime and their combination.



of the electronic and muonic τ decay parameters, the ratio

$$\frac{g_\tau}{g_e} = 1.0004 \pm 0.0023$$

can also be obtained. A more precise test of $\tau - (\mathrm{e}, \mu)$ universality can be performed with the derived value $B_{e,\mu}$:

$$\frac{g_\tau}{g_{e,\mu}} = 1.0002 \pm 0.0020.$$

Thus lepton universality is tested for the charged current in the τ sector at the 0.2% level. Equivalent tests with on-shell W's have a precision of $\mathcal{O}(1-2\%)$. These tests can also be visualised in the τ_τ versus $B_{e,\mu}$ plane in Fig 5. The two lines surround the permitted region for the world average value of the tau mass of $m_\tau = 1776.99^{+0.29}_{-0.26}$ MeV, together with the assumption $g_\tau{=}g_\mu$. The lifetime and branching ratio measurements are consistent.

The impact of LEP on these measurements can be seen in Fig 6 which shows the current situation together with the situation as it stood in summer 1990 before any LEP measurements were included in the world averages. The 2001 situation is given by the small cross with the two dashed lines passing through it. The 1990 situation is given by the large cross and the two lower dashed lines and

Figure 5. Predicted τ_τ versus electronic branching ratio.

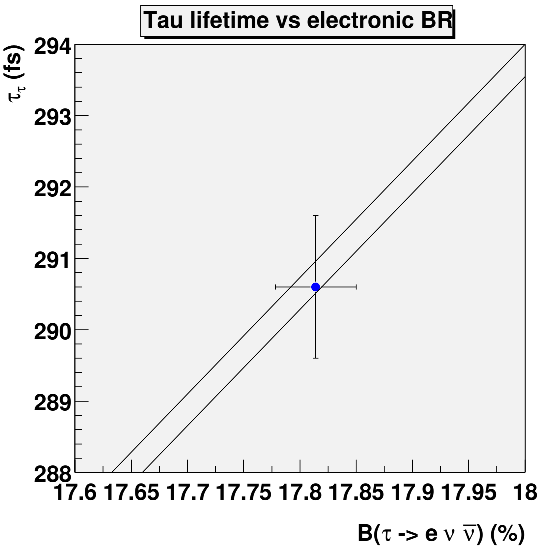
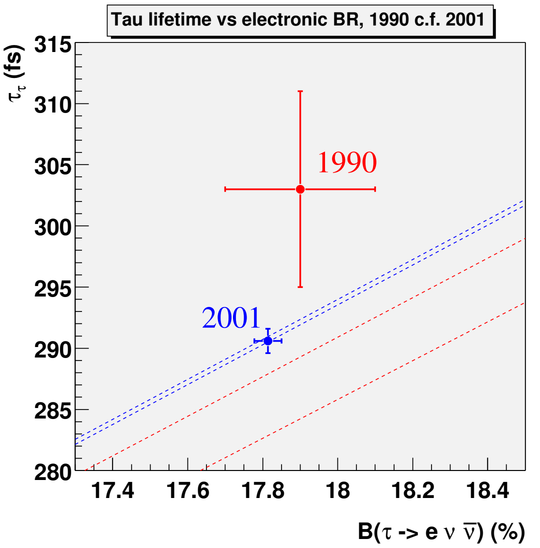
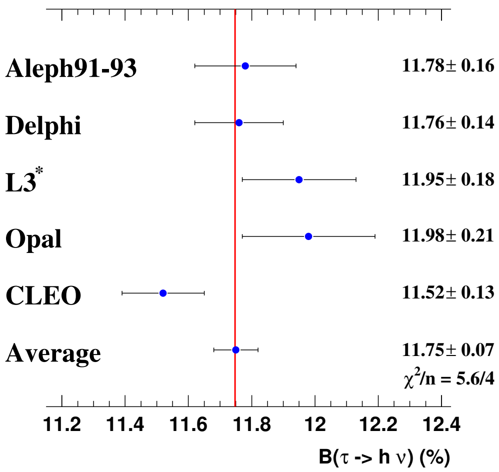


Figure 6. Predicted τ_τ versus B_e: status 2001 c.f. status 1990.



agreement is at about the two standard deviation level. This plot shows clearly that the branching ratios have improved by a factor six in precision, while the lifetime has improved by a factor eight and has shifted by over one standard devi-

Figure 7. Combination of measurements of $B(\tau^- \to h^- \nu_\tau)$.



ation to lower values. These improvements can be attributed to improved detectors and analysis techniques. In particular the emergence of silicon vertex detectors at LEP, together with other analysis techniques such as the missed distance method, have had a major influence in the measurement of the lifetime. Note that the biggest change in this period of time has been the large improvement in the precision of m_τ coming from the BES accelerator. Not only has there been an improvement of a factor of 10 in precision, but the central value of the band defined by the tau mass is shifted by over two standard deviations with respect to the 1990 value.

