LEP Measurements of $V_{\rm cb}$ and $V_{\rm ub}$

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The magnitude of the CKM matrix element $V_{\rm cb}$ has been measured using $\bar{\rm B}^0 \to {\rm D}^{*+}\ell^-\bar{\nu}$ decays recorded on the $\rm Z^0$ peak using the OPAL, ALEPH and DELPHI detectors at LEP. The $\rm D^{*+} \to D^0\pi^+$ decays were reconstructed both in particular decay modes and via an inclusive technique. The product of $|V_{\rm cb}|$ and the decay form factor of the $\bar{\rm B}^0 \to {\rm D}^{*+}\ell^-\bar{\nu}$ transition at zero recoil $\mathcal{F}(1)$ was measured to be $\mathcal{F}(1)|V_{\rm cb}| = (35.6 \pm 1.7) \times 10^{-3}$. $|V_{\rm cb}|$ is obtained by using Heavy Quark Effective Theory calculations for $\mathcal{F}(1)$.

The semi-leptonic branching ratio $BR(b \to cl\nu)$ is also used to extract $V_{\rm cb}$. The combined result is $|V_{\rm cb}| = (40.7 \pm 1.9) \times 10^{-3}$.

The semi-leptonic branching ratio $BR(b \to ul\nu)$ is measured and used to extract the magnitude of $V_{\rm ub}$, $|V_{\rm ub}| = (4.09^{+0.59}_{-0.69}) \times 10^{-3}$.

1. Introduction

The existence of the third generation of fermions was originally predicted to accommodate CP violation into the Standard Model via non-trivial phase of the quark mixing matrix [1]. The coupling between up and down type quarks is described by the 3X3 unitary CKM matrix. In the Wolfenstein parametrization:

$$
\begin{array}{c c}
 & \begin{array}{ccc} d & s & b \end{array} \\
\begin{array}{c} u \\ c \\ t \end{array} &
\begin{pmatrix}
1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1-\lambda^2/2 & A\lambda^2 \\
A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1
\end{pmatrix}
\end{array}
$$

where $\lambda = \sin\theta_c = 0.2205 \pm 0.0018$ and A is determined from $V_{\rm cb}$.

CKM elements are free parameters to be measured. Vcb is the main CKM element allowing b decay. Its smallness accounts for the long b lifetime and our ability to observe higher order effects such as B oscillations. $V_{\rm ub}$ is about 10 times smaller than $V_{\rm cb}$ making the branching ratio $BR(b \to ul\nu)$ about one percent of $BR(b \to cl\nu)$.

2. $V_{\rm cb}$

$V_{\rm cb}$ can be measured from the inclusive semileptonic decay rate of B hadrons to charm states or from the study of the decay rate of $B^0 \to D^{*+}l^-\nu$ as a function of $\rm D^{*+}$ recoil kinematics.

The decay rate $B^0 \to D^{*+}l^-\nu$ is parameterized as a function of the variable ω, the product of the 4-velocities of the $\rm D^{*+}$ and the $\overline{\rm B}^0$, which is related to the square of the 4-momentum transfer from the $\overline{\rm B}^0$ to the $\ell^-\overline{\nu}_\ell$ system, q^2,

$$\omega = \frac{m^2_{\rm D^{*+}} + m^2_{\rm B^0} - q^2}{2m_{\rm B^0} \cdot m_{\rm D^{*+}}}, \qquad (1)$$

and is 1.0 when the $\rm D^{*+}$ is produced at rest in the $\overline{\rm B}^0$ rest frame. Using HQET, the differential partial width for this decay is given by

$$\frac{{\rm d}\Gamma(\bar{\rm B}^0 \to {\rm D}^{*+}\ell^-\bar{\nu})}{{\rm d}\omega} = \mathcal{K}(\omega)\mathcal{F}^2(\omega)|V_{\rm cb}|^2 \qquad (2)$$

