Tests of the Standard Model in the τ sector at LEP

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- Introduction
- Neutral current
  - Universality of $\tau\tau Z$ coupling with lineshape and tau polarisation
- Charged current
  - Universality tests with τ lifetime and branching ratios
  - Lorentz structure
- Conclusions

Introduction

- The τ is a sequential heavy lepton: e, μ, τ
- Production in $\mathrm{e}^+\mathrm{e}^- \longrightarrow \tau^+\tau^-$ via neutral current

  Test of $\tau\tau Z$ vertex
- Decay via charged current (also production in $W \longrightarrow \tau\nu$)

  Test of $\tau\nu_\tau W$ vertex
- Various τ decay modes:

  Leptonic: $\mu^-\nu_\tau\bar{\nu}_\mu$, $\mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\tau$

  Hadronic: $\pi^-\nu_\tau$, $\rho^-\nu_\tau$, $a_1^-\nu_\tau$, etc.
- Can compare τ decay properties to μ decay, and also π, K decay
- Rare decays: domain of B-factories
- Note: will include non-LEP τ decay results where competitive.

τ neutral current couplings

- production process: $e^+e^- \rightarrow Z \rightarrow \tau^+\tau^-$
- axial-vector coupling dominant: $a_\tau \gg v_\tau$
- Cross-section $\propto a_\tau^2 + v_\tau^2$
- Polarisation parameter $\mathcal{A}_l$:

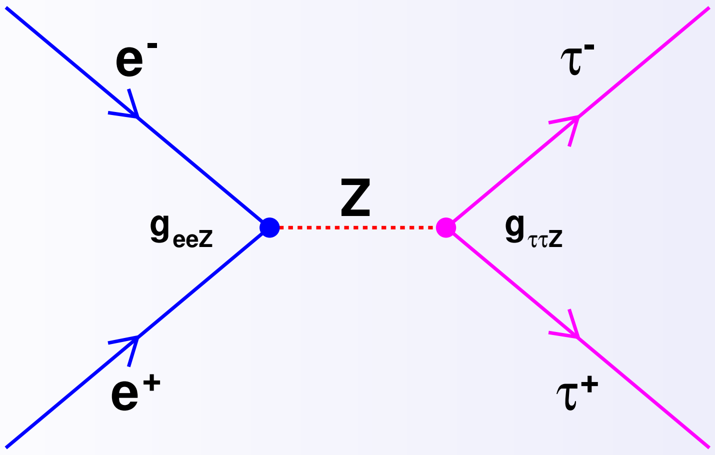
$$\boxed{\mathcal{A}_l = \frac{2v_l a_l}{a_l^2 + v_l^2} \approx \frac{2v_l}{a_l}}$$

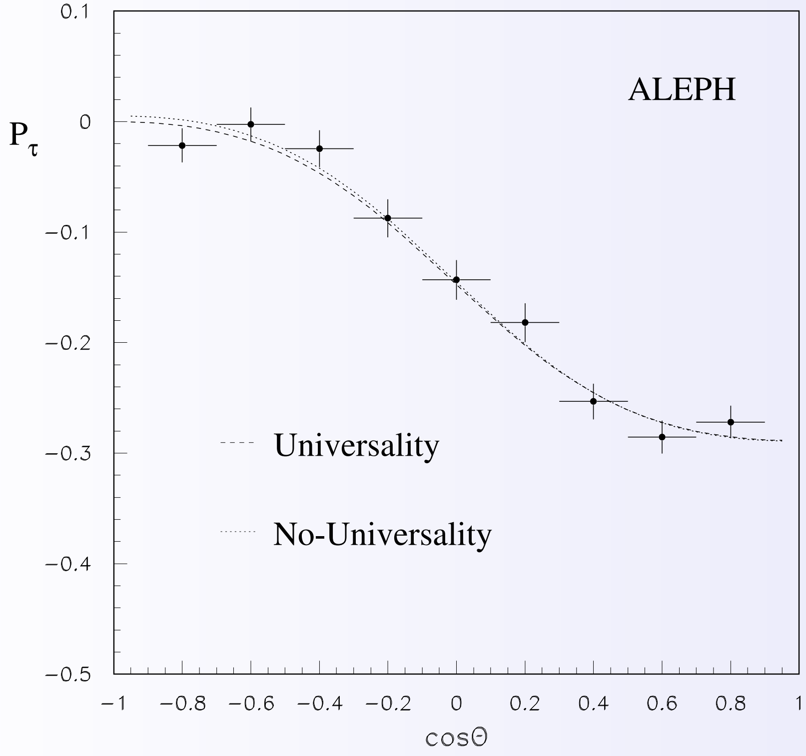
- Forward backward asymmetry:

$$\boxed{A_{FB} \approx \frac{3}{4}\mathcal{A}_e\mathcal{A}_\tau}$$

- Tau polarisation:

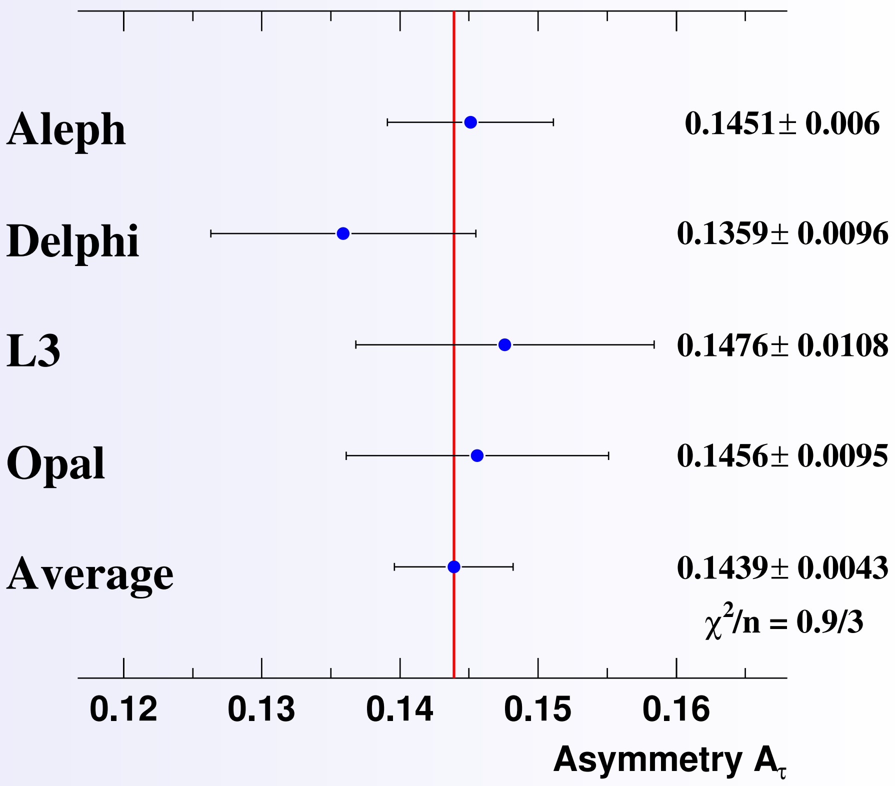
$$\boxed{\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau + \mathcal{A}_e \frac{2\cos\theta}{1+\cos^2\theta}}{1+\frac{4}{3}A_{FB}\frac{2\cos\theta}{1+\cos^2\theta}}}$$



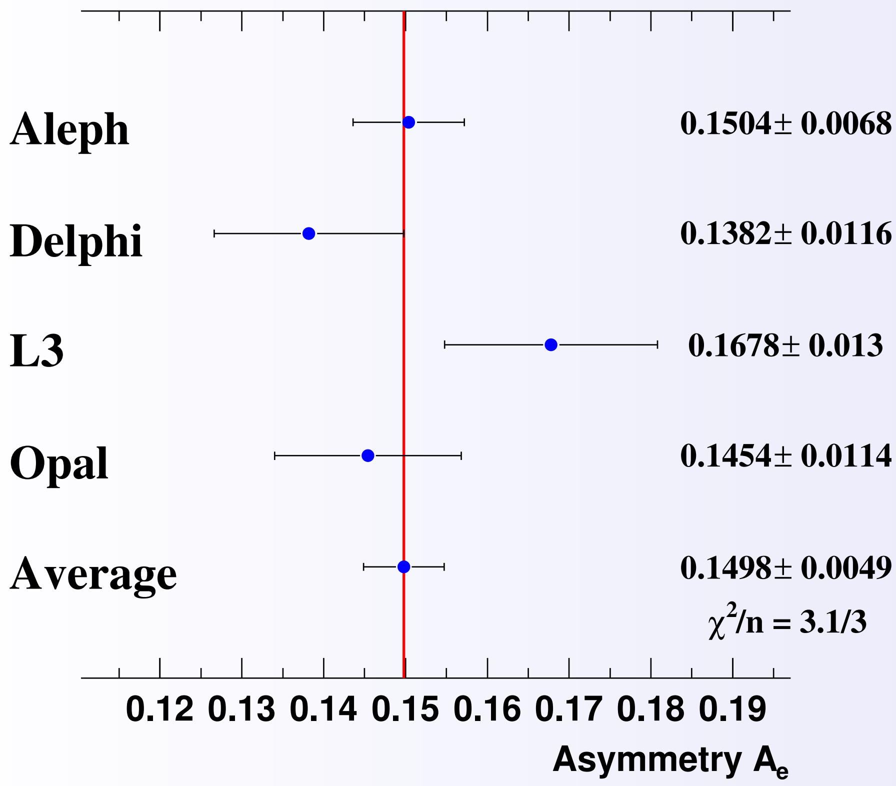


Tau polarisation: combinations

$$\boxed{\mathcal{A}_\tau = \frac{2v_\tau a_\tau}{a_\tau^2+v_\tau^2}}$$



$$\boxed{\mathcal{A}_e = \frac{2v_e a_e}{a_e^2+v_e^2}}$$



$$\mathcal{P}_\tau(\cos\theta) \Longrightarrow \boxed{\mathcal{A}_l = 0.1465 \pm 0.0033}$$

