QCD Tests

at Present and Future Colliders

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Overview:

1) Comments on Event Shape Distributions.
2) The Evolution of Event Shape Means with Energy
   The QCD β-Function
3) Massive 3-Jets – The Running Mass of the b-Quark
5) 3-Jets – Gluon and Quark-Jet Multiplicities
4) 4-Jets – QCD Colour Factors
5) Summary

Describing Event Shape Spectra

Topology of hadronic events measured by event shapes f (e.g. $f = 1 - \mathrm{Thrust}$).
Distributions described by different calculations:

- $\mathcal{O}(\alpha_s^2)$; renorm. scale factor $x_\mu = \mu/E_{CM}$

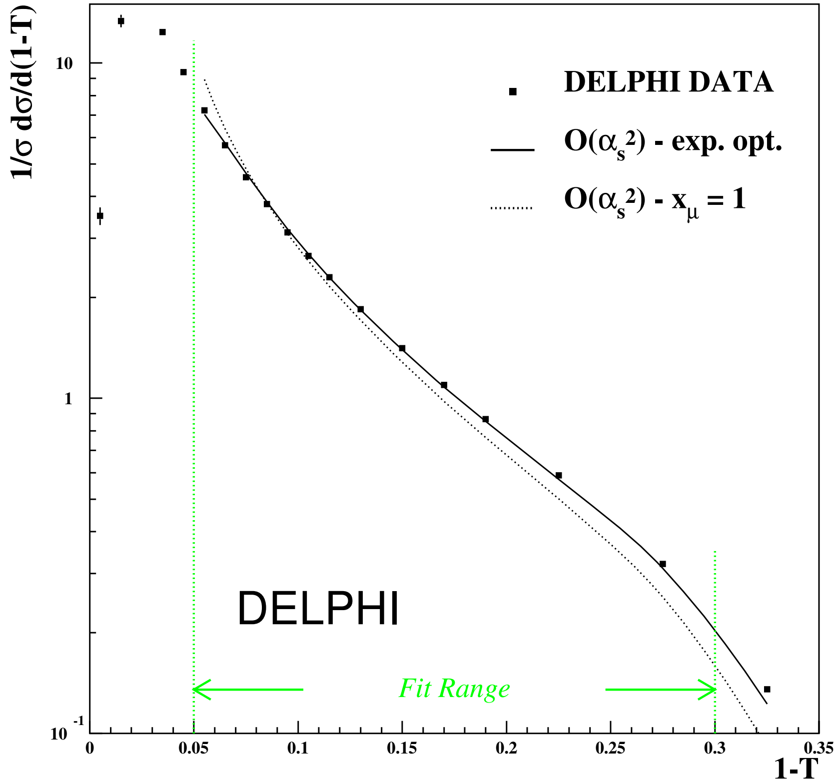
$$\frac{1}{\sigma}\frac{d\sigma}{df} = D(f)_{\mathrm{pert}} =$$

$$A_f \cdot \bar{\alpha}_s(\mu) + (A_f \cdot \beta_0 \log x_\mu + B_f)\,\bar{\alpha}_s(\mu)^2$$

- 2 jet regime: resummed LLA/NLLA
- 2/3 jet regime: matched $\mathcal{O}(\alpha_s^2|x_\mu = 1)$/NLLA
- for all apply MC hadronisation corrections

$$C = \frac{D_{parton}(f)}{D_{hadron}(f)}$$

- very precise data; $\mathcal{O}(10^6)$ events

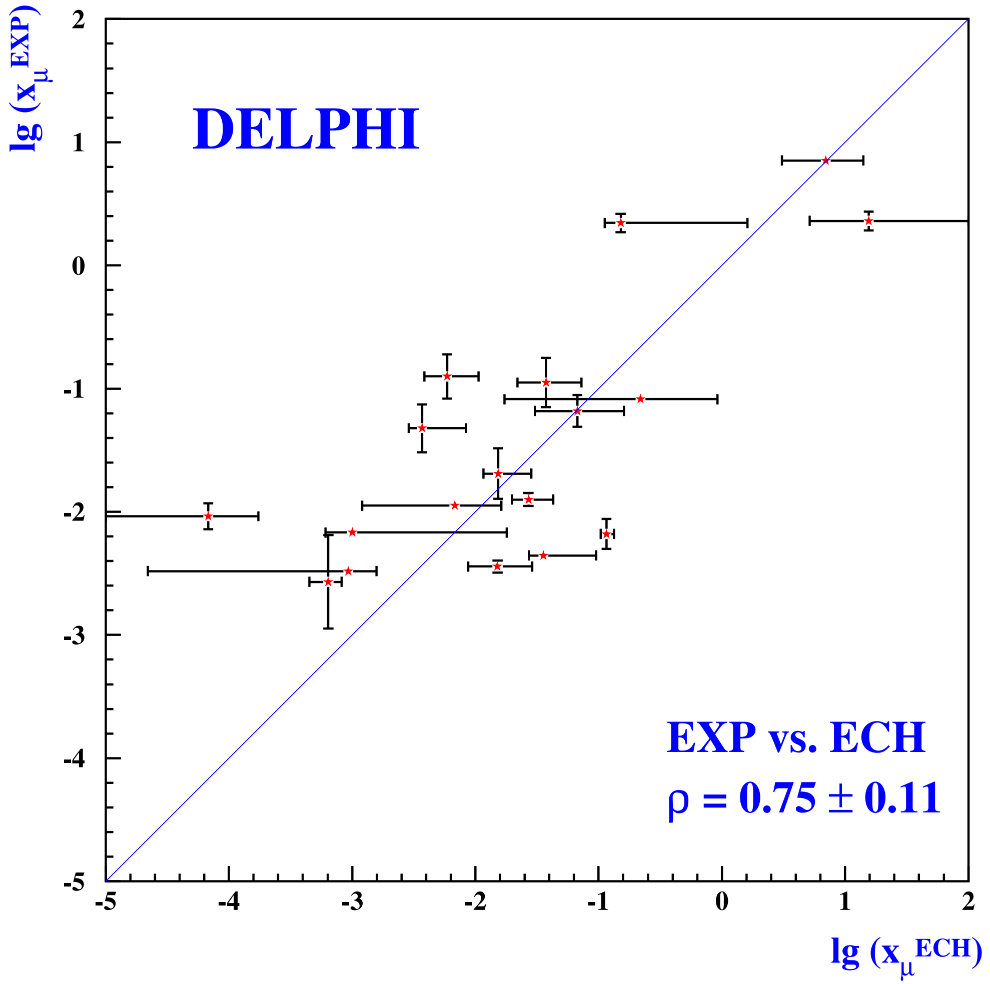


$\mathcal{O}(\alpha_s^2)$ prediction in general inconsistent with data except x_μ is optimised!