A further universality test is possible using the branching ratio of the process $\tau^- \to h^- \nu_\tau$. The measurements of this are shown in Fig. 7. CLEO is competitive with the LEP experiments. Using masses, lifetimes and muonic branching ratios of the π and K, the following relation can be constructed:

$$\frac{g_\tau^2}{g_\mu^2} = \frac{2m_\mu^2}{m_\tau^2} \cdot \frac{B(\tau^- \to h^- \nu_\tau)}{H_\pi + H_K} \qquad (13)$$

where

$$H_h = (1+\delta_h) \cdot \frac{\tau_\tau}{\tau_h} \cdot \frac{m_\tau}{m_h} \cdot \frac{1 - m_h^2/m_\tau^2}{1 - m_\mu^2/m_h^2}, \qquad (14)$$

and δ_h accounts for radiative corrections. The derived result is

$$\left(\frac{g_\tau}{g_\mu}\right)_L = 1.0037 \pm 0.0037,$$

giving a precision of better than 0.4% for longitudinal W exchange, and again consistent with lepton universality.

In a generalisation to the assumption of a V-A type of charged current, one can describe the differential decay spectrum of the outgoing lepton energy using the Michel parameterisation [4], adding tensor, scalar and pseudo-scalar currents. If the daughter lepton polarisation is unmeasured, this has the form

$$\begin{aligned} \frac{1}{\Gamma}\frac{d\Gamma}{dx_l} &= H_0(x_l) - \mathcal{P}_\tau H_1(x_l) && (15) \\ &= [h_0(x_l) + \eta h_\eta(x_l) + \rho h_\rho(x_l)] && (16) \\ &\quad -\mathcal{P}_\tau[\xi h_\xi(x_l) + \xi\delta h_{\xi\delta}(x_l)] \end{aligned}$$

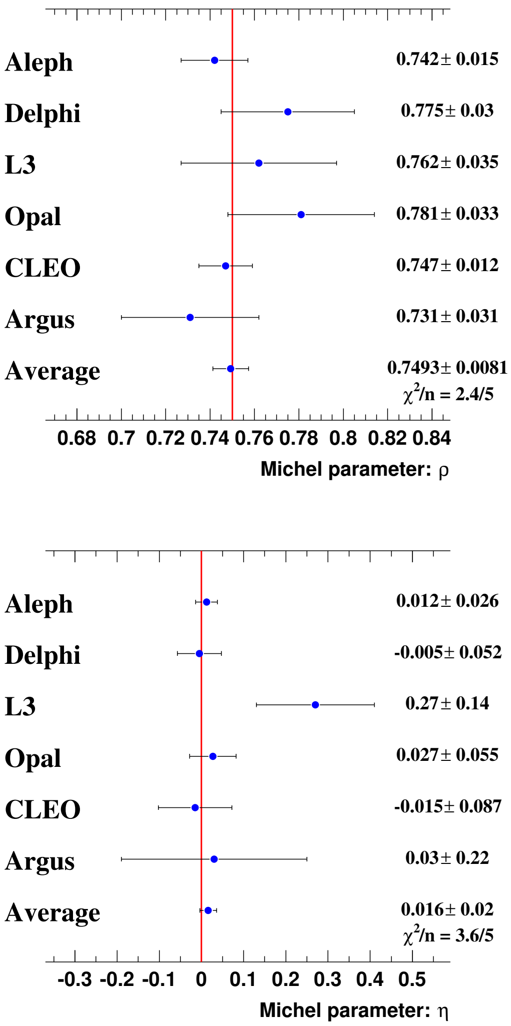
with four independent parameters, ρ, η, ξ and δ. These have standard model values of 3/4, 0, 1, and 3/4 respectively. Any deviation from these values would be indicative of some non-Standard Model effect. All four LEP experiments have measured these parameters, using multidimensional fits to both τ decays in an $e^+e^- \to \tau^+\tau^-$ event. CLEO and Argus have also produced precise measurents of these parameters, in the case of CLEO the precision on some parameters being greater than for LEP. Fig. 8 shows the combinations for the ρ and η parameters.

In Fig. 9 the combined results of the τ decay measurements are shown together with those for muon decay. The muon parameters are currently known with a precision of typically three times better than those of the tau. Precision on both the muon and tau parameters can be expect to be greatly improved with data from the B-factories and with new muon decay experiments. No deviations from the Standard Model values are observed.