Tau NC couplings: comparison

From LEP electroweak fits:

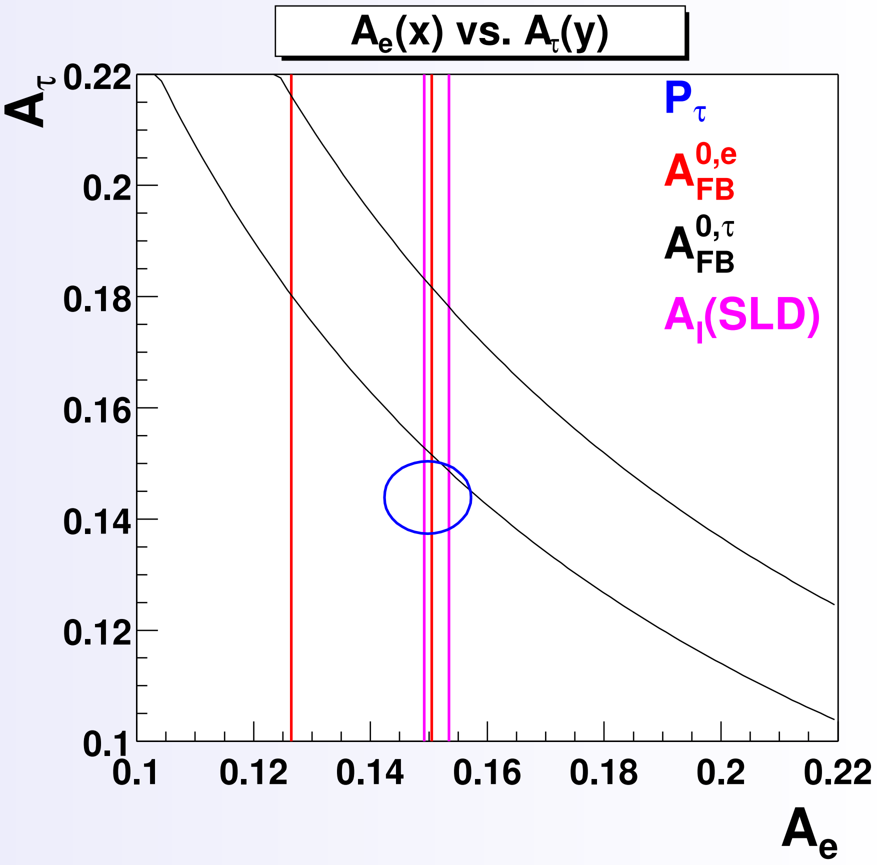
$$\boxed{\frac{\bar{a}_\tau}{\bar{a}_e} = 1.0018 \pm 0.0015}$$

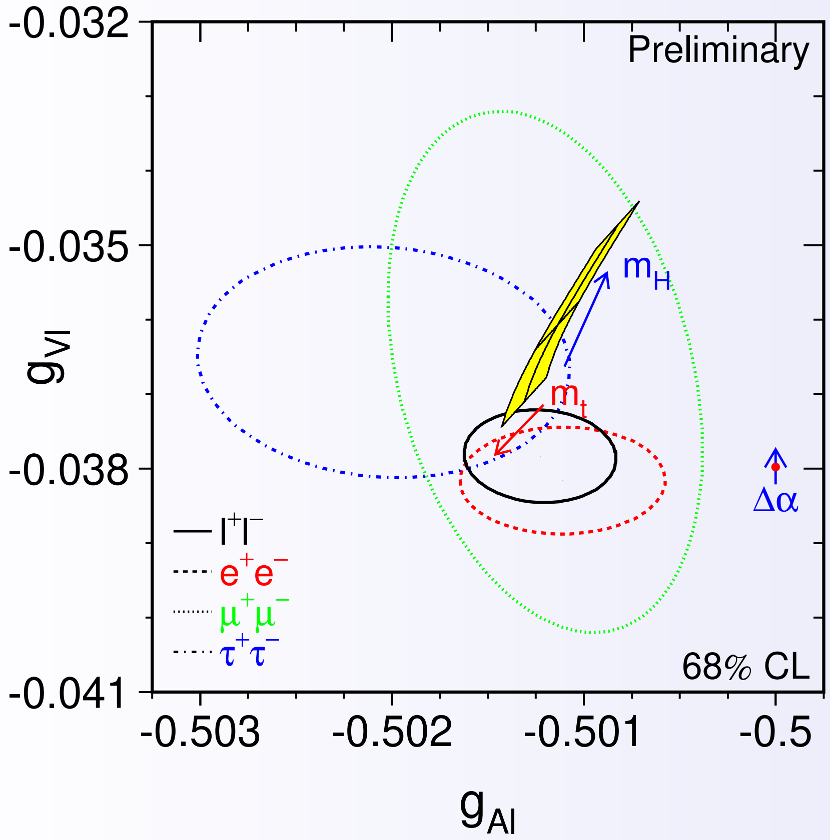
$$\boxed{\frac{\bar{v}_\tau}{\bar{v}_e} = 0.973 \pm 0.041}$$

Compare with muon couplings:

$$\boxed{\frac{\bar{a}_\mu}{\bar{a}_e} = 1.0001 \pm 0.0014}$$

$$\boxed{\frac{\bar{v}_\mu}{\bar{v}_e} = 0.995 \pm 0.096}$$





Neutral weak dipole moments

New $\tau\tau Z$ interaction:

$$\mathcal{L} = \frac{1}{2}\frac{e a_\tau^W}{2m_\tau}\bar{\phi}\sigma^{\mu\nu}\phi Z_{\mu\nu} - \frac{i}{2} e d_\tau^W \bar{\phi}\sigma^{\mu\nu}\phi Z_{\mu\nu}$$

SM (loops):

$$d_\tau^W \approx 3 \cdot 10^{-37} \;\; e \cdot \mathrm{cm}, \quad a_\tau^W = (-2.1 + 0.6i) \cdot 10^{-6}$$

LEP (M. Pilar Casado, HEP2001):

$$Re(d_\tau^W) = (-0.46 \pm 1.48) \cdot 10^{-18} \;\; e \cdot \mathrm{cm}$$
$$Im(d_\tau^W) = (-1.78 \pm 3.29) \cdot 10^{-18} \;\; e \cdot \mathrm{cm}$$
$$Re(a_\tau^W) = (+0.23 \pm 0.47) \cdot 10^{-3}$$
$$Im(a_\tau^W) = (-0.42 \pm 1.03) \cdot 10^{-3}$$

Compatible with zero

Tau decay

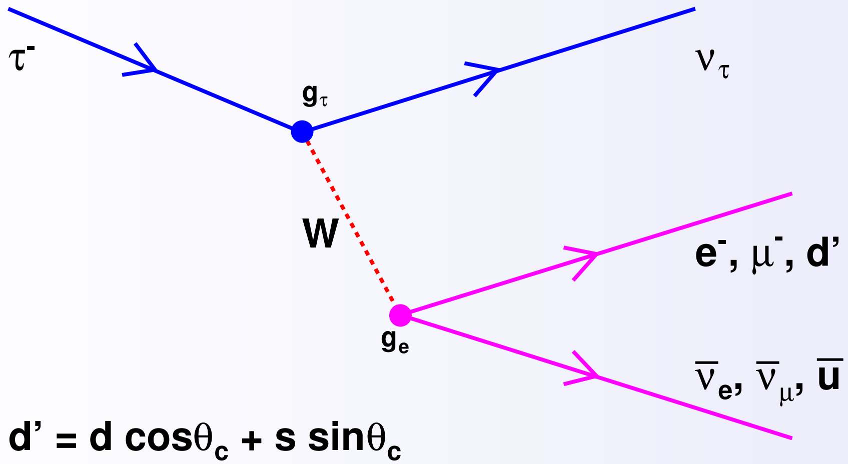
$$\Gamma(\tau \rightarrow l\nu_\tau\overline{\nu}_l) \equiv \frac{B(\tau\rightarrow l\nu_\tau\overline{\nu}_l)}{\tau_\tau} = \frac{G^2_{l\tau} m^5_\tau}{192\pi^3} \cdot P(\frac{m_l}{m_\tau}) \cdot r^\tau_{RC}$$

where:

- $G_{l\tau} = \frac{g_\tau g_l}{4\sqrt{2}m^2_W} \quad (\equiv G_F)$

- $P(x) = 0.97256$ for $\tau \rightarrow \mu$
  $= 1.00000$ for $\tau \rightarrow$ e

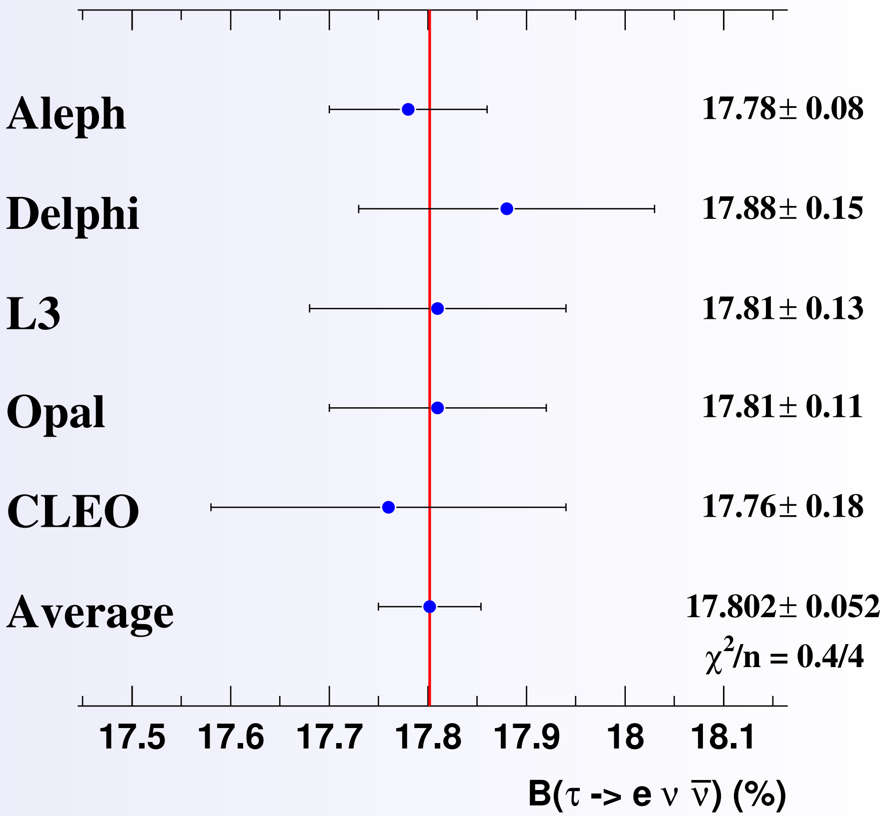
- $r^\tau_{RC} = \left[1 + \frac{3m^2_\tau}{5m^2_W}\right] \times \left[1 + \frac{\alpha(m_\tau)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]$
  ≈ 0.9960 for τ decay



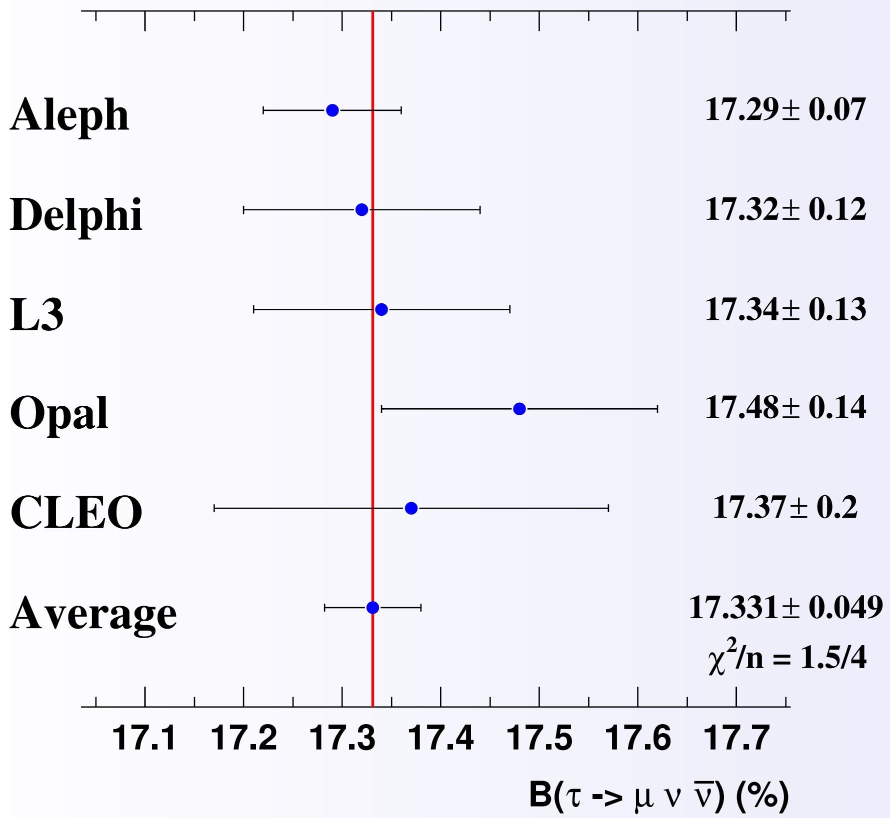
d' = d cosθ_c + s sinθ_c

Tau leptonic branching ratios

$B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$



$B(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu)$



Lep dominant - excellent lepton identification (low systematics)

each has 0.3% relative precision

Universality tests from leptonic BRs

$\mu - e$ universality from leptonic BR's:

$$\frac{g_\mu^2}{g_e^2} = \frac{P(m_e/m_\tau)}{P(m_\mu/m_\tau)} \cdot \frac{B_\mu}{B_e} \implies \boxed{\frac{g_\mu}{g_e} = 1.0005 \pm 0.0022}$$

Comparable precision to the ratio obtained from pion branching ratios $B(\pi^- \to \mu^- \bar{\nu}_\mu)$ and $B(\pi^- \to e^- \bar{\nu}_e)$:

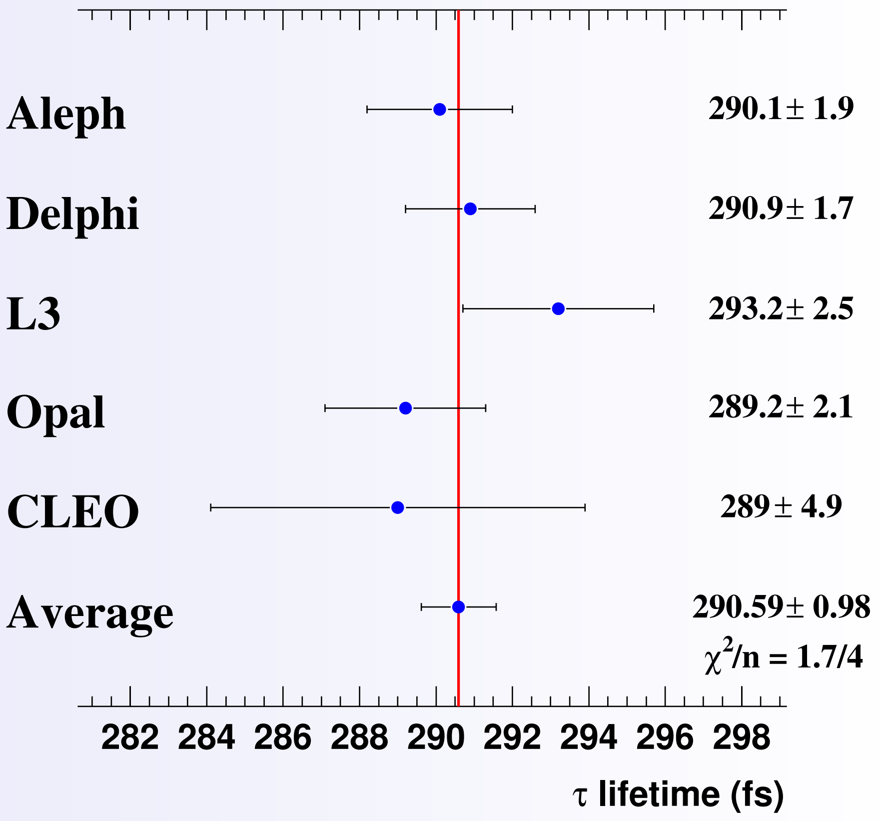
$$\left(\frac{g_\mu}{g_e}\right)_{Long} = 1.0017 \pm 0.0015$$