Only then: $\alpha_s(M_Z)$ values obtained from different observables consistent!

Describing Event Shape Spectra

Observe correlation of exp. optim. scales and theor. ECH/PMS/FAC scale choices



Theoretical scales choices:

- FAC fastest apparent convergence
  truncated = all orders prediction
- PMS principle of minimal sensitivity
  makes x_μ dependence minimal
- ECH effective charge
  resums collinear divergencies using β-function

- values of these scales are similar

Theoreticians dislike scale fixing.
No room for theoretical uncertainty.

Correlation remains to be understood!

Describing Event Shape Spectra

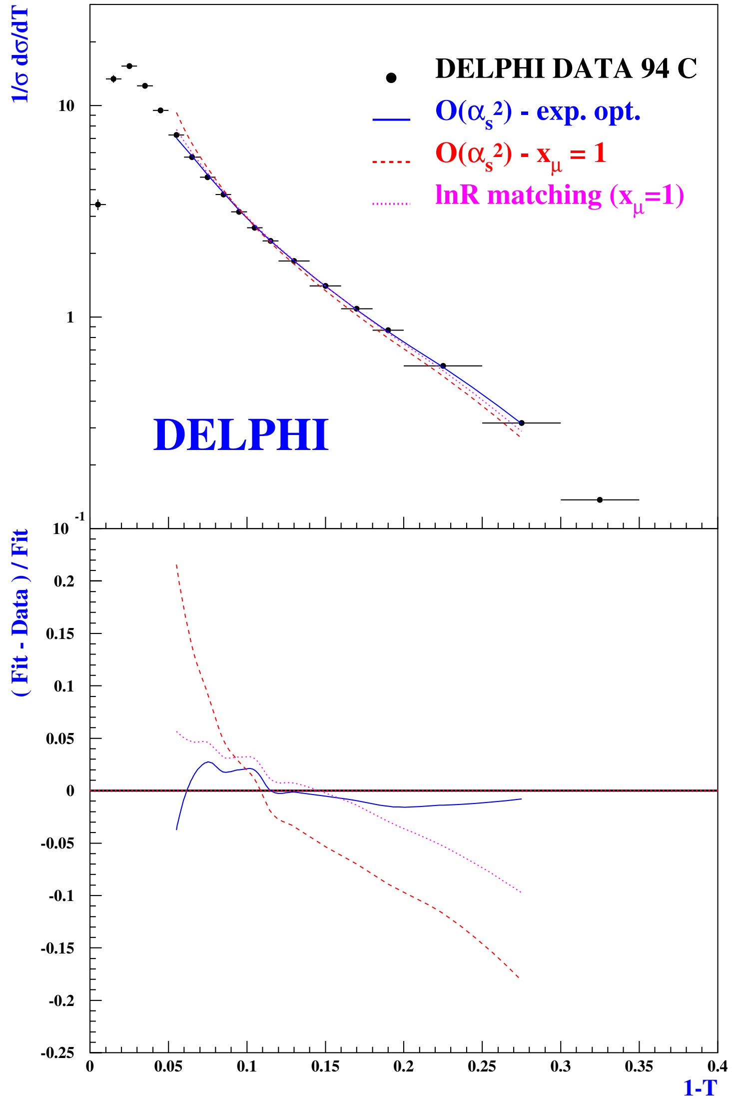
Matching NLLA and $\mathcal{O}(\alpha_s^2|x_\mu = 1)$ predictions:

Expected to improve prescription

Observe residual shortfall à la $\mathcal{O}(\alpha_s^2|x_\mu = 1)$

- Extracted $\alpha_s(M_Z)$ fit range dependend
- Implies bias (↑) of $\alpha_s(M_Z)$

Thrust is worst case $\longrightarrow$



Improvements required

$\mathcal{O}(\alpha_s^3)$ predictions?

Matching including optimised scales?

Other more complete calculations.

Energy Evolution of Means of Event Shape Observables

Alternatively to MC models: Power Corrections
Hadronisation and perturbative gluon radiation
assumed independent

$$\langle p_\perp^2 \rangle = \langle p_\perp^2 \rangle^{pert.} + \langle p_\perp^2 \rangle^{hadr.}$$

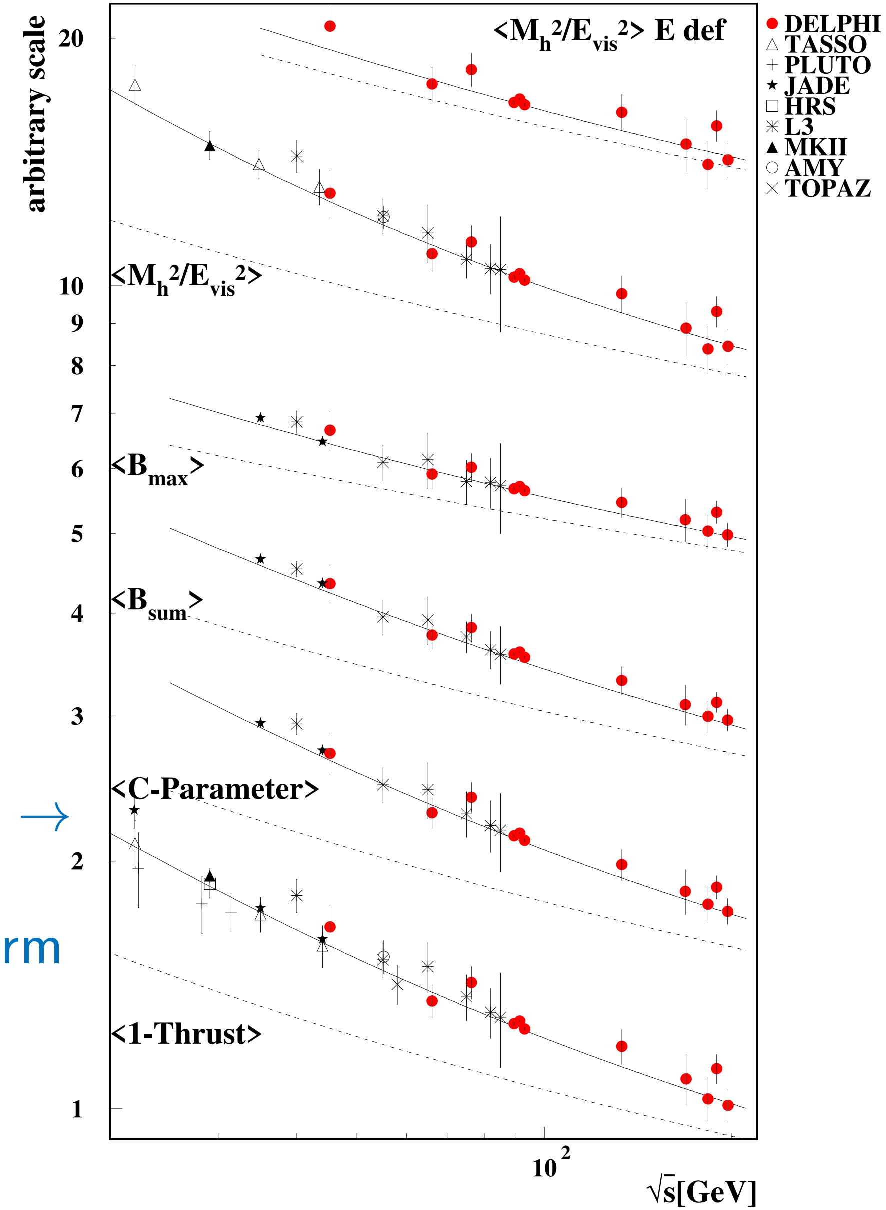
$\langle p_\perp^2 \rangle^{pert.} \propto E_{cm}^2 \quad ; \quad \langle p_\perp^2 \rangle^{hadr.} \simeq const. \rightarrow$
Hadronisation effects vanish $\propto 1/E_{cm}$

