A Minimal See-Saw Model for Hierarchical Neutrino Masses with Large Mixing

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The atmospheric and solar neutrino oscillation data suggest hierarchical neutrino masses with at least one large mixing. The simplest see-saw models for reconciling the two features are $U(1)$ extensions of the SM with flavour dependent gauge charges. I discuss a minimal model of this type containing two heavy right-handed neutrinos, which have normal Dirac couplings to ν_μ and ν_τ but suppressed ones to ν_e. It can naturally account for the large (small) mixing solutions to the atmospheric (solar) neutrino oscillation data.

The recent Superkamiokande data has provided convincing evidence for atmospheric neutrino oscillation and confirmed earlier results of solar neutrino oscillation [1]. The atmospheric neutrino data seems to imply a large mixing between ν_μ and ν_τ, $\sin^2 2\theta_{\mu\tau} > 0.86$, along with $\Delta M^2 = (1.5 - 6) \times 10^{-3}$ eV^2 at 90% CL. They correspond to

$$\theta_{\mu\tau} = 45 \pm 11^\circ \tag{1}$$

and

$$\Delta M \simeq 0.06\ eV, \tag{2}$$

the latter representing the central value of ΔM for hierarchical masses and an upper limit on this quantity for degenerate ones. By far the simplest explanation of the solar neutrino oscillation data is provided by the small mixing angle MSW solution although one can get equally good descriptions in terms of the large mixing angle MSW or vacuum oscillation solutions. The SMA solution corresponds to a small mixing of ν_e with one of the above states, $\sin^2 2\theta_e = 10^{-3} - 10^{-2}$, along with a small $\Delta m^2 = (0.5 - 1) \times 10^{-5}$ eV^2. They correspond to

$$\sin\theta_e = (1 - 5) \times 10^{-2} \tag{3}$$

and

$$\Delta m \simeq 0.003\ eV. \tag{4}$$

By itself the atmospheric neutrino oscillation result of Eqs. (1,2) could be naturally explained in terms of a nearly degenerate pair of ν_μ and ν_τ. Indeed a pseudo-Dirac mass matrix for this pair would lead to degenerate masses and maximal mixing on diagonalisation, i.e.

$$\begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix} \rightarrow \begin{pmatrix} M & 0 \\ 0 & -M \end{pmatrix}, \quad \theta = 45^\circ. \tag{5}$$

But explaining the solar neutrino oscillation result of Eqs. (3,4) would then imply an even finer level of degeneracy between ν_e and one of this pair, which is totally ad-hoc. Therefore it is generally considered more natural to interpret them as hierarchical states, i.e.,

$$\begin{aligned} m_1 \simeq \Delta M &\simeq 0.06\ eV, \\ m_2 \simeq \Delta m &\simeq 0.003\ eV, \\ m_3 \ll m_2 &\simeq 0, \end{aligned} \tag{6}$$

where the first two states are large admixtures of ν_μ and ν_τ and the third one is dominantly ν_e. Indeed much of the recent literature on neutrino physics is focussed on theoretical models, mainly in the see-saw frame work, which can naturally reconcile such hierarchical masses with large mixing [2]. Note that the mass of the 3rd state can be exactly zero as far as the atmospheric and solar neutrino oscillation data are concerned. Thus a minimal see-saw model for explaining these oscillations requires two right-handed neutrinos with normal Dirac couplings to ν_μ and ν_τ, but suppressed ones to ν_e.