Can derive more precise "electronic" BR assuming $e - \mu$ universality:

$$B_{e,\mu} = (17.811 \pm 0.033)\%$$

Tau lifetime

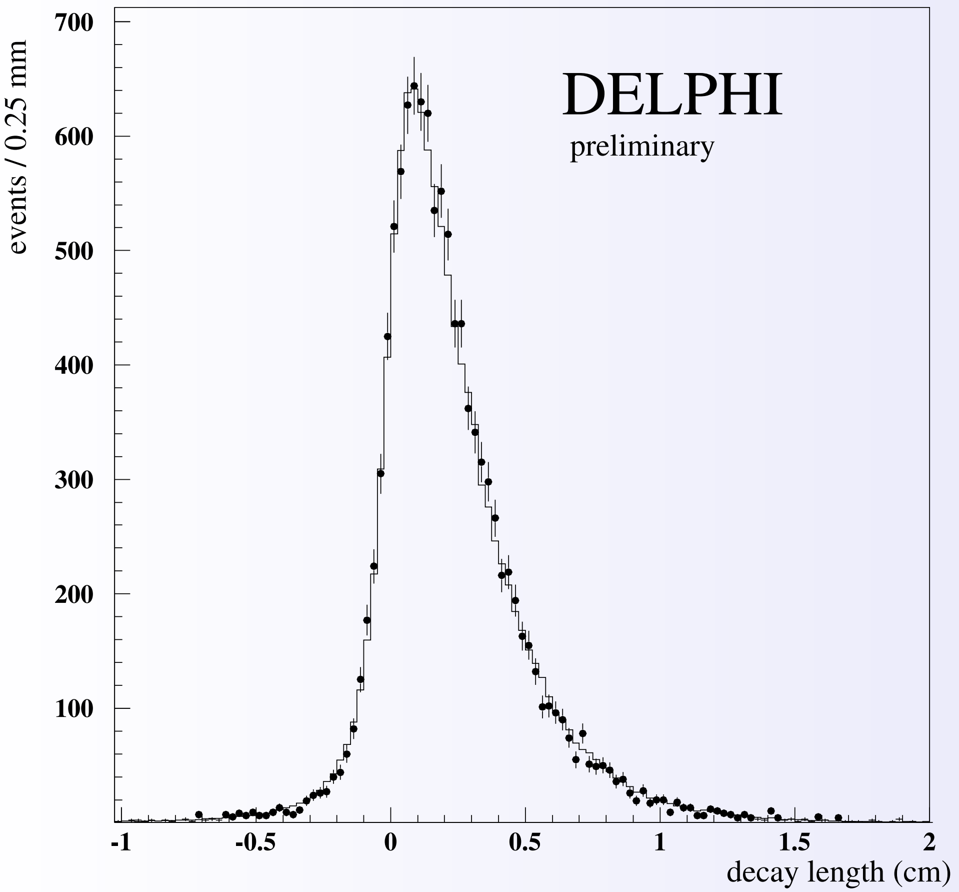
Lifetime combination



Lep dominates: Silicon VD + boost

0.35% relative precision

Typical analysis:
DELPHI 3-prong decay distance



Universality tests with lifetimes and BRs

Test $\tau - e$ and $\tau - \mu$ universality from leptonic BR's and lifetimes:

$$\frac{g_\tau^2}{g_\mu^2} = \frac{P(m_e/m_\mu)}{P(m_e/m_\tau)} \cdot \frac{r_{RC}^\mu}{r_{RC}^\tau} \cdot \frac{m_\mu^5}{m_\tau^5} \cdot \frac{\tau_\mu}{\tau_\tau} \cdot B_e \implies \boxed{\frac{g_\tau}{g_\mu} = 0.9999 \pm 0.0023}$$

$$\frac{g_\tau^2}{g_e^2} = \frac{P(m_e/m_\mu)}{P(m_\mu/m_\tau)} \cdot \frac{r_{RC}^\mu}{r_{RC}^\tau} \cdot \frac{m_\mu^5}{m_\tau^5} \cdot \frac{\tau_\mu}{\tau_\tau} \cdot B_\mu \implies \boxed{\frac{g_\tau}{g_e} = 1.0004 \pm 0.0023}$$

Test $\tau - (e, \mu)$ universality with derived $B_{e,\mu}$:

$$\boxed{\frac{g_\tau}{g_{e,\mu}} = 1.0002 \pm 0.0020}$$

Universality tested to 0.2% level

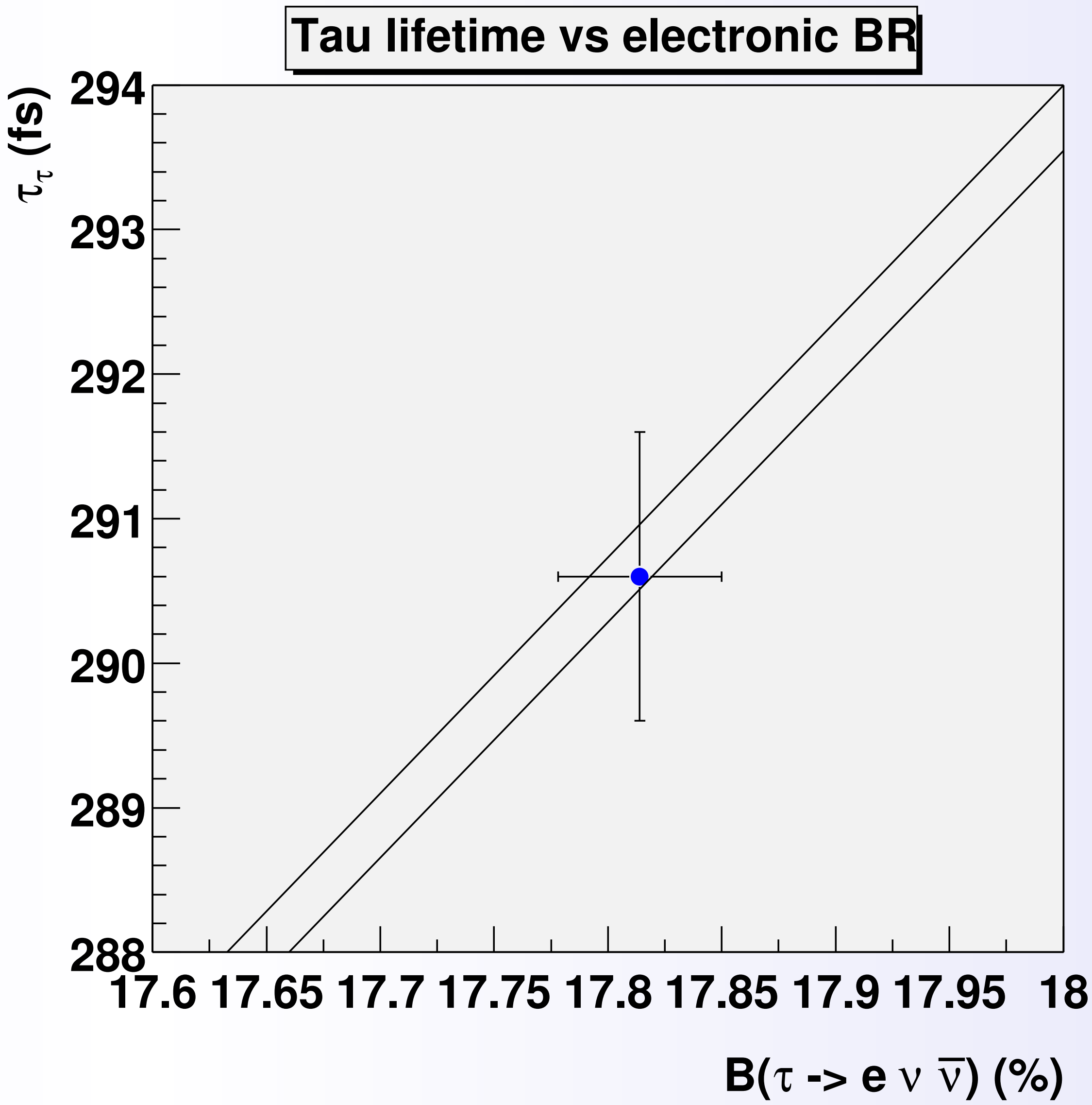
W-decays: test universality at $\mathcal{O}(1 - 2\%)$ level

Universality check: tau lifetime vs electronic BR

$$\tau_\tau = 0.9996 \cdot \frac{g_\mu^2}{g_\tau^2} \cdot \frac{m_\mu^5}{m_\tau^5} \cdot \tau_\mu \cdot B_{e,\mu}$$