Fit for non-perturbative constant C

$$\langle f \rangle = \langle f \rangle^{pert.} + \frac{C}{E_{CM}}$$

full fit $\rightarrow$

$\overline{MS}$ term



More elaborated: pert. relate different observ.
aim for universal non-perturbative constant C

Energy Evolution of Means of Event Shape Observables

However:

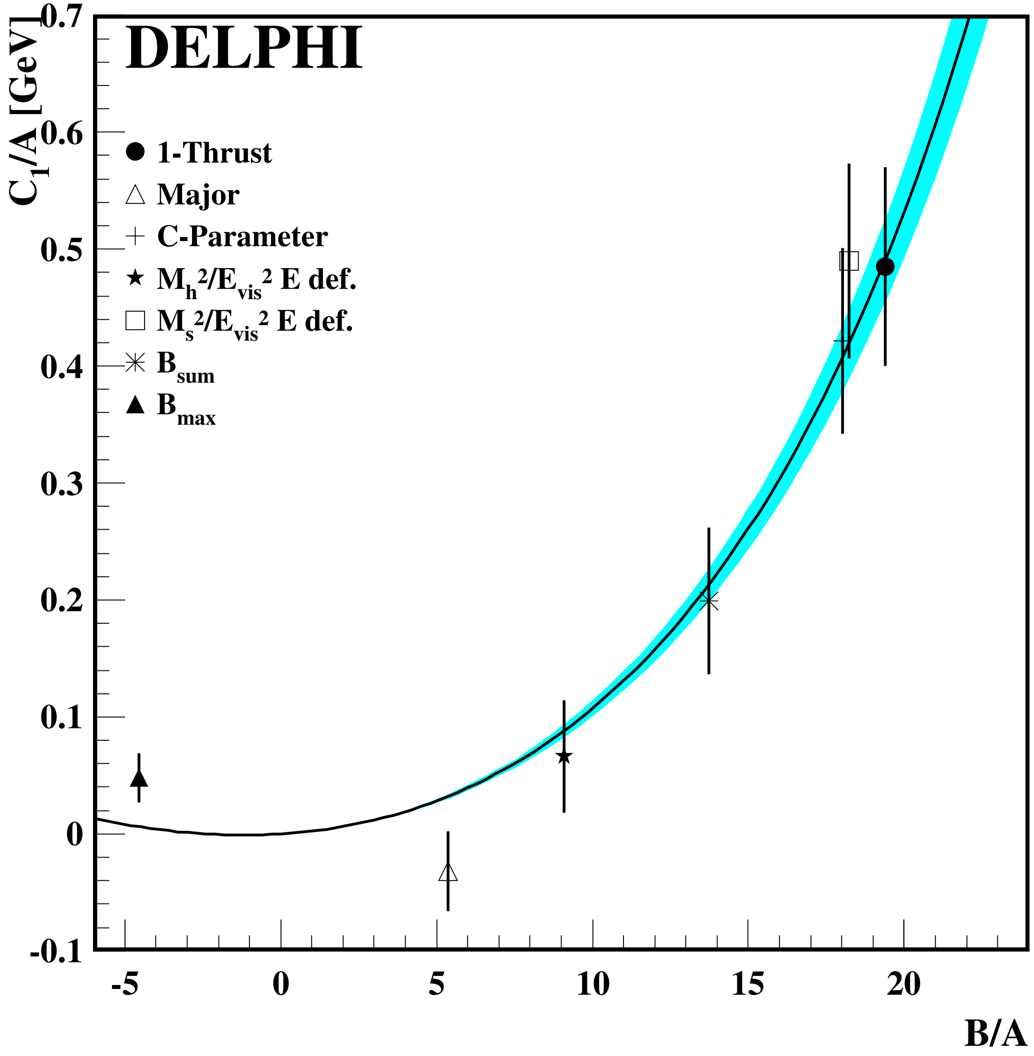
observe strong correlation of
non-pert. C and $2^{\rm nd}$ order pert. B_f

C not purely non-perturbative!

partly expected:
pert./non-pert. borderline is ambigous

Can C be predicted perturbatively?

