Experimental Status of α_s Measurements

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During the period of LEP running, the precision of determinations of the strong coupling, $\alpha_s(M_Z)$, has improved from $\sim 10\%$ to $\sim 2-3\%$. The contribution made by measurements of α_s using the data recorded at LEP, together with progress in theoretical calculations, is reviewed. Consistent results are obtained using a variety of different techniques.

1. INTRODUCTION

Since the mid 1970s, QCD has been the theory governing the strong interaction sector of the Standard Model. Once the SU(3) group structure is chosen, the predictions of the theory depend only on the coupling strength, α_s. Therefore the measurement of α_s in many different processes and at different energies is a fundamental test of QCD, and of our ability to perform reliable calculations in the theory. In this talk we review the substantial contribution which LEP has made to this field.

2. α_s FROM THE Z^0 LINESHAPE

The emission of gluon radiation affects the Z^0 hadronic width, multiplying it by a factor (in the case of massless quarks) of

$$R_{\rm QCD} = 1 \quad + \quad \frac{\alpha_s}{\pi} + 1.409\left(\frac{\alpha_s}{\pi}\right)^2 \quad - \quad 12.805\left(\frac{\alpha_s}{\pi}\right)^3 + \mathcal{O}({\alpha_s}^4) \qquad (1)$$

This is considered to be one of the most reliable predictions of QCD; since it concerns an inclusive quantity, and the calculation is already available to a reasonably high order, higher order effects are expected to be small.

Several of the lineshape observables are sensitive to α_s; for example the total width Γ_Z, the hadron/lepton ratio $\Gamma_{\rm had}/\Gamma_\ell$ and the pole cross-section σ_0. Each of these is also sensitive, in differing degrees, to other parameters such as the Higgs or top quark mass. Consequently, the most accurate measurement of α_s comes from a global fit to all electroweak data, which constrains m_t, m_H etc. using other observables. The value currently obtained by the LEP electroweak working group is [1]:

$$\boxed{\alpha_s(M_Z) = 0.1183^{+0.0026}_{-0.0027}};$$

excluding the theoretical error, which is variously estimated to be $\mathcal{O}(0.002)$.

3. α_s FROM τ-LEPTON DECAYS

Analogously to the Z^0, QCD radiation affects the hadronic width of the τ-lepton, multiplying it by a factor

$$R_{\rm QCD} = 1 \quad + \quad \frac{\alpha_s}{\pi} + 5.2\left(\frac{\alpha_s}{\pi}\right)^2 \quad + \quad 26.6\left(\frac{\alpha_s}{\pi}\right)^3 + \mathcal{O}({\alpha_s}^4) \qquad (2)$$

Hence, α_s can be extracted from measurements of the τ branching ratio $B(\tau^- \to e^-\nu_\tau\overline{\nu}_e)$, either determined directly, or inferred from the τ lifetime assuming lepton universality. However, at such a low energy, non-perturbative (NP) effects are likely to be a concern. They have been estimated using the operator product expansion, and turn out to make only a small contribution to $R_{\rm QCD}$, $\sim -0.014 \pm 0.005$ [2]. Using the PDG value of the branching ratio, a value of

$\alpha_s(m_\tau) = 0.35 \pm 0.03$ is obtained, corresponding to $\alpha_s(M_Z) = 0.121 \pm 0.003$ [3]. However one can constrain the NP terms using measurements of spectral moments (i.e. moments of the mass spectrum in hadronic τ decays), which are predicted in terms of the same NP coefficients. Such analyses have been performed by ALEPH and OPAL [4], yielding $\alpha_s(m_\tau) = 0.334 \pm 0.022$ and 0.348 ± 0.021 respectively. Overall, an average result based on τ-lepton decays is:

$$\boxed{\alpha_s(M_Z) = 0.1210 \pm 0.0025}$$

4. α_s FROM JET RATES / EVENT SHAPES

The determination of α_s from the 3-jet rate, or other related "event shape" variables is one of the most intuitive techniques, which has been pursued since the earliest days of LEP/SLC. The observables used must be stable in the regions of phase space where QCD predictions diverge, owing to collinear and soft gluon emission. More than twenty different observables have been used at LEP, of which the best known include the following:

- The 3-jet rate (or the resolution parameter y_{23} at which an event changes from 2- to 3-jet). Many schemes are available, of which the Durham or k_T scheme is the most important.
- Thrust: $T = \max_{\hat{\mathbf{n}}} (\sum_i |\mathbf{p}_i \cdot \hat{\mathbf{n}}| / \sum_i |\mathbf{p}_i|)$
- Heavy jet mass M_H^2/s; the larger of the masses in the hemispheres defined by the plane normal to the Thrust axis.
- Jet Broadenings B_T, B_W; scaled transverse momenta w.r.t. the Thrust axis.

4.1. α_s using $\mathcal{O}(\alpha_s^2)$ QCD calculations

At LEP startup, QCD matrix elements to $\mathcal{O}(\alpha_s^2)$ were known [5], allowing predictions to be made of the distribution of a generic observable y of the form:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{\alpha_s(\mu)}{2\pi}A(y) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 [A(y)\tfrac{1}{2}\beta_0 \ln({x_\mu}^2) + B(y)] \quad (3)$$

where μ is the renormalization scale, and $x_\mu = \mu/\sqrt{s}$. The leading order coefficient $A(y)$ may be computed analytically; $B(y)$ by numerical integration using the programs `EVENT` [6] or `EVENT2` [7]. A calculation to all orders would be independent of the choice of renormalization scale μ, but fixed-order predictions depend on x_μ. It is natural to expect $\mu \sim \sqrt{s}$, or $x_\mu \sim 1$. However, data generally favour a value $x_\mu \ll 1$ (see Fig. 1), which is taken to be an indication of importance of higher orders, and hence leads to a systematic error. The other important systematic uncertainty in these measurements arises from hadronization, because the data need to be corrected from the hadron-level to the parton-level before fitting QCD predictions. These correction normally rely on Monte Carlo models.

All the LEP and SLC experiments have published measurements of $\alpha_s(M_Z)$ using $\mathcal{O}(\alpha_s^2)$ QCD calculations. The most recent is illustrated in Fig. 1. If x_μ is allowed to vary in the fit as well as α_s, significantly smaller values of $\alpha_s(M_Z)$ are generally obtained, with better consistency between observables. The results quoted by the experiments, using various observables, and different ways of handling the renormalization scale uncertainty, are

ALEPH [8]	0.121 ± 0.008
DELPHI [9]	0.117 ± 0.003
L3 [10]	0.118 ± 0.007
OPAL [11]	0.122 ± 0.006
SLD [12]	0.123 ± 0.011

from which my average would be:

$$\boxed{\alpha_s(M_Z) = 0.120 \pm 0.005}$$

4.2. α_s using resummed QCD calculations

The $\mathcal{O}(\alpha_s^2)$ QCD predictions diverge in the two-jet region. Specifically, the leading order prediction for observable y behaves, as $y \to 0$, like

$$R(y) = \int_0^y (1/\sigma)(\mathrm{d}\sigma/\mathrm{d}y')\mathrm{d}y' \propto \alpha_s L^2 \quad (4)$$

where $L = \ln(1/y)$. In general, the leading term at $\mathcal{O}(\alpha_s^n)$ is of the form $(\alpha_s L^2)^n$. At small y, $\alpha_s L^2$ is not small, so the predictions are not reliable

unless they are summed to all orders. Such "resummation" of leading and next-to-leading logarithms (NLLA) has proved possible for a few event shape variables[13] giving a prediction of the form:

$$R_{\rm NLLA}(y) = (1 + C_1\alpha_{\rm s} + C_2\alpha_{\rm s}^2) \times \exp[Lg_1(\alpha_{\rm s}L) + g_2(\alpha_{\rm s}L)] \quad (5)$$

By careful use of a "matching scheme", $\mathcal{O}(\alpha_{\rm s}^2)$ and NLLA calculations can be combined without double counting common terms. These are the state-of-the-art calculations for e^+e^- event shapes at present, and have largely superseded the $\mathcal{O}(\alpha_{\rm s}^2)$ approach.