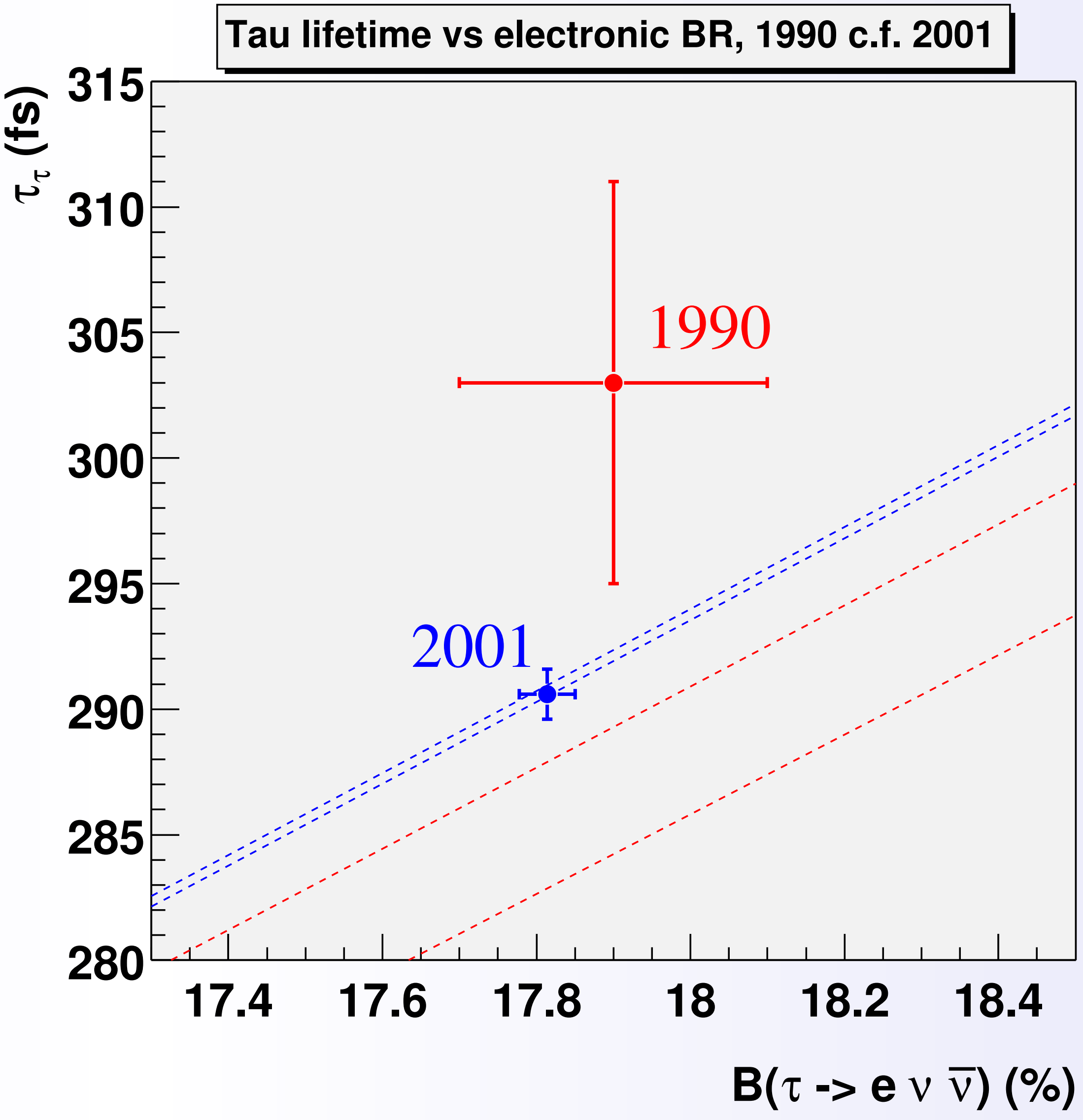
Assume $g_\tau = g_\mu$: get prediction for lifetime in terms of leptonic BR.

$$m_\tau = 1776.99^{+0.29}_{-0.26}\ \mathrm{MeV}$$



Tau lifetime vs electronic BR: history

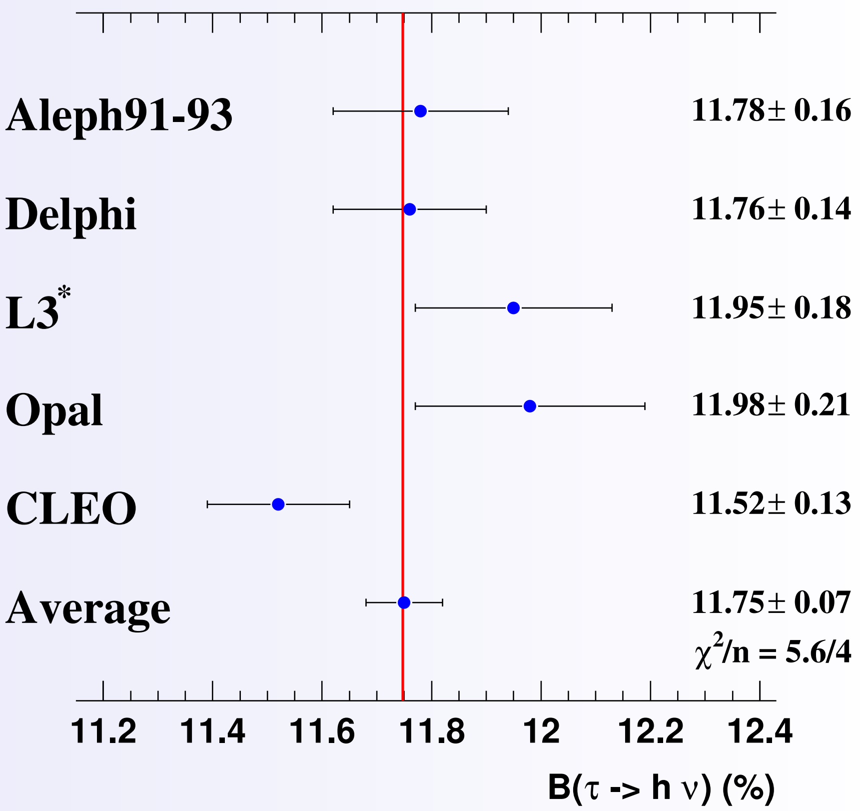
- Big improvement in precision due to LEP measurements.
- Improved τ_τ-B_e agreement in 2001 c.f. 1990.
- Important change in m_τ due to BES measurement.
- Improvement in precision:
  - lifetime: factor 8
  - BRs: factor 6



Universality test with hadronic branching ratios

$\tau - \mu$ universality with π, K lifetimes and muonic BRs

$$\boxed{B(\tau^- \to h^- \nu_\tau)}$$



$$\frac{g_\tau^2}{g_\mu^2} = \frac{2m_\mu^2}{m_\tau^2} \cdot \frac{B(\tau^- \to h^- \nu_\tau)}{H_\pi + H_K}$$

where

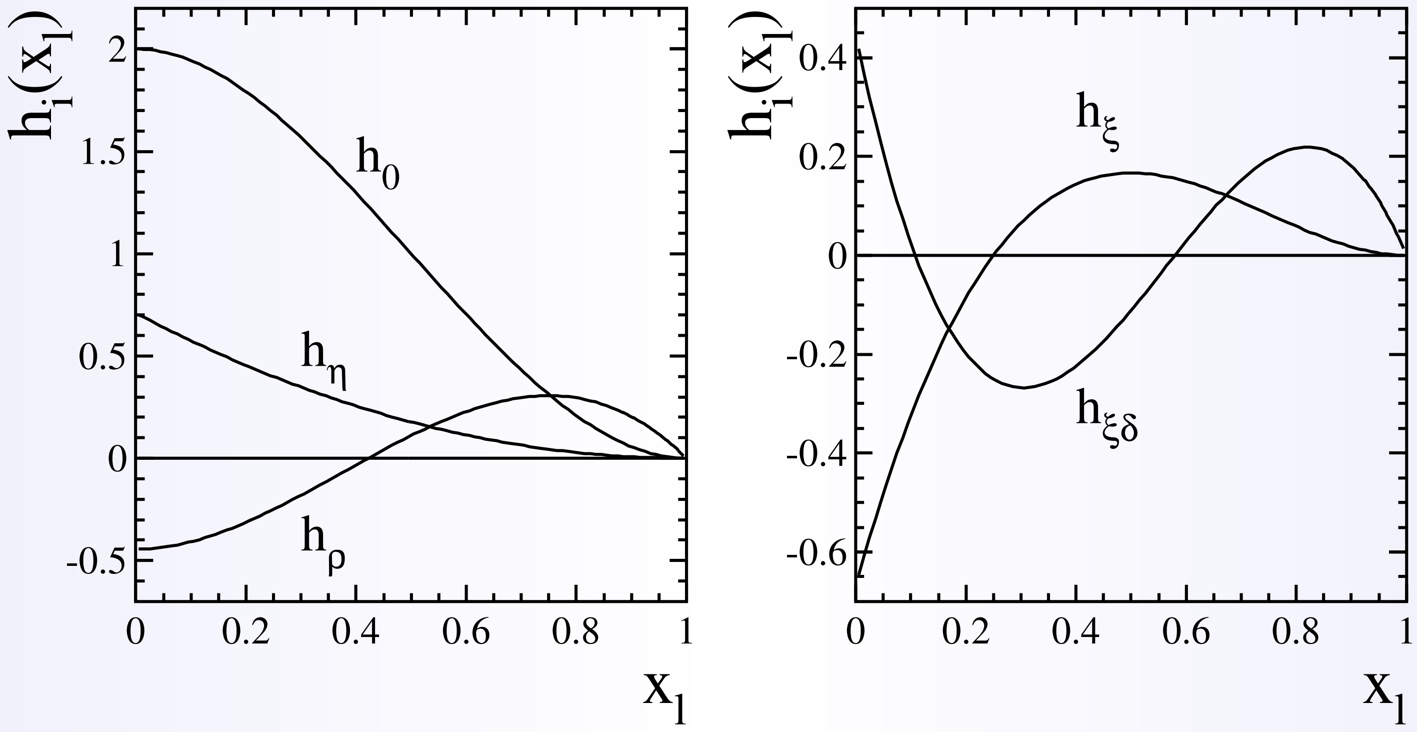
$$H_h = (1 + \delta_h) \cdot \frac{\tau_\tau}{\tau_h} \cdot \frac{m_\tau}{m_h} \cdot \frac{1 - m_h^2/m_\tau^2}{1 - m_\mu^2/m_h^2} \times B(h^- \to \mu^- \bar{\nu}_\mu)$$