Yes: Renormalisation Group Invariant (RGI) pert. theory



Energy Evolution of Means of Event Shape Observables – RGI

Require observable $R = \langle f \rangle / A_f$ to fulfill Renormalisation Group Equation (RGE):

$$\frac{dR^{-1}}{d\log Q^2} = -\beta_0 \cdot (1 + \rho_1 \cdot R + \rho_2 \cdot R^2 + \ldots) = \rho(R)$$

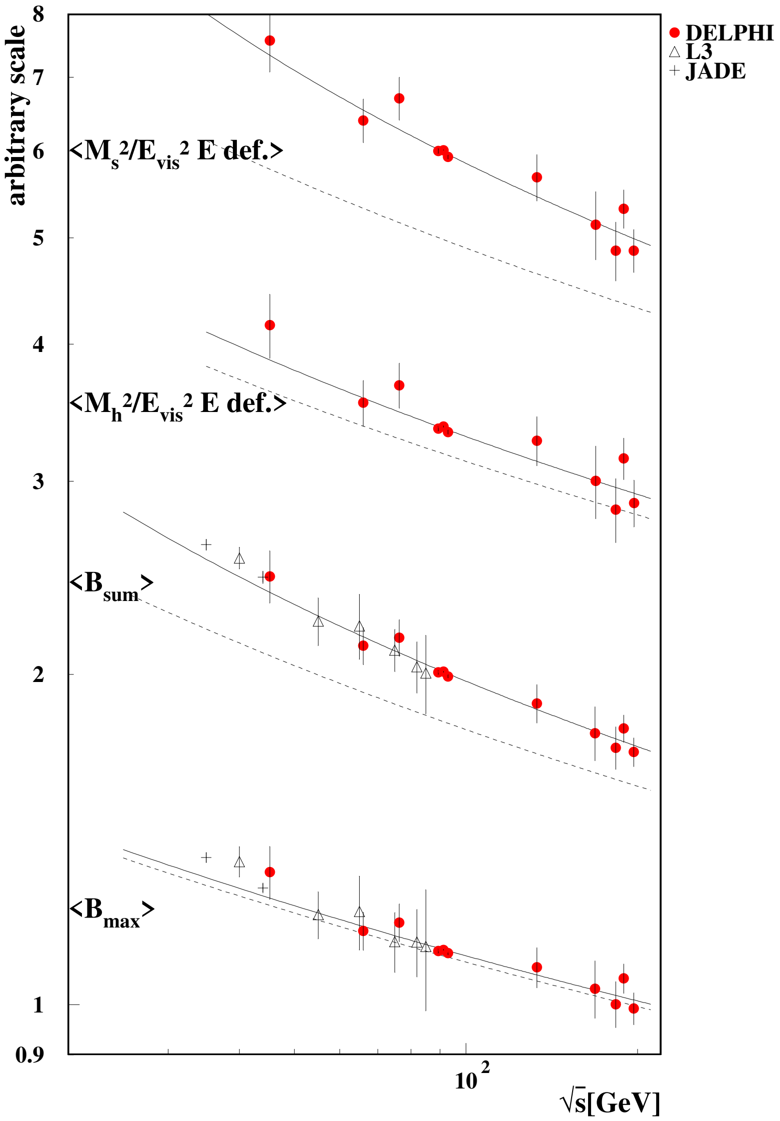
$\beta_0 = (11C_A - 2n_f)/3$ and ρ_i scheme inv. const.

Solution: equiv. to implicit eqn. for $\alpha_s(\mu)$

$$\beta_0 \log \frac{Q^2}{\Lambda_R^2} = \frac{1}{R} - \rho_1 \log \left\{ 1 + \frac{1}{\rho_1 R} \right\} + \ldots$$

Λ_R directly related to $\Lambda_{\overline{\mathrm{MS}}}$, $\alpha_s(M_Z)$

- RGI here only for obs. depending on one E scale
- RGI equivalent to ECH scale choice
- Can be enhanced by power term
- Data corrected for (b) mass effects



RGI describes E dep. of shape obs. means well
$\alpha_s(M_Z) \sim 0.120 \pm 0.002_{RMS}$ values consistent. Observe power terms insignificant!

β-Function from Energy Evolution of Shape Means

RGI applies for individual measurements

$\rightarrow$direct measurement of QCD β-function
Rem: β_0 and ρ_i universal

β-function: slope of e.g. $1/\langle 1 - T \rangle$ plot
From final data of low E expts + DELPHI

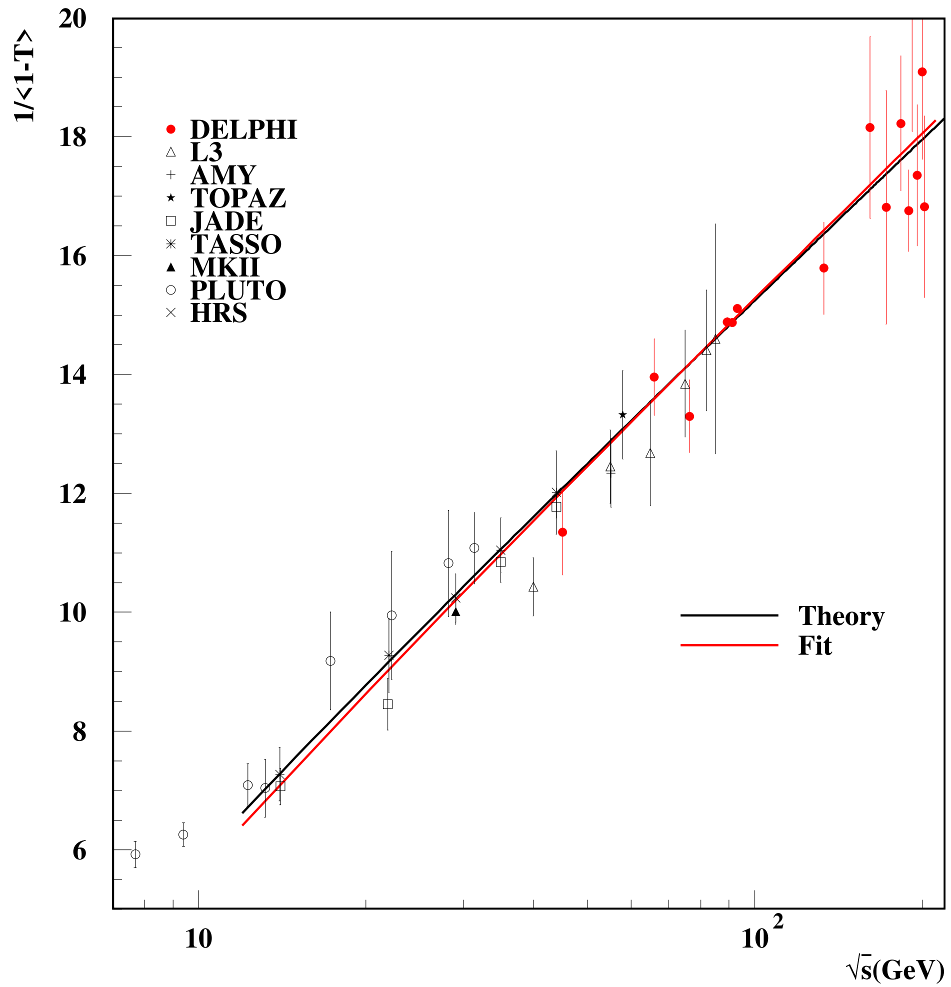
$\beta_0 = 7.86 \pm 0.32$ QCD :7.67

assuming QCD: number of active flavours

$n_\mathrm{f} = 4.75 \pm 0.44$ QCD :5

From α_s running $(R_\tau, F_2, F_3, R_\mathrm{Z}, \dots)$

$\beta_0 = 7.76 \pm 0.43$



Measurement impies limits on new physics e.g. light Gluinos
Repeat with full data of all LEP experiments; HERA ?
Residual power terms: more low energy $\mathrm{e^+e^-}$ data (Cornell?)