It may be noted here that the standard see-saw model [3] represents a $U(1)$ extension of the standard model (SM) gauge group into

$$SU(3)_C \times SU(2) \times U(1)_Y \times U(1)_{Y'} \tag{7}$$

with the gauge charge [4]

$$Y' = B - L = B - (L_e + L_\mu + L_\tau). \tag{8}$$

Then the requirement of anomaly cancellation implies the existence of three right-handed singlet neutrinos (N_i) with $Y' = -1$ to match the three left-handed neutrinos ($\nu_{e,\mu,\tau}$) carrying this gauge charge. Cancellation of the axial parts of the Y' current between the left and right handed fermions ensures purely vector coupling for Y', which in turn ensures that the model is anomaly free [5]. The flavour independence of Y' implies however that the singlet neutrinos have normal Dirac couplings to all the left-handed doublets $\nu_{e,\mu,\tau}$ along with the SM Higgs doublet ϕ instead of preferential couplings to $\nu_{\mu,\tau}$ as suggested by data. In order to accomplish the latter one has to invoke a horizontal symmetry with flavour dependent charges [2]. In other words one first takes a flavour blind step beyond the SM and then applies correctives via additional symmetry groups with flavour dependent charges. Let us consider instead a one-step process, where the desired flavour depence is incorporated into the gauge charge Y' of the $U(1)$ extension of SM (Eq. 7). While such flavour dependent $U(1)$ extensions of the SM gauge group are hard to embed in the familiar GUTs they can arise naturally from string theories [6].

We have studied two such $U(1)$ extensions of the SM [7,8], corresponding to the gauge charges

$$Y' = B - 3L_e \tag{9}$$

and

$$Y' = B - \frac{3}{2}(L_\mu + L_\tau), \tag{10}$$

in the context of the atmospheric and solar neutrino oscillations. I shall concentrate on the simpler of the two models [8], corresponding to the gauge charge (10). Indeed it seems to represent a minimal see-saw model for explaining these neutrino oscillation data. In this case the anomaly cancellation requirement implies the existence of two right-handed singlet neutrinos ($N_{1,2}$) with $Y' = -\frac{3}{2}$ to match the two left-handed neutrinos ($\nu_{\mu,\tau}$) carrying this gauge charge.

The minimal Higgs sector of this model consists of

$$\begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}_{Y'=0} \quad \& \quad \chi^0_{Y'=-3}, \tag{11}$$

i.e. the SM Higgs doublet along with a singlet carrying non-zero Y' charge. The Y' symmetry is spontaneously broken via the vacuum expectation value of χ, $<\chi>$, at a high mass scale. The coupling of this χ to $\bar{N}_1^C N_1$ and $\bar{N}_2^C N_2$ gives them large Majorana masses $\sim< \chi >$. Moreover the coupling of ϕ to $\bar{\nu}_\mu N_{1,2}$ and $\bar{\nu}_\tau N_{1,2}$ gives them Dirac masses $\sim< \phi >$, while there is no such coupling to ν_e. Thus the see-saw mechanism would generate two non-zero mass states, which are large admixtures of ν_μ and ν_τ, while ν_e remains massless.

One can generate a small mixing of ν_e with the non-zero mass states, as required by the SMA solution (3) to the solar neutrino oscillation, by expanding the Higgs sector. For this purpose we add another doublet and a singlet with

$$\begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix}_{Y'=-3/2} \quad \& \quad \zeta^0_{Y'=-3/2}. \tag{12}$$

The coupling of the doublet η to $\bar{\nu}_e N_{1,2}$ generates Dirac mass terms $\sim< \eta >$. The singlet ζ^0 does not couple to fermions; but it is required to avoid an unwanted Goldstone boson. The latter comes about because there are 3 global $U(1)$ symmetries, corresponding to rotating the phases of ϕ, η and χ^0 independently in the Higgs potential, while only 2 local $U(1)$ symmetries are spontaneously broken. The addition of the singlet ζ^0 introduces two more terms in the Higgs potential, $\eta^+\phi\zeta^0$ and $\chi^0\zeta^0\zeta^0$, so that the phases can no longer be rotated independently. While the ζ^0 is expected to acquire a large vev at the $U(1)_{Y'}$ symmetry breaking scale, the doublet η must have a positive mass squared term in order to avoid $SU(2)$ breaking at this scale. Nonetheless it can acquire a small but non-zero vev at the $SU(2)$ symmetry breaking scale, which can be es-

timated from the relevant part of the potential

$$m_\eta^2 \eta^\dagger \eta + \lambda (\eta^\dagger \eta)(\chi^\dagger \chi) + \lambda' (\eta^\dagger \eta)(\zeta^\dagger \zeta) - \mu \eta^\dagger \phi \zeta . \quad (13)$$