$$\boxed{\left(\frac{g_\tau}{g_\mu}\right)_L = 1.0037 \pm 0.0037}$$

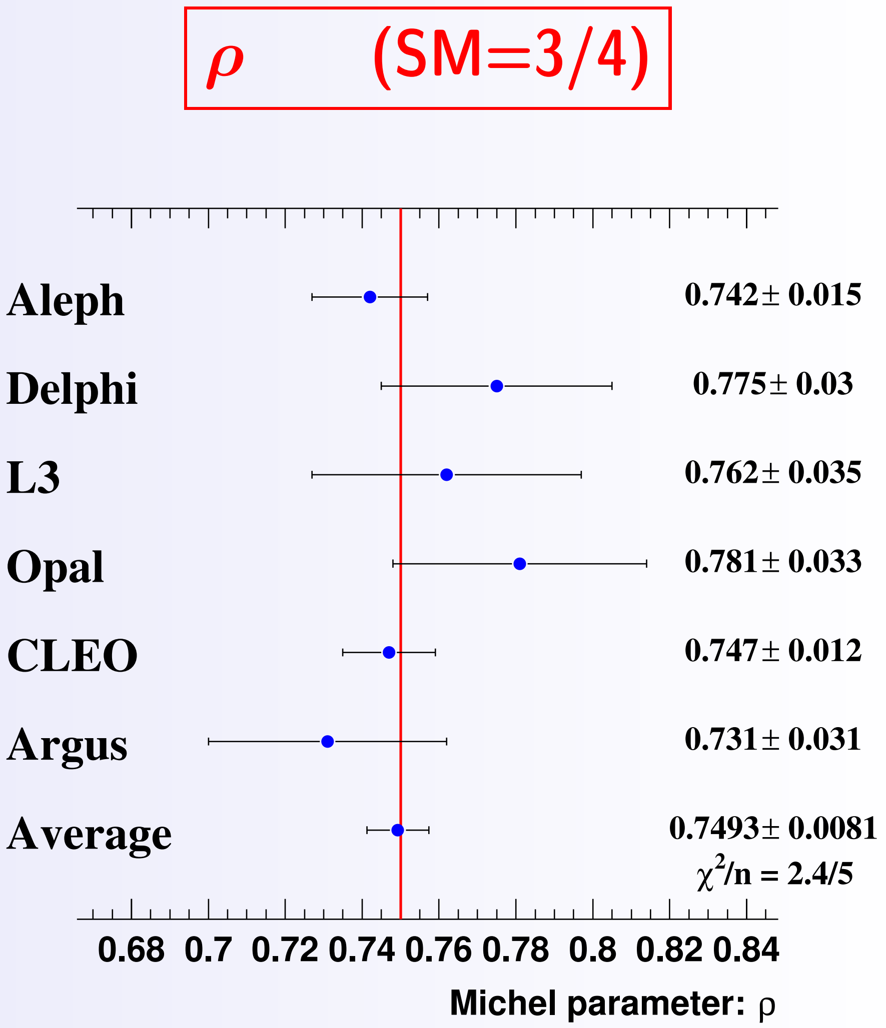
Michel parameterisation

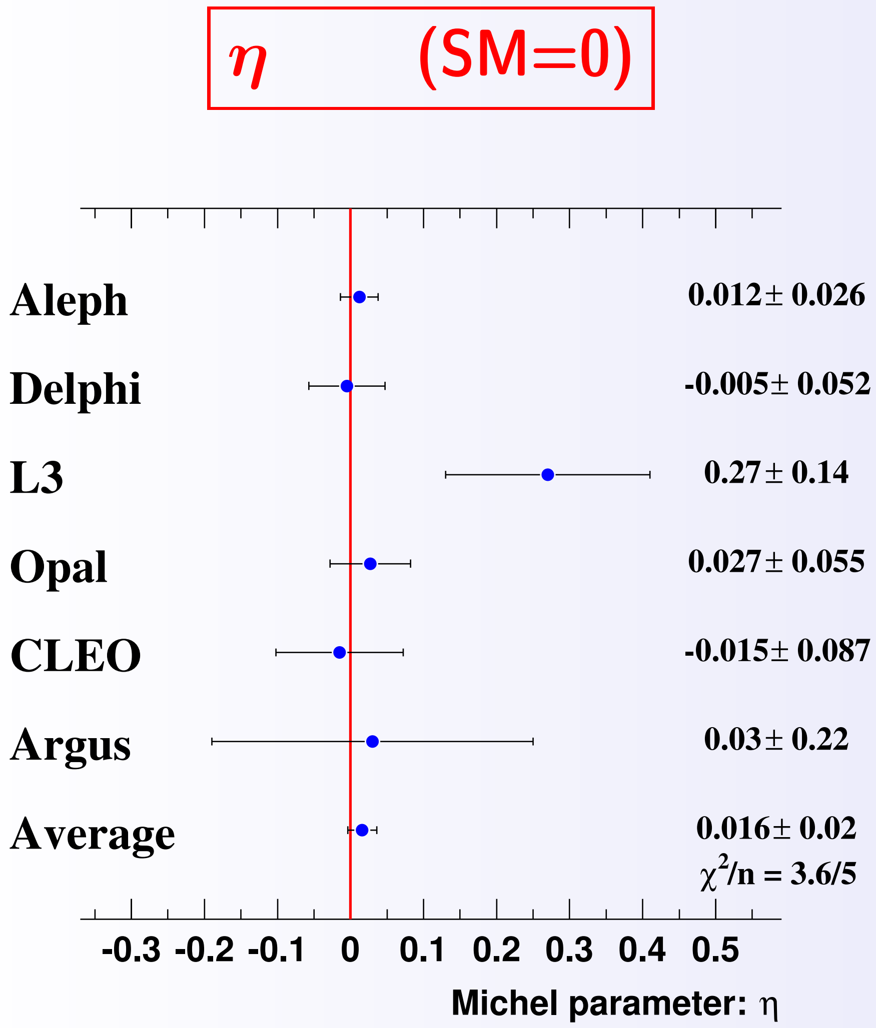
Differential decay spectrum of $x_l = E_l/E_{max}$ has 4 independent parameters ρ, η, ξ and δ if the daughter lepton polarisation is unmeasured:

$$\begin{aligned}\frac{1}{\Gamma}\frac{d\Gamma}{dx_l} &= H_0(x_l) - \mathcal{P}_\tau H_1(x_l)\\ &= [h_0(x_l) + \eta h_\eta(x_l) + \rho h_\rho(x_l)] - \mathcal{P}_\tau[\xi h_\xi(x_l) + \xi\delta h_{\xi\delta}(x_l)]\end{aligned}$$