Measurement of the b Quark Mass at the Z

- Quark masses are free parameters of the QCD Lagrangian
- Known only to limited precision as quarks bound inside hadrons
- Access to quark masses only via dynamical relations
- Mass effects in jet like x-sections enlarged: $\propto \frac{m_b^2}{p_{\perp g}^2} = \frac{m_b^2}{M_Z^2} \cdot \frac{1}{y_{cut}} \sim 0.3\% \cdot \mathcal{O}(10)$
- Besides pert. pole mass M_b define renormalised mass $m_b(Q)$ in $\overline{\mathrm{MS}}$ scheme. This running mass absorbs radiative corrections like $\alpha_s(Q)$.

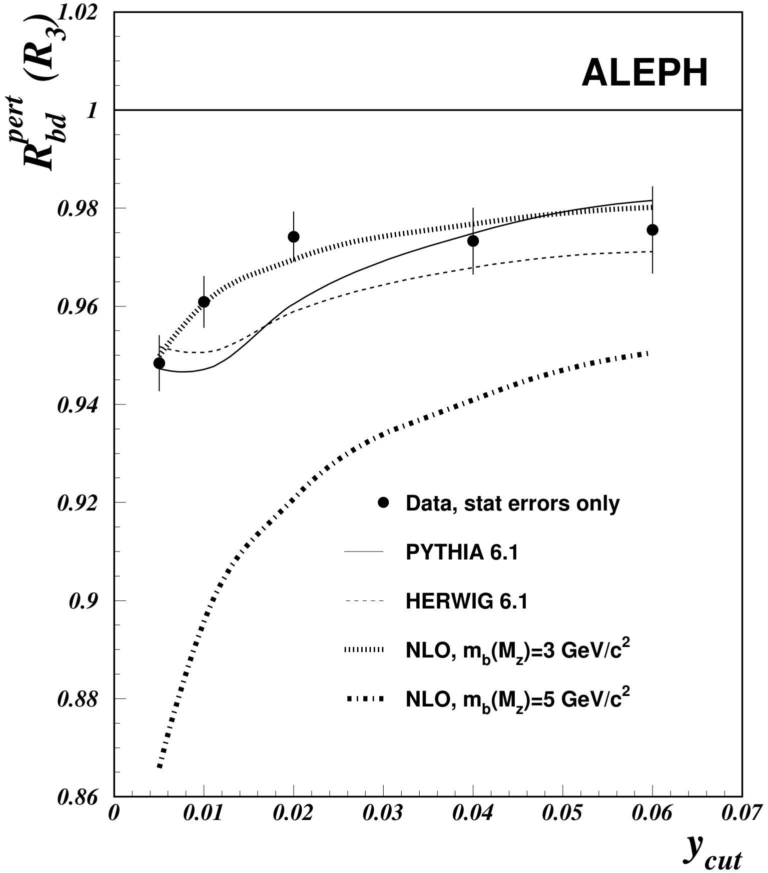
Measurement:

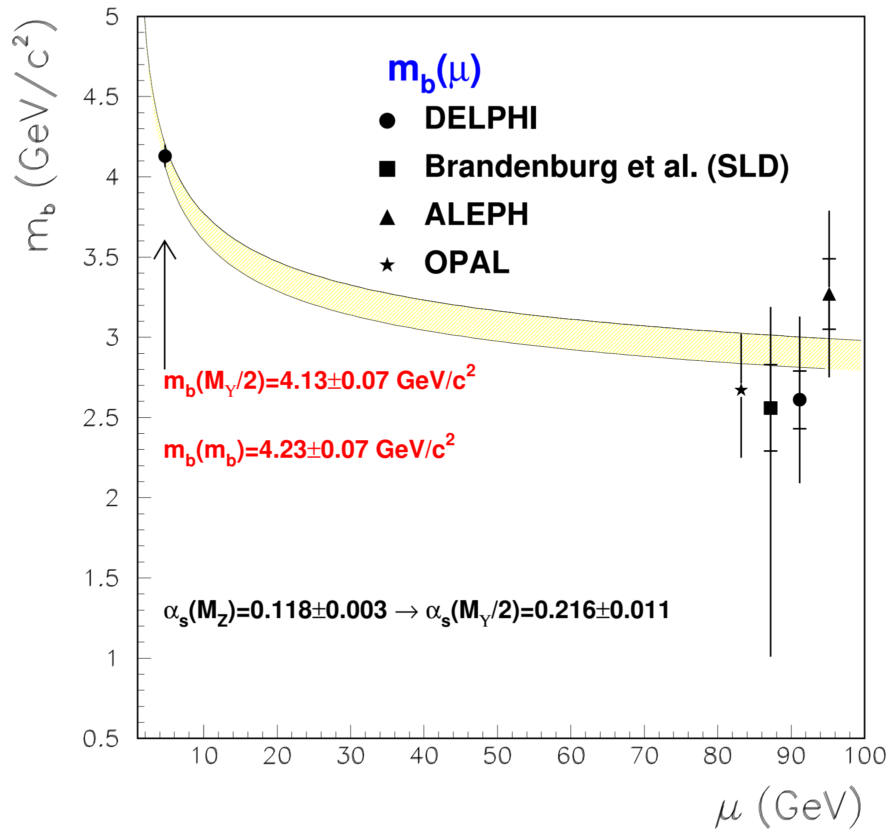
- Compare b/inclusive ratio of jet rates/event shapes obs.
- b-identificaton using impact param. tag. Purity 65-90%
- Corrections: limited acceptance, tag bias, hadronisation (partly neglected)

- ALEPH: several shapes use R_3, $\langle y \rangle$
- Brandenburg et al (SLD): several jet algos (assign syst. for discr.)
- DELPHI: R_3 Durham, Cambridge
- Opal: T, M_h, B_w, y_3, C use correlation between observables (???)