where $\mathcal{K}(\omega)$ is a known phase space term and $\mathcal{F}(\omega)$ is the hadronic form factor for the decay [2]. Although the shape of this form factor is not known, its magnitude at zero recoil, $\omega = 1$, can be estimated using HQET. Thus, an accurate determination of $\mathcal{F}(1)|V_{\rm cb}|$ can be made by measuring ${\rm d}\Gamma/{\rm d}\omega$ and extrapolating to $\omega = 1$, with $\mathcal{F}(\omega)$ approximated by a power series expansion around $\omega = 1$. Since the decay rate vanishes at $\omega = 1$, the accuracy of the extrapolation relies on achieving a reasonably constant reconstruction efficiency in the region around $\omega = 1$.

2.1. Selecting the samples

Each event in the selected samples includes an identified lepton consistent with coming from b decay. Two main D^{*+} selection methods have been used: those where both the D^{*+} and the D^0 are reconstructed exclusively, leading to clean but small samples, and those where the D^0 is reconstructed inclusively and the D^{*+} identified by the "slow" π from the D^{*+} decay to D^0.

Due to the small mass difference between D^{*+} and D^0, the "slow" pions, from the D^{*+} decay can be selected by finding pion very close in phase space to inclusively reconstructed D^0. The $B^0 \to D^{*+} l^- \nu$ events are selected by also requiring a lepton with charge opposite to the "slow" π and a separated secondary vertex. The resulting samples have large statistics and large background. For example, the DELPHI sample [4] has 7,075 $D^{*+}\ell^-$ with 4,278 combinatorial background and a signal purity of 50%. Figure 1 shows the spectrum of D^{*+}–D^0 mass difference from that analysis.

Another selection method relies on exclusive reconstruction of the D^0 and D^{*+}. D^0 are reconstructed in the modes $K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-\pi^+\pi^-\pi^+$ and $K_s\pi^+\pi^-$. In this case, after combining with the opposite sign lepton, the secondary vertex is not required. This method yields smaller but cleaner event samples, e.g the ALEPH sample [3] has 579±32 with a signal purity of 78%.

2.2. Backgrounds

The amount of combinatorial is estimated from data using the shape from "wrong sign" $D^{*+}\ell^+$ ($\pi^+\ell^+$) combinations, normalized using the events with $D^{*+}D^0$ mass difference outside the signal region.

The most problematic background are $D^{*+}\ell^-$ events from b semileptonic decays involving orbitally excited charm mesons (generically referred to as D^{**}), *e.g.* $B^- \to D^{**0}\ell^-\bar{\nu}$ followed by $D^{**0} \to D^{*+}\pi^-$. These decays will be denoted collectively by $\bar{B} \to D^{*+} h\, \ell^-\bar{\nu}$. This background comprises 11-24% of $D^{*+}\ell^-$ events. It can be estimated, and partially removed from the sample, by fitting the missing mass, $\mu^2 = M^2_{B^0} + M^2_{D^{*+}\ell^-} - 2P_{B^0} \cdot P_{D^{*+}\ell^-}$, to D^{*+} and D^{**} contributions. Another method of determining the fraction of this background is searching for additional tracks going out of the secondary vertex.

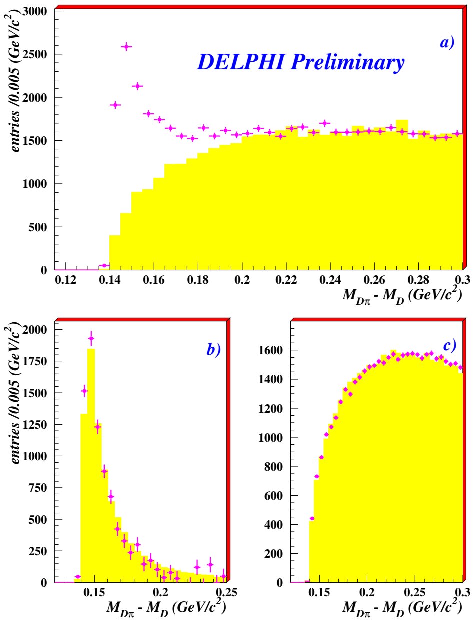

Figure 1. D^{*+}–D^0 mass difference from [4]. The shaded area is the background estimate from the"wrong sign" combination shown in c). The background subtracted data is in b)

The background from fake leptons combined with a π from D^{*+} and the background from $\bar{B}^0 \to D^{*+}X_c$ with $X_c \to \ell^-\bar{\nu}$ or $\bar{B}^0 \to D^{*+}\tau$ with $\tau \to \ell^-\bar{\nu}$ are small and estimated from simulated events.