In a further generalisation, tensor derivative terms can be included in the Lagrangian, of the form

$$\begin{aligned} \mathcal{L} &= \frac{g}{\sqrt{2}} W^\alpha \frac{i}{2m_\tau} \overline{\tau} \sigma_{\alpha\beta} q_\beta (\kappa_\tau^W + i\tilde{\kappa}_\tau^W \gamma^5) \frac{1-\gamma^5}{2} \nu \\ &\quad + \text{h.c.} && (17) \end{aligned}$$

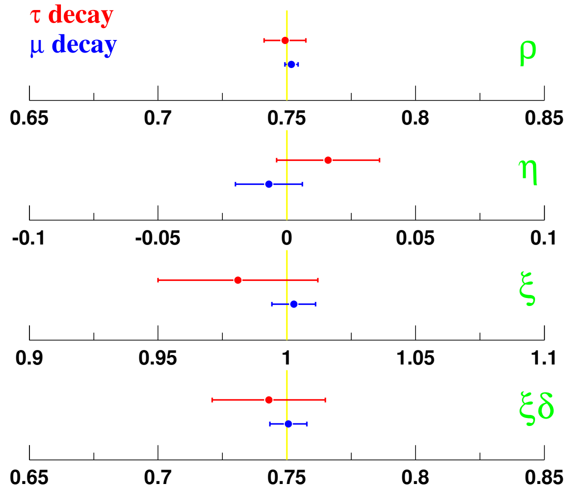
Figure 8. Combinations of ρ and η Michel parameters



where κ_τ^W is the anomalous magnetic form factor and $\tilde{\kappa}_\tau^W$ the anomalous electric form factor. These can be estimated using universality relations and the τ leptonic decay width [5]. The following results are obtained [6]:

$$\kappa_\tau^W = 0.001 \pm 0.008,$$
$$\tilde{\kappa}_\tau^W = 0.00 \pm 0.16.$$

Figure 9. Comparison of Michel parameters measured in τ decay (upper points) with those from μ decay (lower points).



A fit to τ decay spectra [7] (DELPHI) gives the result

$$\kappa_\tau^W = 0.019 \pm 0.036 \pm 0.018,$$

compatible with zero and hence the Standard Model expectation.

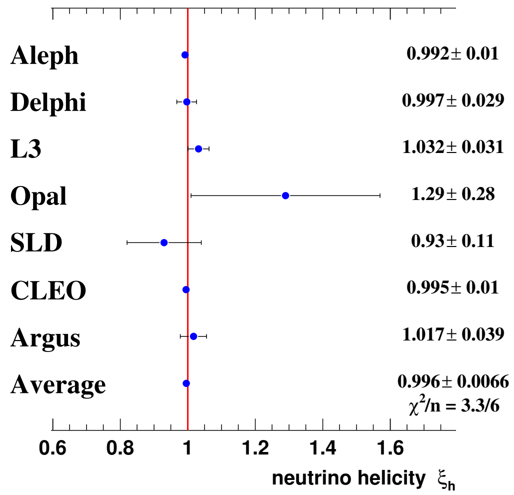
4. TAU NEUTRINO

The fits to the Michel parameters in τ decay also permit simultaneously a measurement to the ν_τ helicity parameter ξ_h, using the τ hadronic decays. In the limit of massless neutrinos and under the assumption that only vector and axial-vector type couplings occur in the τ hadronic decays, this can be equated to the (negative) chirality of the ν_τ. The combination of the different measurements is shown in Fig. 10 This gives an upper limit on the right-handed tau-neutrino coupling:

$$\frac{g_R^2}{g_L^2} < 0.007 \text{ at } 90\% \text{ CL.}$$

Note that the big difference in the precision of different experiments is due to the use of different decay modes in the different experiments' analyses.

Figure 10. Comparison of ν_τ helicity parameter measured in τ decay.



5. SUMMARY

The four LEP experiments have produced many results on τ physics and related topics. In this domain a total of about 100 papers and letters have been published to date by the four experiments, and many of the results shown here are as yet preliminary or do not yet contain results from the full LEP data sample. Results from other sources (CLEO, Argus, SLAC) have been included where they are of a similar precision to those of the LEP experiments.

The results presented here show the derived results on tests of the Standard Model in the tau sector, in particular test concerning lepton universality in the weak charged and neutral currents.

The current status of universality tests in the neutral current is as follows:

- axial-vector coupling - universality tested to 0.2%;
- vector coupling tested to 4%;
- no evidence for non-Standard Model couplings.

For the charged current:

- no evidence for deviation from V-A theory;
- μ-e universality good to 0.2% level;
- τ-e, τ-μ universality tested to ≈0.2%;
- the right-handed τ neutrino coupling is tightly constrained.

Due to the quality of the LEP experiments coupled with the clean experimental environment of the LEP machine, during the lifetime of LEP the precision on such basic τ decay parameters as the lifetime and the leptonic decay partial widths have been improved by almost an order of magnitude. The neutral current coupling has been measured with a similar precision to the charged current - an improvement of a few orders of magnitude over what existed previously. This is due to the large samples of $Z \to \tau^+\tau^-$ decays produced on the Z resonance.

Now that LEP has closed, the immediate future lies with the B-factories, where copious production of τ pairs will allow far greater precision on measurements of the Michel parameters and perhaps, if systematic effects can be sufficiently mastered, on the tau lifetime and leptonic branching ratios. In the longer term, a prospective Z-factory built into a future e^+e^- linear, such as that mooted for TESLA, would allow a very high precision on the tau lifetime, perhaps of order 10^{-4} or better, as well as a very high precision on the branching ratios of the major tau decay modes.

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