Although we start with a positive m_η^2 term, after minimization of the potential with respect to η we see that this field has acquired a small vev,

$$< \eta >= \mu < \phi >< \zeta > /2M_\eta^2, \quad (14)$$

where $M_\eta^2 = m_\eta^2 + \lambda < \chi >^2 + \lambda' < \zeta >^2$ represents the physical mass of η. The size of the soft term is bounded by the Y' symmetry breaking scale, i.e. $\mu \leq < \zeta >$. Thus with a choice of $M_\eta \sim 5 < \zeta >$, we get

$$< \eta > / < \phi > \sim 1/50, \quad (15)$$

which will account for the small mixing angle of ν_e (3).

Let us write down the 5×5 neutrino mass-matrix in this model. We shall be working in the basis where the charged lepton mass matrix, arising from their couplings to the SM Higgs boson ϕ, is diagonal. This defines the flavour basis of the doublet neutrinos. Since the two singlet neutrinos do not couple to the charged leptons, their Majorana mass matrix can be independently diagonalised in this basis. While the overall size of their masses will be at the Y' symmetry breaking scale, it is reasonable to assume a modest hierarchy between them,

$$M_1/M_2 \sim 1/20, \quad (16)$$

in analogy with those observed in the quark and the charged lepton sectors. This will account for the desired mass ratio for the doublet neutrinos (6). Thus we have the following 5×5 mass matrix $\mathcal{M}$ in the basis $(\nu_e, \nu_\mu, \nu_\tau, N_1^C, N_2^C)$:

$$\begin{pmatrix} 0 & 0 & 0 & f_e^1 < \eta > & f_e^2 < \eta > \\ 0 & 0 & 0 & f_\mu^1 < \phi > & f_\mu^2 < \phi > \\ 0 & 0 & 0 & f_\tau^1 < \phi > & f_\tau^2 < \phi > \\ f_e^1 < \eta > & f_\mu^1 < \phi > & f_\tau^1 < \phi > & M_1 & 0 \\ f_e^2 < \eta > & f_\mu^2 < \phi > & f_\tau^2 < \phi > & 0 & M_2 \end{pmatrix}, \quad (17)$$

where the $f_{e,\mu\tau}^{1,2}$ are the Higgs Yukawa couplings. We shall assume these couplings to be of similar order of magnitude, i.e. the elements of a mass-matrix arising from the same Higgs vev are expected to be of similar size. There is of course no conflict between such democratic mass-matrix elements and the hierarchical mass eigen-values assumed above (16). In fact they are closely related - the former implies large cancellation in the determinant as required by the latter.

The resulting 3×3 mass-matrix for the doublet neutrinos is given by the see-saw formula in this basis,

$$m_{ij} = \frac{D_{1i} D_{1j}}{M_1} + \frac{D_{2i} D_{2j}}{M_2}, \quad (18)$$

where D is the 2×3 Dirac mass matrix at the bottom left of (17). One can then calculate the corresponding mass eigen-values $m_{1,2,3}$ and mixing-angles by diagonalising this matrix [8]. Alternatively we can read off the approximate magnitudes of these quantities directly from the mass matrix (17), i.e.

$$\begin{aligned} \tan\theta_{\mu\tau} &\simeq \mathcal{M}_{42}/\mathcal{M}_{43} \simeq f_\mu^1/f_\tau^1 \sim 1, \\ \sin\theta_e &\simeq \mathcal{M}_{51}/\mathcal{M}_{52} \simeq < \eta > / < \phi > \sim \frac{1}{50}, \\ m_2/m_1 &\simeq M_1/M_2 \sim 1/20. \end{aligned} \quad (19)$$