2.3. Reconstruction of ω

The recoil variable ω is estimated in each event using the reconstructed four-momentum transfer to the $\ell\bar{\nu}$ system:

$$q^2 = (E_{B^0} - E_{D^*})^2 - (\mathbf{p}_{B^0} - \mathbf{p}_{D^*})^2.$$

This is estimated from one or more of the following combinations: p_ℓ and E_ν, the B direction and E_ν or the B and D^{**} momenta.

The B direction is estimated from either the direction from the primary to secondary vertex or from the reconstructed direction of the B momentum. The D^{*+} momentum is estimated from either exclusive reconstruction or from scaling the momentum of the "slow" π by the factor $M_{D^{*+}}/M_{\pi}$. E_{ν} is estimated from the corrected missing energy in the event. The neutrino has a mean energy of $\sim$ 8 GeV and the resolution on E_{ν} is 2.6–2.8 GeV. The ω resolution is much better for the exclusive method than for the inclusive method. Figure 2 shows reconstructed ω in bins of true ω from the OPAL inclusive analysis [5]. The calculation of Leibovich et al.[8] was used to simulate the recoil spectrum of $\bar{B} \to D^{*+} h\, \ell^{-} \bar{\nu}$ decays, where h is any hadron.

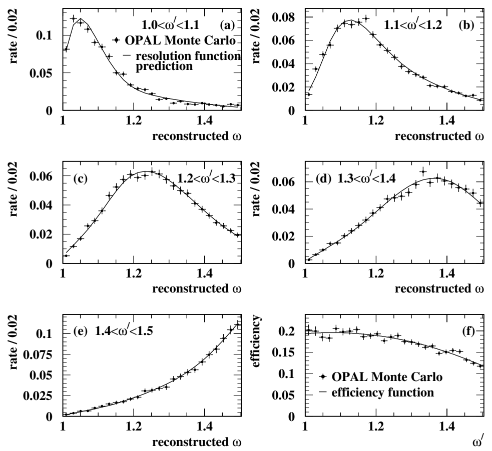

Figure 2. reconstructed ω in bins of true ω (ω') from the OPAL inclusive analysis [5]

2.4. Fit for V_{cb}

The ω distribution is fit to

$$\frac{d\Gamma}{d\omega} = \mathcal{K}(\omega)\mathcal{F}^2(\omega)|V_{cb}|^2$$

using the expansion of [7]. The fit gives an estimate of $\mathcal{F}(1)V_{cb}$ and ρ^2, the slope of $\mathcal{F}^2(\omega)$ at zero recoil. Figure 3 shows the fit to the ω spectrum from the OPAL inclusive analysis.

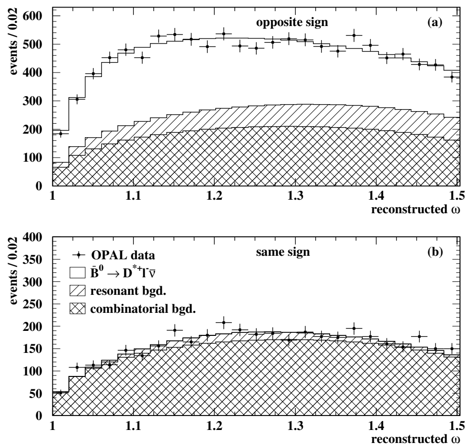

Figure 3. ω distribution from [5] with fit for $\mathcal{F}(1)V_{cb}$.