Again, all the experiments have presented measurements using resummed QCD calculations; an example is given in Fig. 2. Note that the data in the low y region are better described using the NLLA calculations. Results from LEP1 using $\mathcal{O}(\alpha_{\rm s}^2)$+NLLA QCD calculations, and various combinations of observables, are:

ALEPH [14]	0.120 ± 0.004
DELPHI [9]	0.119 ± 0.005
L3 [10]	0.122 ± 0.007
OPAL [15]	0.120 ± 0.006
SLD [12]	0.118 ± 0.006

from which my average would be

$$\boxed{\alpha_{\rm s}(M_{\rm Z}) = 0.120 \pm 0.005}$$

4.3. $\alpha_{\rm s}$ at LEP2

All LEP experiments have produced measurements of $\alpha_{\rm s}$ at many energies at LEP2, based on $\mathcal{O}(\alpha_{\rm s}^2)$+NLLA QCD calculations. The main interest is to test the running of $\alpha_{\rm s}$ with energy, which is a characteristic prediction of QCD. Hadronization errors are much smaller than at LEP1, but statistical (and some experimental systematic) errors are sizeable because of the small data samples. It is therefore profitable to combine results from the four experiments. A LEP QCD Working Group has been combining $\alpha_{\rm s}$ measurements, and results from October 2000 are shown in Fig. 3. My average of the LEP2 results, evolved to the scale M_Z, is

$$\boxed{\alpha_{\rm s}(M_{\rm Z}) = 0.120 \pm 0.006}$$

while a combined fit to all LEP1 and LEP2 data [16] yields

$$\boxed{\alpha_{\rm s}(M_{\rm Z}) = 0.1195 \pm 0.0007(\text{expt.}) \pm 0.0046(\text{theo.})}$$

The measurements are clearly consistent with the evolution expected from QCD.

The LEP QCD Working Group is working actively to improve the combination of LEP results. Detailed checks and comparisons of theoretical calculations have been performed and (minor) bugs fixed. Improved calculations for B_T, B_W, y_{23}, C have become available since the LEP1 era, and will be applied retrospectively. More complete coverage of observables and energies will be possible. Consistent use of the $\log(R)$ matching scheme, with treatment of kinematic constraints for NLLA calculations will be used by all experiments and treatment of systematic errors will be harmonised. This work is nearly complete, and it appears that the net changes will be small compared to the total errors.

5. $\alpha_{\rm s}$ FROM SCALING VIOLATIONS

Scaling violations in fragmentation functions permit the determination of $\alpha_{\rm s}$ (c.f. scaling violations in DIS). Such an analysis depends on comparing data at different energy scales, which therefore involves a varying flavour mix. We thus need measurements of *flavour-dependent* fragmentation functions. Fortunately these are available, exploiting the powerful b- and c-tagging capabilities of the LEP experiments. We also need information on gluon fragmentation, which is available using 3-jet events or the "longitudinal fragmentation function". Results using LEP1 and PETRA/PEP data have been published by ALEPH [17] and DELPHI [18], obtaining $\alpha_{\rm s}(M_{\rm Z}) = 0.126 \pm 0.009$ and 0.124 ± 0.011 respectively. My average is therefore:

$$\boxed{\alpha_{\rm s}(M_{\rm Z}) = 0.125 \pm 0.009}$$

No results from LEP2 have yet been published.

6. FLAVOUR DEPENDENCE OF $\alpha_{\rm s}$

QCD requires the flavour independence of $\alpha_{\rm s}$. The flavour tagging techniques developed for LEP

electroweak measurements can also be used to test the flavour dependence of α_s. Using tagged samples, the measurement of α_s can then be made using usual observables. Quark mass effects are expected to suppress gluon radiation from heavy quarks, so it is necessary to take these into account. The most precise published results (from SLD[19]/OPAL [20]) validate the flavour independence of α_s to $\sim 1.5\%$ for b-quarks and to $\sim 5\%$ for c-quarks. An example is given in Fig. 4. Note that the measured values of α_s are flavour independent so long as mass effects are accounted for.