They are clearly in good agreement with the corresponding experimental quantities of Eqs. (1), (3) and (6). Note that in this model the ν_e mixing with the higher mass (m_1) eigen-state is also expected to be of similar size as above, i.e.,

$$\mathcal{M}_{41}/\mathcal{M}_{42} \simeq < \eta > / < \phi > \sim 1/50. \quad (20)$$

This prediction is well within the present experimental limit on this quantity (≤ 0.2) from CHOOZ data [9]; but can be tested by future long base line experiments.

Finally, the scale of the Y' symmetry breaking can be estimated from the larger Majorana mass M_2, i.e.

$$\begin{aligned} M_2 \sim f^2 < \phi >^2 / m_2 &\sim f^2 \, 10^{16} GeV \\ &\sim 10^{12-16} GeV. \end{aligned} \quad (21)$$

The lower limit corresponds to $f \sim 10^{-2}$ as in the case of τ Yukawa coupling, while the upper limit corresponds to $f \sim 1$ as in the case of top. Thus the observed scale of neutrino masses (6) can be explained if one assumes the Y' symmetry breaking scale to be in the range of $10^{12} - 10^{16}$ GeV.

One can get a more exact derivation of the masses and mixing angles via the 3×3 mass-matrix of the doublet neutrinos (18), i.e.

$$\begin{pmatrix} c_1^2 + c_2^2 & c_1 a_1 + c_2 a_2 & c_1 b_1 + c_2 b_2 \\ c_1 a_1 + c_2 a_2 & a_1^2 + a_2^2 & a_1 b_1 + a_2 b_2 \\ c_1 b_1 + c_2 b_2 & a_1 b_1 + a_2 b_2 & b_1^2 + b_2^2 \end{pmatrix}, \quad (22)$$

where

$$\begin{aligned} a_{1,2} &= \frac{f_\mu^{1,2} < \phi >}{\sqrt{M_{1,2}}}, \\ b_{1,2} &= \frac{f_\tau^{1,2} < \phi >}{\sqrt{M_{1,2}}}, \\ c_{1,2} &= \frac{f_e^{1,2} < \eta >}{\sqrt{M_{1,2}}}. \end{aligned} \quad (23)$$

Note that the assumed hierarchies of (15) and (16) imply

$$a_1, b_1 \gg a_2, b_2, c_1 \gg c_2. \quad (24)$$

The determinant of (22) vanishes identically, ensuring that one of the mass eigenvalues is zero. The other two are

$$m_1 \simeq a_1^2 + b_1^2, \quad m_2 \simeq \frac{(a_1 b_2 - a_2 b)^2}{a_1^2 + b_1^2}. \quad (25)$$

The corresponding mixing matrix U between the flavour and the mass eigenstates is

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & \frac{-c_2\sqrt{a_1^2+b_1^2}}{a_1 b_2 - b_1 a_2} & \frac{c_1}{\sqrt{a_1^2+b_1^2}} \\ \frac{b_1 c_2 - c_1 b_2}{a_1 b_2 - b_1 a_2} & \frac{b_1}{\sqrt{a_1^2+b_1^2}} & \frac{a_1}{\sqrt{a_1^2+b_1^2}} \\ \frac{c_1 a_2 - a_1 c_2}{a_1 b_2 - b_1 a_2} & \frac{-a_1}{\sqrt{a_1^2+b_1^2}} & \frac{b_1}{\sqrt{a_1^2+b_1^2}} \end{pmatrix} \begin{pmatrix} \nu_3 \\ \nu_2 \\ \nu_1 \end{pmatrix}. \quad (26)$$

One can easily check that the Eqs. (23-26) lead to the masses and mixing angles of Eqs. (19-21).

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