2.5. Systematic errors

Systematic errors arise from the uncertainties in the fit input parameters, the Monte Carlo modelling of the signal ω resolution, the recoil spectrum of $b \to D^{**}\ell\bar{\nu}$ decays and selection efficiencies, and possible biases in the fitting method.

2.6. Results from $B^0 \to D^{*+}l^-\nu$

The experimental $\mathcal{F}(1)V_{cb}$ results of the LEP experiments using $B^0 \to D^{*+}l^-\nu$ are given in Table 2,corrected to use common inputs. The LEP average is $\mathcal{F}(1)V_{cb} = (35.6 \pm 1.7)10^{-3}$. Using the theoretical estimate $\mathcal{F}(1) = 0.88 \pm 0.05$ we obtain $V_{cb} = 40.5 \pm 1.9(exp) \pm 2.3(theory)$

2.7. Determination of V_{cb} from inclusive semileptonic b decay

The b semileptonic branching ratio was measured by the LEP experiments $BR(b \to Xl\nu) = 10.654 \pm 0.23$. From this we subtract $BR(b \to ul\nu)$ to obtain $BR(b \to cl\nu)$. From the rate

Table 1
Dominant systematic errors on $\mathcal{F}(1)V_{\rm cb}$

Source of error	$\mathcal{F}(1)V_{\rm cb}$ (%)	ρ
Branching ratios		
Br(D $\rightarrow$ K$n\pi$)	1.0	.01
Br(D^{*+} $\rightarrow$ D$^0\pi$)	1.0	-
Br(B $\rightarrow$ B^0)	1.6	-
Background		
B$^+$ $\rightarrow D^{*+}\ell^-\bar{\nu}X$	4.5	0.16
Detector	2.1	0.10
Total systematic	5.7	0.19

Table 2
Experimental results corrected for common inputs

experiment	$\mathcal{F}(1)V_{\rm cb}$ ($\times 10^{-3}$)
ALEPH	$33.0 \pm 2.1 \pm 1.6$
DELPHI	$34.5 \pm 1.4 \pm 2.5$
OPAL inclusive	$37.9 \pm 1.3 \pm 2.4$
OPAL exclusive	$37.5 \pm 1.7 \pm 1.8$

$\Gamma(b \rightarrow cl\nu) = \frac{BR(b\rightarrow cl\nu)}{\tau_B}/$ we obtain $|V_{\rm cb}| = (40.9 \pm 0.5 \pm 2.4) \times 10^{-3}$.

Combining the two measurements gives the LEP average

$$|V_{\rm cb}| = (40.7 \pm 1.9) \times 10^{-3}$$

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3. $V_{\rm ub}$

Measuring the branching ratio for $b \rightarrow ul\nu$ provides the most precise way to determine the $V_{\rm ub}$element of the CKM matrix. $BR(b \rightarrow ul\nu)$ is about 1% of $BR(b \rightarrow cl\nu)$ so it is important to make the selection as efficient as possible; identify quantities which $b \rightarrow ul\nu$ from $b \rightarrow cl\nu$; and model precisely how much of each decay mode is included in the final selection.

3.1. Event selection and $b \rightarrow ul\nu$ separation

In the LEP measurements the event selection is an efficient selection of semileptonic b decays, using secondary decay vertices, energetic lepton selection and cascade decays. Inclusive B hadron reconstruction is used to obtain the B decay kinematics.

There are a number of experimental quantities that help distinguish $b \rightarrow ul\nu$ from $b \rightarrow cl\nu$:

- The decay kinematics depend on the recoil particle mass. The lepton from $b \rightarrow ul\nu$ has higher momentum in the B rest-frame. The recoil mass is smaller and the lepton carries a larger part of the B energy.

- The light quark final state tends to have an energetic leading hadron. This leads to a larger lepton-hadron invariant mass, larger hadron p_T and lower hadron rapidity with respect to the lepton for $b \rightarrow ul\nu$.