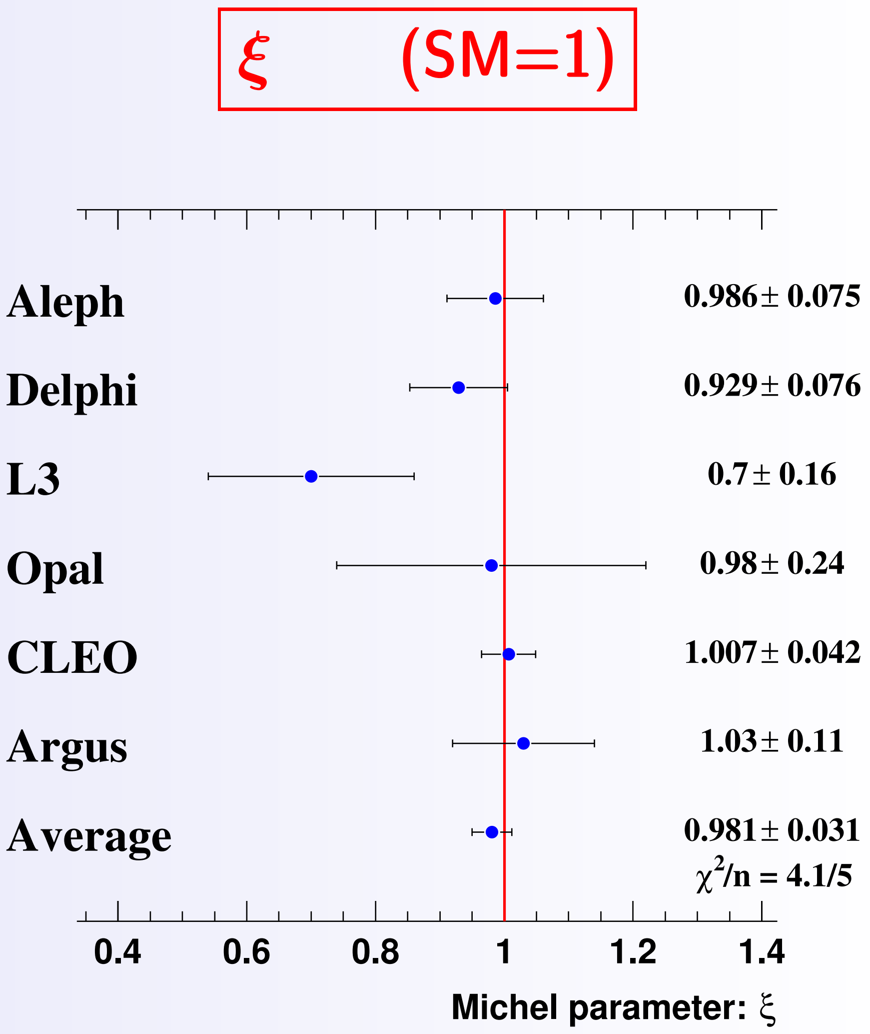
Michel parameters combinations: ρ and η

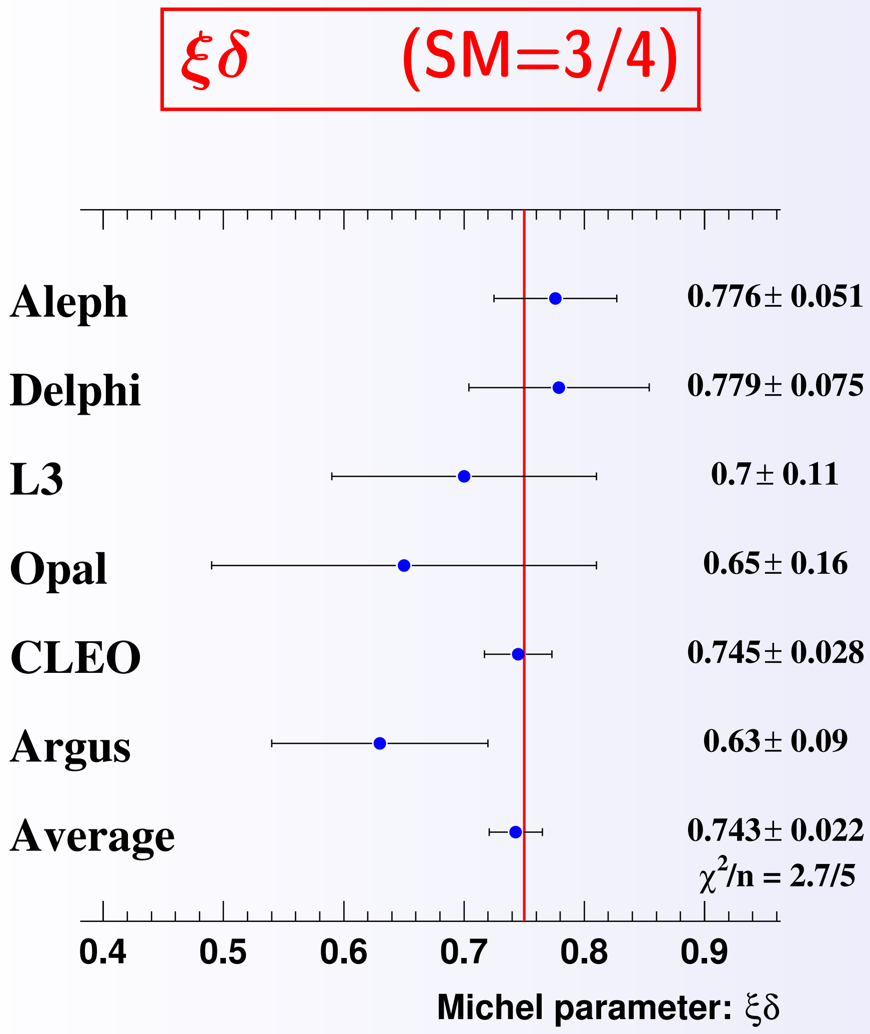




(Note: different experiments use different combinations of τ decay modes)

Michel parameters combinations: ξ and $\xi\delta$





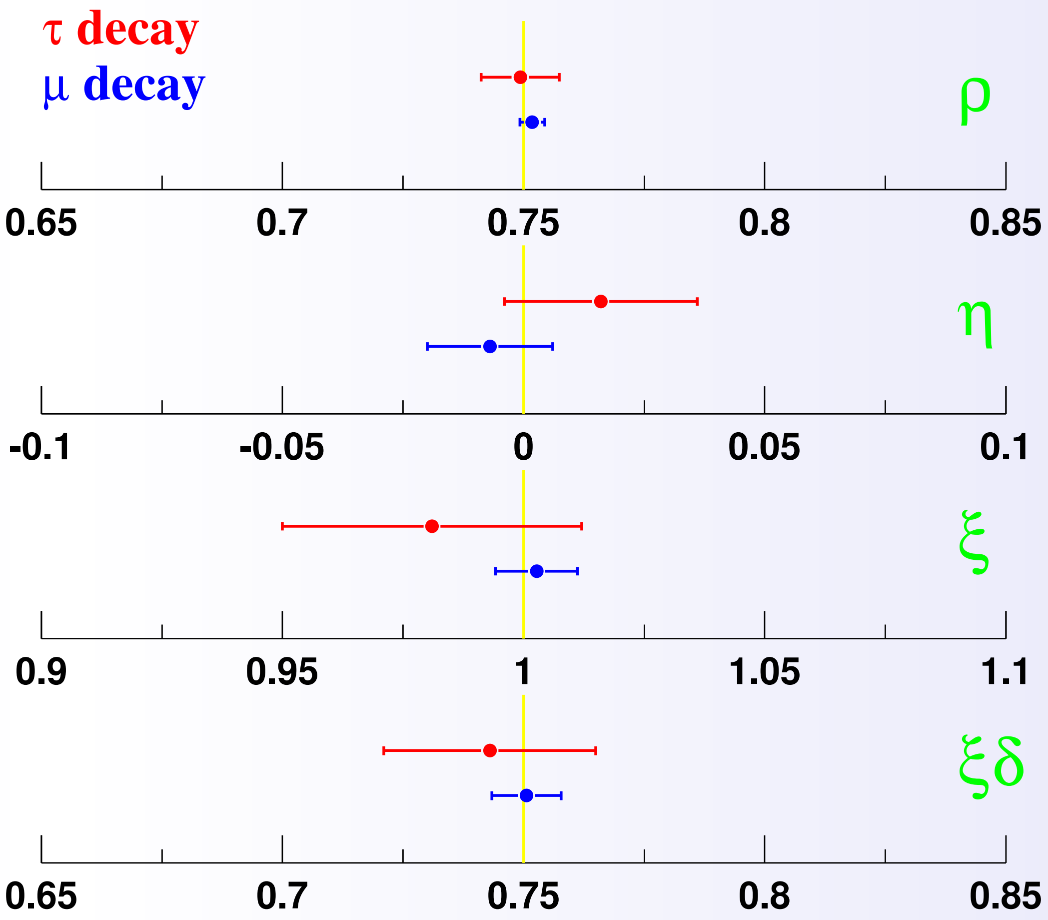
Michel parameters: comparison μ and τ decay

RED: $\tau^- \to l^- \nu_\tau \bar{\nu}_l$

BLUE: $\mu^- \to e^- \nu_\mu \bar{\nu}_e$

Muon decay parameters have a precision of about 3 or 4 times better than those for taus.