7. POWER CORRECTIONS AND α_s

The standard approach to the measurement of α_s using event shapes is to use Monte Carlo models (JETSET, HERWIG etc.) to account for hadronisation, i.e. to account for non-perturbative contributions at scales where α_s diverges. On general grounds, non-perturbative effects are expected to be suppressed by powers of the energy scale $Q(=\sqrt{s})$. In this case the effect $\propto \frac{1}{Q}$ (or $\propto \frac{1}{Q^2}$ in the case of y_{23}). One approach [21] is to introduce a non-perturbative parameter

$$\alpha_0(\mu_I) = \frac{1}{\mu_I}\int_0^{\mu_I} \alpha_s(q)\mathrm{d}q \qquad (6)$$

to parametrise the low energy behaviour of α_s, where $\mu_I = 2$ GeV. Predictions for event shape observables y then take the form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{\mathrm{d}\sigma_{\mathrm{Pert}}}{\mathrm{d}y}(y - \mathcal{P}D_y) \qquad (7)$$

where $\mathcal{P}$ is a universal factor, $\propto \frac{1}{Q}$, and D_y depends on the observable. The prediction is thus a simple shift of the distribution. One can therefore fit data – either differential distributions, or moments such as $\langle 1 - T\rangle$ – to determine α_s and $\alpha_0(\mu_I)$. A recent comprehensive analysis using all available data[22], fitting both moments and distributions, quotes:

$$\boxed{\alpha_s(M_Z) = 0.1171^{+0.0032}_{-0.0020}}$$

More details on this topic can be found elsewhere in these proceedings [23].

8. α_s FROM 4-JET EVENTS

The NLO (i.e. $\mathcal{O}(\alpha_s^3)$) corrections to 4-parton final states have recently been computed [24], though the $\mathcal{O}(\alpha_s^3)$ contributions to 2- and 3-jet observables are not yet known. This provides new possibilities for α_s measurements using 4-jet rates, or other observables sensitive to 4-jet production. Preliminary results have been presented by ALEPH[25] and DELPHI[26], obtaining $\alpha_s(M_Z) = 0.117 \pm 0.004$ and 0.118 ± 0.002 respectively, in good agreement with the measurements based on 3-jet observables. The errors are small because optimised scales were employed.

All the measurements of α_s mentioned so far have assumed the SU(3) group structure of QCD. Measurements of QCD colour factors may also be performed based on 4-jet events using $\mathcal{O}(\alpha_s^2)$ matrix elements, and these are independent of α_s. However, we can now use NLO 4-parton calculations and simultaneously determine both α_s and the colour factors. The first such measurements appeared recently from OPAL [27] and ALEPH [28], obtaining $\alpha_s(M_Z) = 0.120 \pm 0.011 \pm 0.020$ and $0.119 \pm 0.006 \pm 0.022$ (prelim.) respectively. The results for α_s (and colour factors) are consistent with other measurements, but the α_s values have large theoretical errors. Other talks in these proceedings discuss measurements of colour factors [29,23].

9. SUMMARY

The high precision of LEP data, together with substantial theoretical developments, have allowed great progress in the precision of α_s measurements. The measurements at LEP/SLC reported in this talk are summarised in Fig. 5, and compared with the world average (which includes LEP data) of $\alpha_s = 0.118 \pm 0.002$ [3]. There is excellent agreement among methods, and with the world average.