Theory: four independent calculations

Measurement of the b Quark Mass at the Z





Typical result (DELPHI prel.)

$m_b(M_Z) =$

$2.61 \pm 0.18 \pm 0.47_{hadc.} \pm 0.18_{b-tag} \pm 0.12_{th}$

Improvement: understand fragmentation

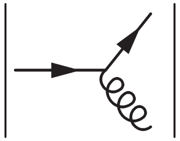
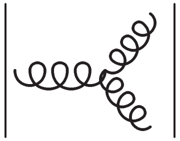
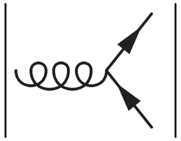
- Good agreement between experiments
- Mass running as expected

Future improvements ?

NNLO with masses

Multiplicity of Three Jet Events or Gluons vs. Quarks

Colour factors (eigenvalues of Casimir operators of gauge group) $\Longleftrightarrow$ gauge group

 $\propto C_F \cdot \alpha_s$  $\propto C_A \cdot \alpha_s$  $\propto T_F \cdot \alpha_s$ not divergent

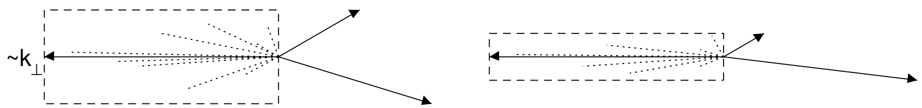
multiplicity ratio $\dfrac{\text{gluon jet}}{\text{quark jet}} = \begin{cases} C_A/C_F = 2.25 & \text{naively expected at high energy} \\ \sim 1.1 - 1.4 & \text{observed PEP/PETRA/LEP} \end{cases}$

Understood: consider gluon coherence, biases due to selection, non. pert. effects

Measure multiplicity of 3 jet $\mathrm{q\bar{q}g}$ events, theory considers:

- coherence ($p_\perp^2$ scales), just add gluon and quark multiplicities,

  $$N^{ch}_{\mathrm{q\bar{q}g}} = N^{ch}_{\mathrm{q\bar{q}}}(s_{\mathrm{q\bar{q}}}, y_{cut}) + \frac{1}{2} \cdot N^{ch}_g(p^2_{\perp g})$$

- 

  phase space restriction of $\mathrm{q\bar{q}}$ system due to g jet

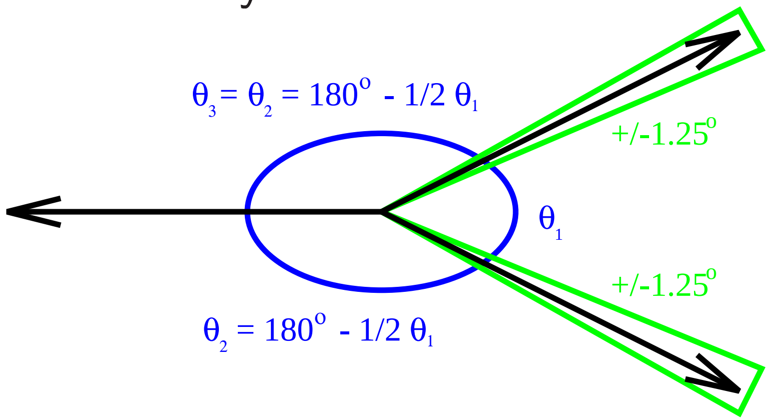
- finite energy (non-pert.) gq differences

  $$\left.\frac{dN_{gg}(L')}{dL'}\right|_{L'=L+c_g-c_q} = \frac{C_A}{C_F}\left(1 - \frac{\alpha_0 c_r}{L}\right)\frac{d}{dL}N_{q\bar{q}}(L)\ , \qquad L = \log\frac{s}{\Lambda^2}\ , \ c_i = \text{const.}$$

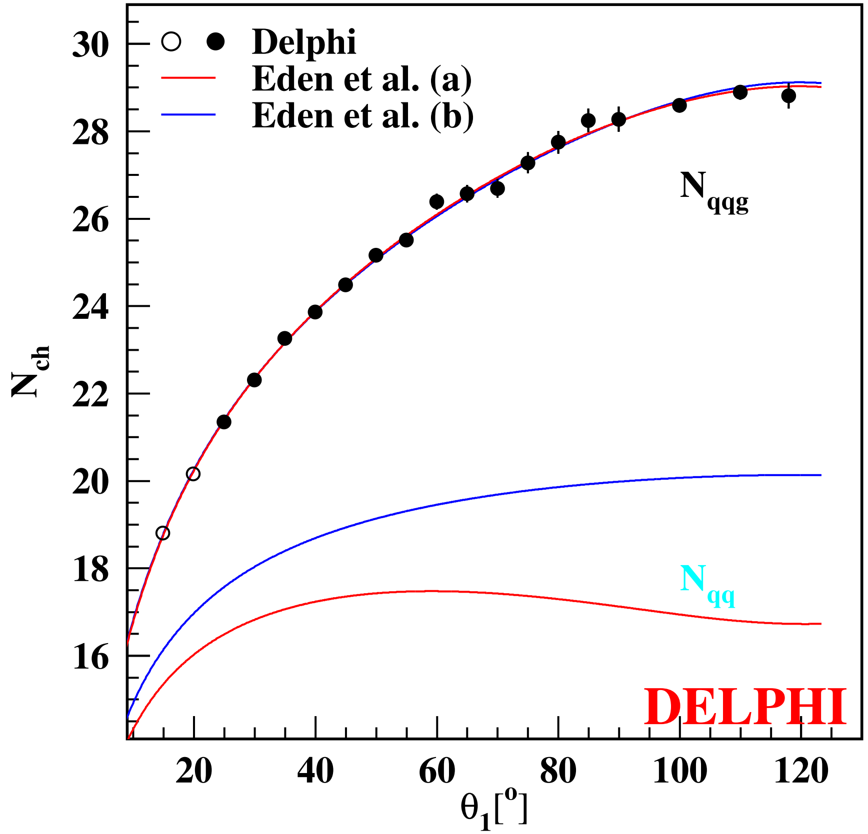
Differential eqn. for slopes, $\Longrightarrow$ non-pert. integration const. α_0 for multiplicity