- The charm decay characteristics can be utilized. In $b \rightarrow cl\nu$ there will be larger multiplicity in the final state, more strange particles and protons, and a worse single vertex fit to lepton and charm due to the charm lifetime.

Figure 4 shows the distribution of quantities used in the OPAL[12] analysis for signal and background. Since no single quantity separates well signal from background, but all provide some separation, they are combined together using an artificial neural network. The distributions of the neural network output from signal and background are used to fit the data and obtain the amount of $b \rightarrow ul\nu$ and $b \rightarrow cl\nu$ in the sample. Figure 5 shows the neural output distribution for the data compared to background simulation spectrum.

DELPHI[10] uses a slightly different approach. The data sample is divided into 4 classes, enriched or depleted in $b \rightarrow ul\nu$ in different ways. A fit to the lepton energy and the number of events in each class gives $b \rightarrow ul\nu / b \rightarrow ul\nu$.

3.2. $V_{\rm ub}$ results

Table 3 shows the measured $BR(b \rightarrow ul\nu)$ from the LEP experiments. The first error on each measurement is the experimental error. The second and third errors come from the $b \rightarrow c$ background description and the $b \rightarrow u$ modelling respectively, and are fully correlated between the experiments.

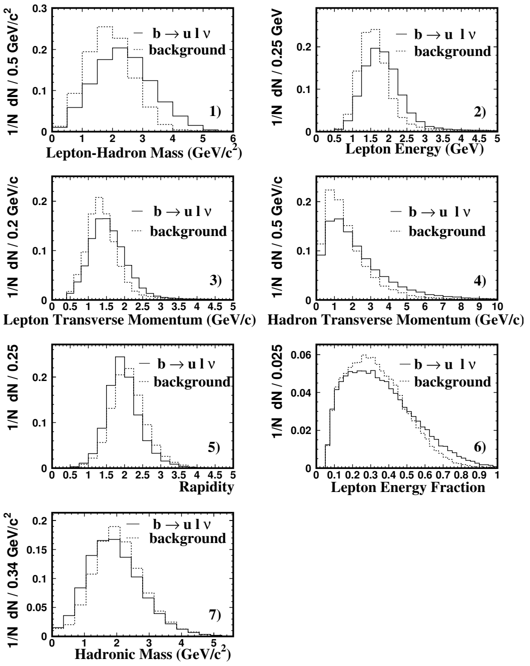

Figure 4. Comparison between the signal $b \to ul\nu$ and the background in the Monte Carlo simulation for the seven neural network input variables. The $b \to ul\nu$ signal and background are normalized to unity.

Table 3
Experimental results of $BR(b \to ul\nu)$

experiment	$BR(b \to ul\nu)$ $(\times 10^{-3})$
ALEPH	$1.73 \pm 0.56 \pm 0.51 \pm 0.21$
DELPHI	$1.69 \pm 0.54 \pm 0.39 \pm 0.25$
L3	$3.3 \pm 1.3 \pm 1.4 \pm 0.5$
OPAL	$1.63 \pm 0.57 \pm 0.48 \pm 0.25$

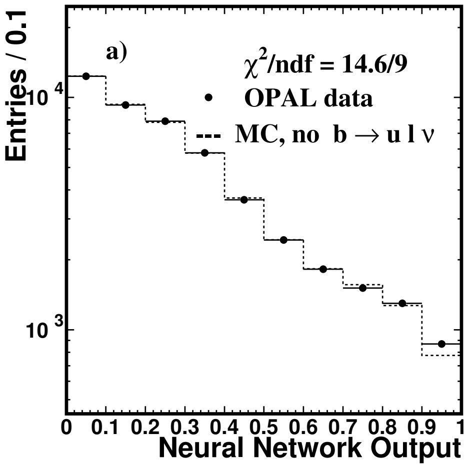

Figure 5. The neural network output distributions for data and background Monte Carlo simulated events

The LEP average of $BR(b \to ul\nu) = 1.71 \pm 0.53$ is used to extract $|V_{\rm ub}| = (4.09^{+0.59}_{-0.69}) \times 10^{-3}$.

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