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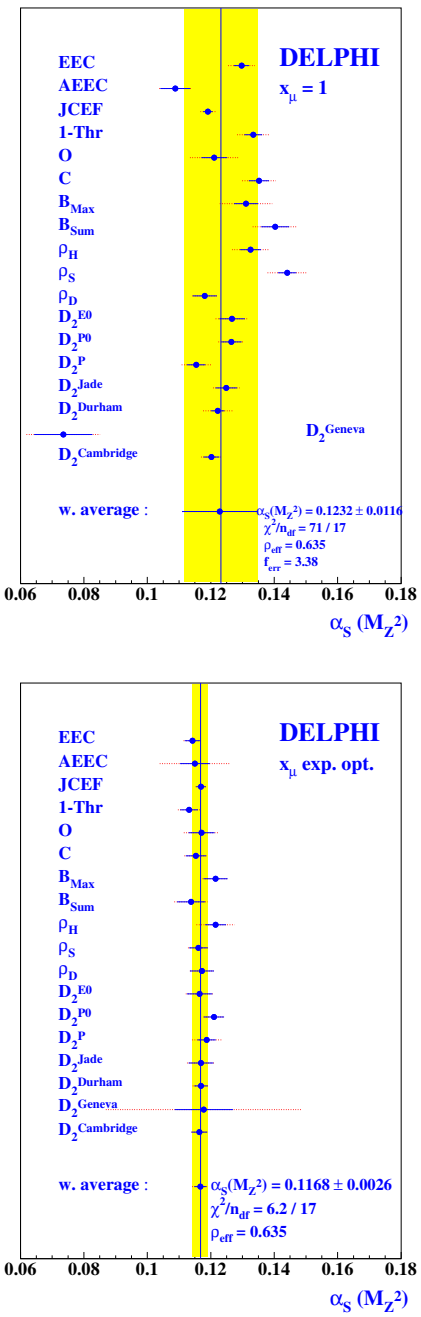


Figure 1. Values of $\alpha_s(M_Z)$ obtained from event shape variables using $\mathcal{O}(\alpha_s^2)$ QCD calculations, with x_μ=1 (above) and with x_μ optimised by fitting to data (below) [9].

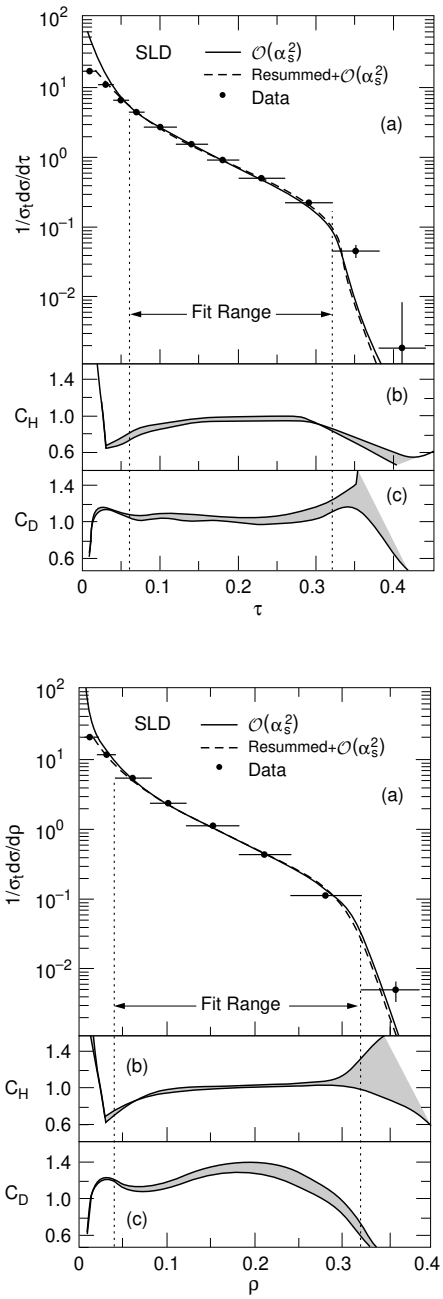
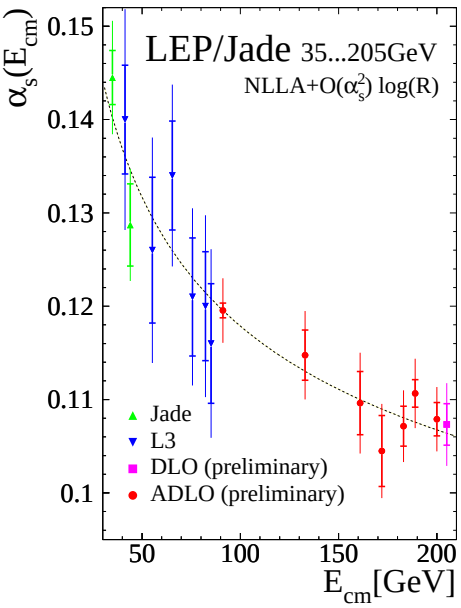


Figure 2. Fits of $\mathcal{O}(\alpha_s^2)$+NLLA QCD to data for $\tau = (1-T)$ and $\rho = M_H^2/s$, with $\mathcal{O}(\alpha_s^2)$ fits for comparison [12] The insets show detector and hadronization corrections.