Multiplicity of Three Jet Events or Gluons vs. Quarks

- Measure symmetric events



  - No jet identification!
  - High statistics
  - Take scales from opening angle
- C_A/C_F from multiplicity slope



$$\frac{C_A}{C_F} = 2.221 \pm 0.032(stat.) \pm 0.047(exp.) \pm 0.058(hadr.c.) \pm 0.075(theo.)$$

Multiplicity of Colour Singlet gg Events

- Extract multiplicity of gg events:

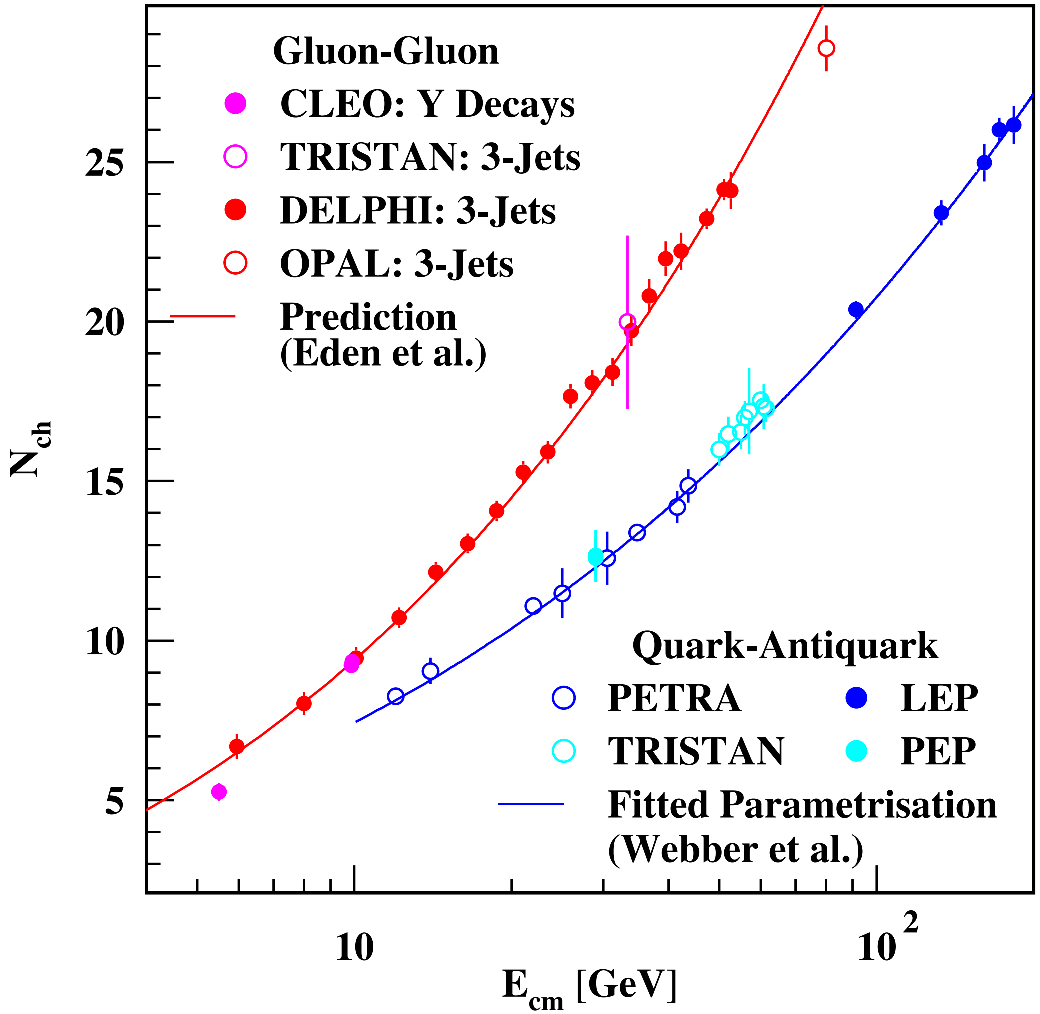
$$N_g^{ch} = 2 \cdot \left(N_{q\bar{q}g}^{ch} - N_{q\bar{q}}^{ch}\right)$$

- Slope for gluons
  $\sim$ twice the slope for quarks

  $\Longrightarrow C_A/C_F$!

- Good agreement with direct gluon measurements (CLEO)

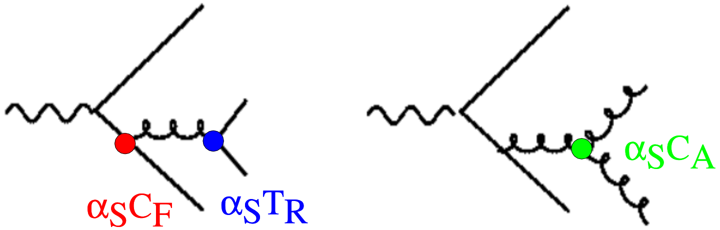
- Recently similar OPAL analysis



Improvements: apply similarly to $p\bar{p}$ multiplicity; requires theoretical calculation
Try to really understand tagging of gluon jets