Both consistent with V-A.



Charged Current dipole moments

$$\mathcal{L} = \frac{g}{\sqrt{2}} W^{\alpha} \left\{ \overline{\tau}\gamma_{\alpha}\frac{1-\gamma^5}{2}\nu + \frac{i}{2m_{\tau}} \left(\overline{\tau}\sigma_{\alpha\beta} q_{\beta} (\kappa_{\tau}^{W} + i\tilde{\kappa}_{\tau}^{W}\gamma^5) \frac{1-\gamma^5}{2}\nu \right) \right\} + \mathrm{h.c.}$$

κ_{τ}^{W}: anomalous magnetic form factor
$\tilde{\kappa}_{\tau}^{W}$: anomalous electric form factor

Anomalous couplings from universality relations (Rizzo)

τ leptonic decay width and $W \to \tau\nu$ decay width give: (J. Swain, Tau2000)

$$\boxed{\kappa_{\tau}^{W} = 0.001 \pm 0.008}$$

$$\boxed{\tilde{\kappa}_{\tau}^{W} = 0.00 \pm 0.16}$$

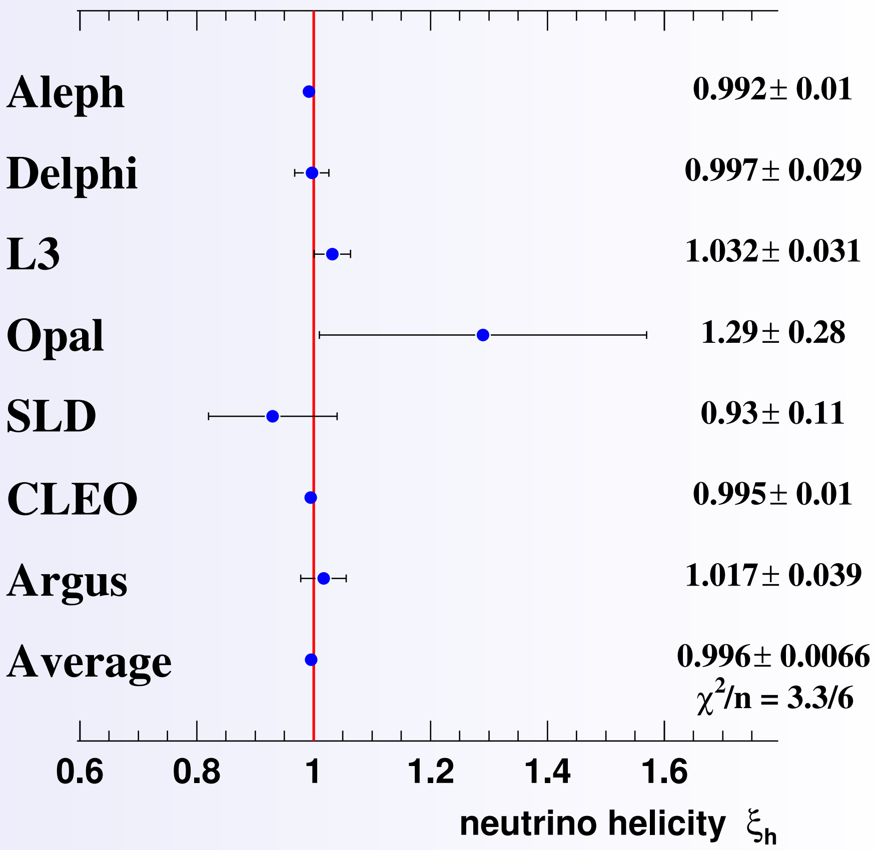
Delphi τ decay spectra: (Chizhov)

$$\boxed{\kappa_{\tau}^{W} = 0.019 \pm 0.036 \pm 0.018}$$

Compatible with zero

Tau neutrino helicity parameter ξ_h ($= -h_{\nu_\tau}$)

ξ_h (SM=1)



$$h_{\nu_\tau} = \frac{g_R^2 - g_L^2}{g_R^2 + g_L^2} = \frac{2\mathrm{Re}(v_\tau a_\tau^*)}{|v_\tau|^2 + |a_\tau|^2} = -\xi_h$$

W.A. gives upper limit on right-handed tau-neutrino coupling:

$$\boxed{\frac{g_R^2}{g_L^2} < 0.007 \quad \text{at} \quad 90\% \text{ CL}}$$

(Note (again): different experiments use different combinations of τ decay modes)

Strong Coupling Constant

With R_τ, plus study of hadronic spectral functions in τ decay, can measure α_s

Aleph: $\alpha_s(m_\tau) = 0.334 \pm 0.007_{exp} \pm 0.021_{th}$

$\alpha_s(m_Z) = 0.1202 \pm 0.0008_{exp} \pm 0.0024_{th} \pm 0.0010_{evol}$

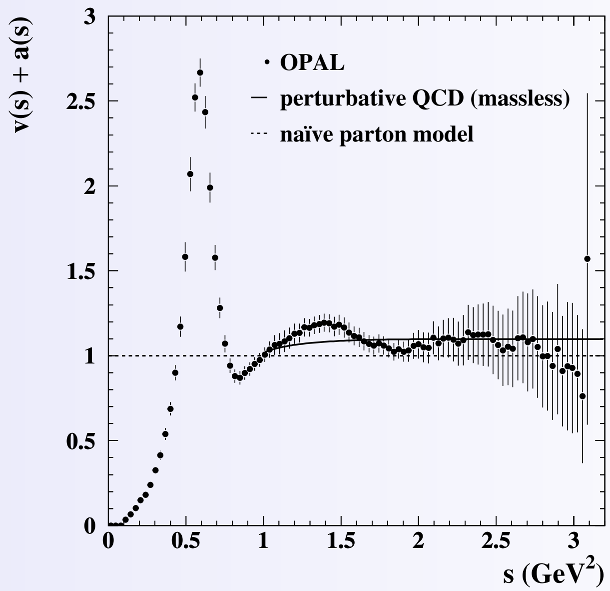
Opal: $\alpha_s(m_\tau) = 0.348 \pm 0.009_{exp} \pm 0.019_{th}$

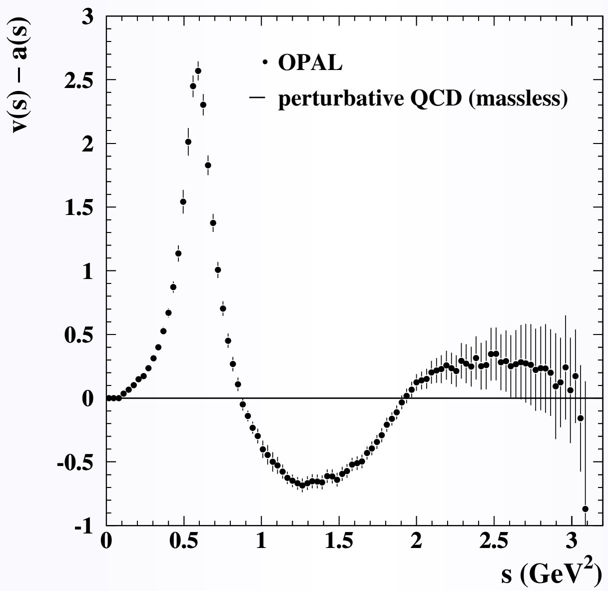
$\alpha_s(m_Z) = 0.1219 \pm 0.0010_{exp} \pm 0.0017_{th}$

For Comparison:

LEP+SLD EWWG full fit: $\alpha_s(m_Z) = 0.118 \pm 0.003$

Delphi HEP2001 shape variables: $\alpha_s(m_Z) = 0.1176 \pm 0.0022$ (just one example.....)





Similar precision.
Good agreement after evolution from $\mathcal{O}$(1 GeV) to $\mathcal{O}$(100 GeV)

Published articles

Not all numbers in this talk are yet published;

Published τ papers and letters to date:

	Experiment			
	A	D	L	O
tau polarisation	6	4	4	4
tau lifetime	5	3	2	4
BRs (non-K)	4	4	4	10
BRs (kaons)	5	1	1	4
Lorentz structure	2	1	2	1
anomalous couplings	2	0	2	6
QCD+hadr. structure	3	1	0	3
rare decays	0	1	0	0
tau neutrino mass	2	0	0	3

Quite a few results still to be published:

e.g., full LEP-1 stats. on hadronic BRs: A,D,L

Conclusions

Neutral current couplings

- axial-vector coupling: lepton universality good to 0.2%
- vector coupling: universality tested to 4% (relative unc.) level
- no evidence for non-SM couplings

Charged current couplings

- no evidence for deviation from V-A
- μ-e universality good to 0.2%
- τ-e, τ-μ universality tested to $\approx$0.2%
- right-handed τ neutrino couplings tightly constrained