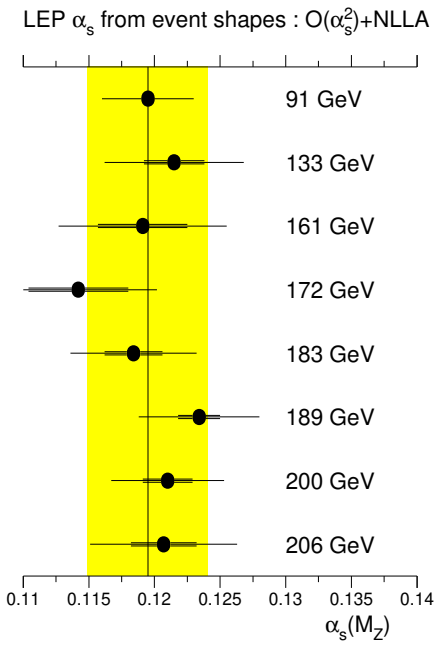


Figure 3. Compilation of α_s measurements at LEP1 and LEP2 [16]. The upper plot shows the values as a function of c.m. energy, the curve corresponding to $\alpha_s(M_Z) = 0.1195$. The lower plot shows the measurements evolved to a common scale M_Z, with the result of the combined fit shown as a band.

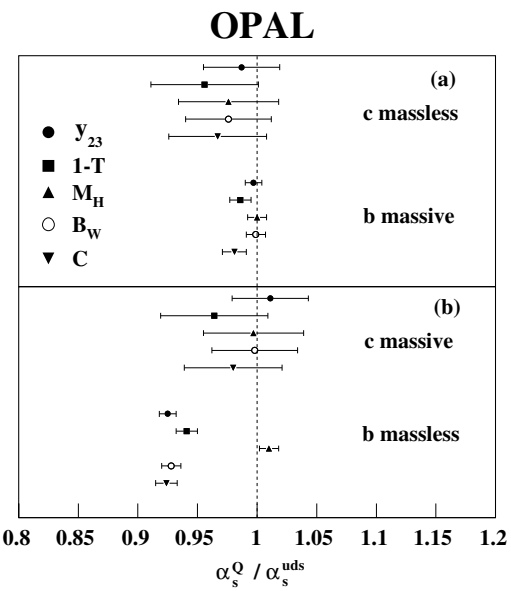


Figure 4. Ratio of α_s for b- or c-quarks to α_s for light quarks [20].

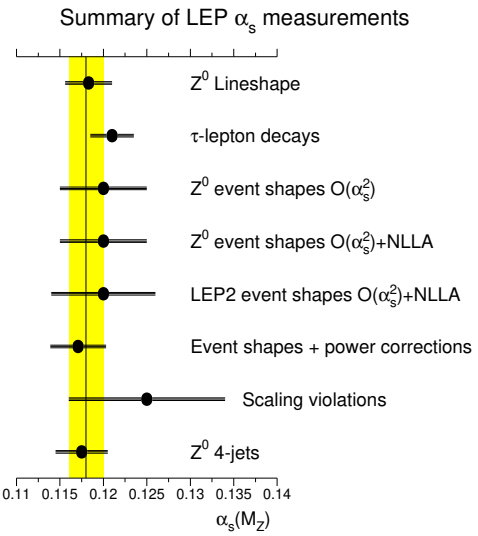


Figure 5. Compilation of measurements of $\alpha_s(M_Z)$ from LEP/SLC. The world average [3] is indicated by a band.