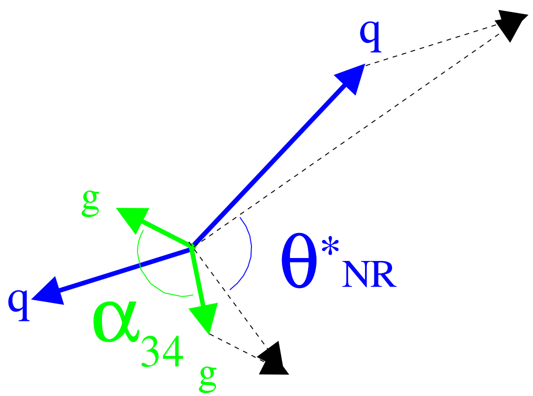
Colour Factors from Four Jet Events

4 jet evts access full QCD gauge structure



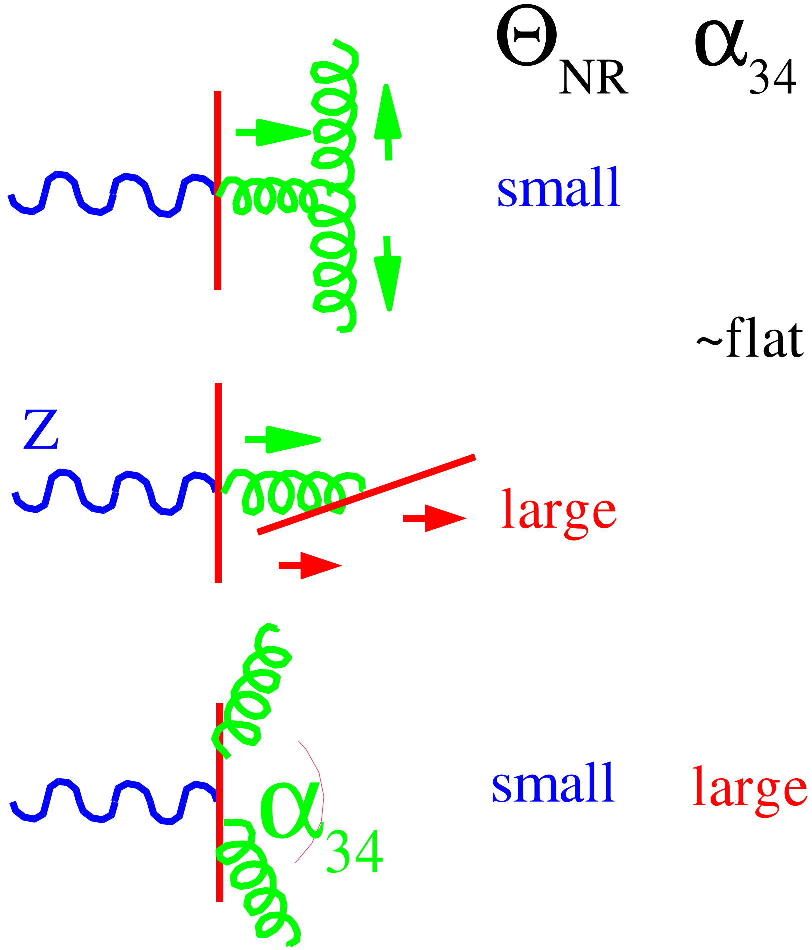
Decompose colour contributions using helicity structure.

Define sensitive angles (e.g.):



$\Theta^*_{NR} \sim \angle$(betw. prim. and sec. partons)

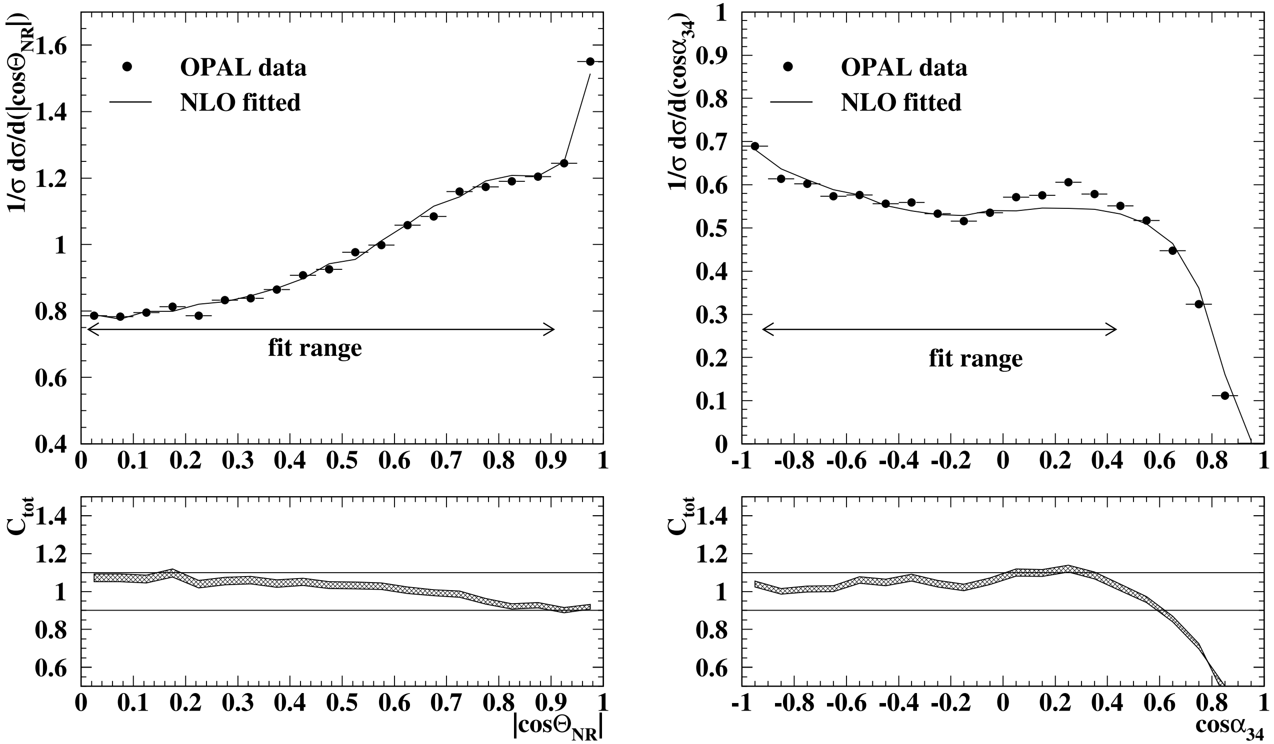
$\alpha_{34} \sim \angle$(betw. secondary partons)



Initial gluon polarised, applies to any vector particle. Spin 0 or 2 case different.

Colour Factors from Four Jet Events

New OPAL & ALEPH: NLO 4 jet calculation, include als resum./matched 3 jet rate.



Reconstruct jets, unfold detector and (some) hadronisation

Primary fit yields: $\frac{C_A}{C_F}$, $\frac{n_f \cdot T_F}{C_F}$ and α_s.

Solve for individual colour factors.

Improvement: Full NNLO 3 jet predictions

Opal :

$$C_A = 3.02 \pm 0.25(stat.) \pm 0.49(syst.)\ ,\ C_F = 1.34 \pm 0.13(stat.) \pm 0.22(syst.)$$

Aleph:

$$C_A = 2.93 \pm 0.14(stat.) \pm 0.49(syst.)\ ,\ C_F = 1.34 \pm 0.07(stat.) \pm 0.22(syst.)$$

But : Correlations are large! QCD: $C_A = 2.25$, $C_F = 1.33$

Summary

Looking back

LEP & SLD produced first QCD "precision" results and obtained many genuinely new results

Some have been discussed

- m_b and its running
- β function
- Colour factors from four jet events and g/q multiplicity

For illustration $\Longrightarrow$